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Research Article

Non-linear Companding Techniques for PAPR Reduction in OFDM

Sonali K.Deshmukh[Ȧ*] and S. D. Chavan[Ȧ]

[Ȧ]Department of Electronics and Telecommunication, Pad. Dr. D. Y. Patil Institute of Engineering and Technology, Pune India

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Abstract

Orthogonal frequency division multiplexing (OFDM) is form of multi-carrier modulation technique. Due to its advantages of high spectral efficiency, robustness to multipath fading effects, immunity to impulse interference are among the property of OFDM. However, one the major drawback of OFDM is very high peak-to-average power ratio (PAPR).Number of techniques has been proposed in literature for reducing the PAPR in OFDM systems. Among the various PAPR reduction techniques, in this paper the non-linear companding scheme is proposed to reduce the PAPR and improve Bit Error Rate(BER) for OFDM systems. The Nonlinear Companding Transform (NCT) distortion technique is proposed for reducing the high Peak –To –Average Power Ratio This proposed scheme reduces the PAPR by compressing the peak signals and expanding the small while maintaining average power constant selecting transform parameters . Finally, simulation results show that the proposed scheme in terms of spectrum side-libes, PAPR reduction and BER performance

Keywords: *Orthogonal frequency Division Multiplexing (OFDM), Peak-to-Average Power Ratio(PAPR), Nonlinear Companding Transform (NCT).*

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1. Introduction

The orthogonal frequency division multiplexing (OFDM),a multi carrier communication scheme, has been a promising candidate for achieving high rate data transmission in mobile environment. OFDM, employing multicarrier modulation (MCM) technique, enables it to transmit signals over multiple subcarriers simultaneously. Unlike single carrier case, where the pulses are short because of which the inter symbol interference (ISI) is dominant, by dividing the total bandwidth into several narrow sub channels, which are transmitted in parallel, the effects of the multipath delay spread can be minimized. The carriers are made orthogonal to each other by precisely choosing the frequency spacing between them.

However, one of the major drawbacks of the OFDM signal is its large envelope fluctuation, likely resulting in large peak-to-average power ratio (PAPR). Because of the high PAPR, the transmitter power amplifier may be driven into saturation. This potentially contaminates the adjacent channels resulting in the co-channel interference. Two different approaches have been suggested in the literature in order to mitigate this problems, namely, either reducing the peak-to-average power ratio (PAPR), or improving the amplification stage of the transmitter by increasing the dynamic range of linear operation of the amplifier. In this paper, we focus on reducing the PAPR of OFDM signals.

Some of the methods proposed in the literature to reduce the PAPR of OFDM signals include block coding techniques, selective mapping, partial transmitting. Also, there are different nonlinear methods which modify the envelope of the original OFDM signal for PAPR reduction. The idea behind these nonlinear methods is that by clipping the peaks of OFDM signal or by modifying the constellation points (ACE) or by applying Hadamard Transformation on the OFDM signal or by transforming the original signal as in the case of μ-law companding, PAPR reduction is achieved. But, by μ-law companding the OFDM signal, the PAPR is reduced at the expense of increase in average power. In order to overcome the problem of increase of average power and to have efficient PAPR reduction, a nonlinear companding technique namely exponential companding has been developed to overcome the problem of increasing average power and to have efficient PAPR reduction. This scheme transforms the Rayleigh distributed OFDM signal into a uniformly distributed signal and achieves better system performance than the µ-law companding scheme. However, the distribution of large amplitude signals is increased by the uniform companding. Hence, we can predict that Bit Error Rate (BER) performance is degraded when the OFDM transmitters employ a power amplifier with heavy nonlinearity.

In this paper, a novel non-linear companding scheme is proposed. This scheme mainly focuses on compressing the large signals, while maintaining the average power constant by properly choosing the transform parameters. Furthermore, theoretical analysis shows that the proposed scheme without de-companding operation at the receiver can also offer a good BER performance. Finally,

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*Corresponding author: **Sonali K.Deshmukh** simulation results show that the proposed scheme offers a better PAPR reduction and BER performance than the EC scheme.

2. Literature Survey

Several algorithms (Muller, S. H., and Huber, J. B.*et al*, 2008;Jayalath A. D. S. and T. ellamburaC, *et al*, 2009; Cimini Jr, L. J. and Sollenberge N.R *et al*, 2000; Chong C,V. and Tarokh,V.,*et al* ,2001 ;Tan, M., *et al.*, 2001) have been proposed to handle this PAPR problem. However, none of these algorithms have produced significant reduction of PAPR in OFDM systems. Partial Transmit Sequence (PTS) was proposed in (Muller, S. H., and Huber, J. B.*et al*, 2008). In PTS, the information bearing subcarrier block is subdivided into disjointed carriers unblocks and introduced rotation factors for each sub-block and modified the subcarrier amplitude vector. Thereby, PAPR was reduced with different rotation factors for different sub blocks. This needs number of iterations find the optimum combination of factors for sub-blocks. Adaptive PTS(JayalathA. D. S. and T. ellambura C *et al*, 2009) was proposed to reduce the number of iterations by setting up a desired threshold and trial for different weighing factors until the PAPR dropped below the threshold with this approach we can reduce the number of iterations and the complexity of the system by only 0.1% loss in reduction of PAPR. In (Cimini Jr, L. J. and Sollenberge N..R. *et al*,2000), 256 subcarriers are considered with QPSK. Results showed that the PAPR can be reduced by 4.1 dB and 4 .0 dB without adaptive PTS and with adaptive PTS respectively. However these two approaches need to send side information to the receiver which implies a reduction in the bandwidth efficiency. PTS with embedded side information (Chong C,V. and Tarokh,V.,*et al* ,2001;Tan, M., *et al.*, 2001) is another approach that can be combined with both conventional and adaptive PTS. This approach embeds a combined knowledge within the transmitted data. So no extra bits are sent But these introduce word errors during detection of the sequence information. A simple Encodable / Decodable OFDM QPSK proposed in (Tan, M., *et al.*, 2001) used Reed-Muller code with QPSK. This could reduce the PAPR to less than 6 dB but it could not be used with higher orders signal constellation OFDM PAPR reduction by a rotation of redundancy bit position in sub-block code word scheme was proposed in(Armstrong, J.,*et al*, 2001) . In this method the redundant it positions of sub-block code words are rotated and the lowest PAPR codeword is chosen by a feedback scheme. However the side information for bit position is required. Oversized IFFT (Huang, X. *et.al*, 2004) is proposed as another scheme to solve this problem. In oversized IFFT, clipping and filtering are done by forward and inverse FFT. This can avoid out of band power but with some in band distortion, overall shrinking of constellation and with the introduction of some noise like components Companding transform (X. Wang, T. T. Tjhung, *el at* 2001; Wang, X., Tjhung, T.T., Ng, C.S.*el at* 2002) compresses large signal while enhancing smalls signal that can achieve a desired PAPR but with a significant increase in the bit error rate (BER).

3. Peak To Average Power ratio

High Peak-to-Average Power Ratio has been recognized as one of the major practical problem involving OFDM modulation. High PAPR results from the nature of the modulation itself where multiple subcarriers /sinusoids are added together to form the signal to be transmitted. When N sinusoids add, the peak magnitude would have a value of N, where the average might be quite low due to the destructive interference between the sinusoids. High PAPR signals are usually undesirable for it usually strains the analog circuitry. High PAPR signals would require a large range of dynamic linearity from the analog circuits which usually results in expensive devices and high power consumption with lower efficiency (for e.g. power amplifier has to operate with larger back-off to maintain linearity).

In OFDM system, some input sequences would result in higher PAPR than others. For example, an input sequence that requires all such carriers to transmit their maximum amplitudes would certainly result in a high output PAPR. Thus by limiting the possible input sequences to a smallest sub set, it should be possible to obtain output signals with a guaranteed low output PAPR. The PAPR of the transmit signal x(t) is the ratio of the maximum instantaneous power and the average power.

$$\text{PAPR}\,\{x\,(t),\,\tau\} = \frac{max_{t=\tau}[x(t)]^2}{E\{[x(t)]^2\}} \qquad (1)$$

where E{.} denotes expectation operator.

4. System Description

The proposed new algorithm based on nonlinear companding scheme with objectives of overcoming the problem of increasing average power, high PAPR, decreasing BER performance. Figure 1 showing the basic block diagram for companding PAPR reduction scheme based OFDM system.

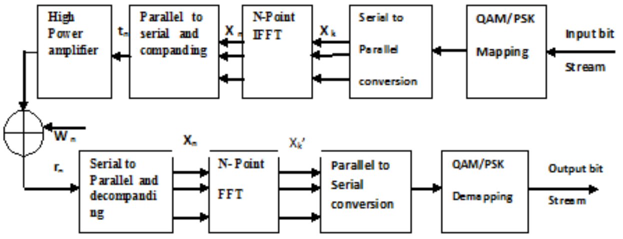

Fig. 1 Proposed Companding Scheme Based OFDM System Model

The whole system bandwidth is divided into many orthogonal sub-channels (with narrow bandwidth), and data symbols typically modulated by Phase Shift Keying (PSK) or Quadrature Amplitude Modulation (QAM) are transmitted independently on the subcarriers.The proposed a non-linear companding technique that can effectively improve the BER performance and reduce the PAPR of transmitted OFDM signals. Moreover, the new scheme has the advantage of maintaining a constant average power through the companding operation. Therefore, the efficiency of the amplifier can be improved.

The figure1shows the new method of companding PAPR reduction scheme based OFDM system. The whole system bandwidth is divided into many orthogonal sub-channels (with narrow bandwidth), and data symbols typically modulated by Phase Shift Keying (PSK) or Quadrature Amplitude Modulation (QAM) are transmitted independently on the subcarriers. We propose a novel non-linear companding technique that can effectively improve the BER performance and reduce the PAPR of transmitted OFDM signals. Moreover, the new scheme has the advantage of maintaining a constant average power through the companding operation. Therefore, the efficiency of the amplifier.

5. System Algorithm

In this section a proposed new companding technique that can effectively reduce the PAPR of transmitted (companded) OFDM signals by transforming the statistics of the amplitudes of these signals into uniform distribution .The new scheme also has the advantage of maintaining a constant average power level in the nonlinear companding operation. The strict linearity requirements on HPA can then be partially relieved.

Assume the input information symbols are statistically independent and identically distributed. Based on the central limit theory, x_n can be approximated as a complex Gaussian process when the number of sub-carrier N is large (e.g. N ≥64). Assume that x_n has zero mean and variance σ^2,so its magnitude $|x_n|$ has Rayleigh distribution with the probability distribution Function (PDF) given by

$$f_{|x_n|}(x) = \frac{2x}{\sigma^2} exp\left(-\frac{x^2}{\sigma^2}\right), \qquad x \geq 0 \tag{2}$$

The cumulative Distribution Function (CDF) of $|x_n|$ can be expressed as

$$\boldsymbol{F}_{|x_n|}(\boldsymbol{x}) = Prob\{|x_n| < x\} \tag{3}$$

$$= \int_0^x \frac{2y}{\sigma^2} \exp\left(-\frac{y^2}{\sigma^2}\right) dy$$

$$= 1 - \exp\left(-\frac{x^2}{\sigma^2}\right), \quad x \geq 0 \tag{4}$$

The proposed companding scheme is given by

$$t_n = h(x_n) \tag{5}$$

Where x_n is the original OFDM signal and t_n is companded signal, the proposed companding function $h(.)$ only changes the amplitude of input signals. Assume the transition point and cutoff point of the PDF of $|t_n|$ are $c\sigma$ and A, respectively. In the interval $[0, c\sigma]$, the PDF of $|tn|$ is similar to that of $|x_n|$; in the interval $[c\sigma, A]$,$|t_n|$ has uniform distribution.

Since$|x_n|$ has a Rayleigh distribution, the coordinates of the transition and cutoff point are $(c\sigma, f_{|x_n|}(c\sigma))$ and $(A, f_{|x_n|}(c\sigma))$, respectively .Thereby, PDF of $|t_n|$ can be expressed as

$$f_{|t_n|}(x) =$$

$$\begin{cases} \frac{2x}{\sigma^2} exp\left(-\frac{x^2}{\sigma^2}\right) & 0 \leq x < c\sigma \\ f_{|x_n|}\ (c\sigma) & c\sigma \leq x \leq A \end{cases} \tag{6}$$

and the CDF of $|\, t_n|$ is

$$F_{|t_n|}(x) =$$

$$\begin{cases} 1 - exp\left(\frac{x^2}{\sigma^2}\right) & 0 \leq x \leq c\sigma \\ 1 - \exp(-c^2) + f_{|x_n|}(c\sigma)(x - c\sigma) & c\sigma < x \leq A \\ 1 & x > A \end{cases} \tag{7}$$

From the definition of the PDF$\int_0^\infty f_{|t_n|}(x)dx = 1$, we have $A = (c + 1/2c)\sigma$.Consider a constant average power level in the non-linear companding operation, the power of OFDM signal x_n can be expressed as

$$\int_0^\infty x^2 f_{|t_n|}(x)dx = \int_0^\infty \frac{x^2.2x}{\sigma^2} exp\left(-\frac{x^2}{\sigma^2}\right) dx \tag{8}$$

The power of the companded signal can be calculated as

$$\int_0^\infty x^2 f_{|t_n\ |}(x)dx = \int_0^{c\sigma} \frac{x^2 2x}{\sigma^2} exp\left(-\frac{x^2}{\sigma^2}\right) dx$$

$$+ \int_{c\sigma}^{(c+1/2c)\sigma} x^2 \frac{2c}{\sigma} \exp(-c^2)dx \tag{9}$$

By combining(7) and (8) obtained value of c =1/$\sqrt{6}$.

Given that $h(x)$ is a strictly monotonic increasing function, we have,

$$F_{|x_n|}(x) = Prob\{x_n \leq x\} \tag{10}$$

$$= Prob\{h(|x_n|) \leq h(x)\} = F_{|t_n|}\big(h(x)\big) \tag{11}$$

Considering the phase of input signal , companding function h(x) is given by

$$h(x) = sgn(x) F_{|t_n|}^{-1}\ (F_{|x_n|}(x)) \tag{12}$$

where sgn(x) denote the sign function. Thus, the companding function can be calculated

$$h(x) = \begin{cases} x & |x| \leq \frac{\sigma}{\sqrt{6}} \\ sgn(x). \sqrt{6}\sigma(\frac{2}{3} - \frac{1}{2}\ \ exp\left(\frac{1}{6} - \frac{|x|^2}{\sigma^2}\right)) & |x| \geq \frac{\sigma}{\sqrt{6}} \end{cases} \tag{13}$$

The de-companding function at the receiver

$$h^{-1}(x) = \begin{cases} x & |x| \leq \frac{\sigma}{\sqrt{6}} \\ sgn(x) \frac{\sigma}{6} \sqrt{6 - 36. \ln\left(\frac{4\sigma - \sqrt{6}|x|}{3\sigma}\right)} & |x| > \frac{\sigma}{\sqrt{6}} \end{cases} \tag{14}$$

A.Mathematical Model of OFDM Signal and PAPR

In OFDM, a block of N symbols, {X_k, k=0, 1, … , N-1}, is formed with each symbol modulating one of a set of subcarriers, {fn ,n=0, 1, ….., N-1} with equal frequency separation 1/T, where T is the original symbol period. An inverse discrete Fourier transform (IDFT) can efficiently generate the multicarrier symbols. The IDFT of vector $X_k=[X_0, X_1,\ldots X_{N-1}]$ results in T/N spaced discrete time signal $x_n=[x_0, x_1,\ldots x_{N-1}]$T. Thus, the transmitted signal is

$$x_n = \frac{1}{\sqrt{N}} \int_{k=0}^{N-1} X_k exp\, j\,\frac{2\pi kn}{N} \qquad 0 \le k \le N-1 \tag{15}$$

Then , the power of OFDM signal x_n can be expressed as

$$|x_n|^2 = \frac{1}{N}\sum_{m=0}^{N-1}\sum_{k=0}^{N-1} X_m\, X_k exp\left(\frac{j.2\pi(m-k)n}{N}\right) \tag{16}$$

And the PAPR of the OFDM signal in terms of power is defined as

$$PAPR = \ 10.log_{10}\frac{Max\{|x_n|^2\}}{E[|x_n|^2]} \ \ (dB) \tag{17}$$

Where $|x_n|$ returns the magnitude of x_n and $E\,[.]$ denotes the expectations operation.

The complementary additive distribution operate (CCDF) is one in all for most of times used performance measures for PAPR reduction techniques, that denotes the chance that the PAPR of a knowledge block exceeds a given threshold z. The CCDF of the PAPR of a knowledge block of N symbols with Nyquist rate sampling comes as

$$P(PAPR > z) = 1 - P(PAPR \le z)$$

$$= 1 - (1 - \ \ e^{-z})^N \tag{18}$$

where, z is the threshold.

6. Simulation Results

In this simulation, the PAPR reduction and BER performance of the proposed scheme with the exponential companding scheme using the AWGN channels are considered.

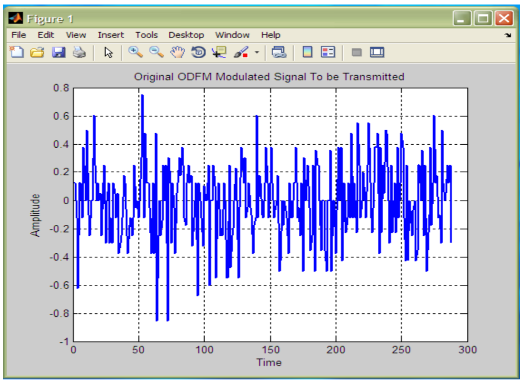

Fig.2 Original OFDM Modulated Signal

The waveform of OFDM signal before companding and the modulated OFDM signal is generated after QAM mapping and IFFT as is plotted in figure.2

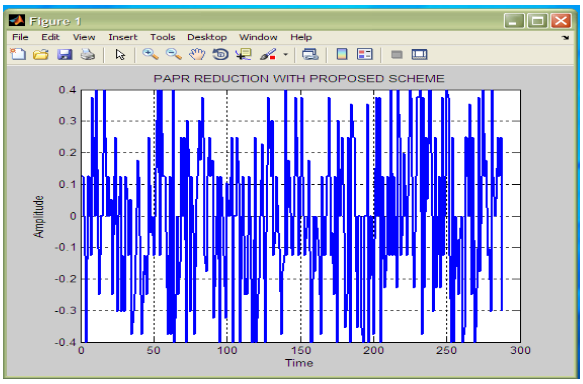

Fig. 3 PAPR Reduction with Proposed Nonlinear Companding Scheme

The figure.3 Proposed Nonlinear Companding Scheme respectively. And Figure 4 depicts the simulated CCDF of the two companding schemes with the original signal (no companding). The new algorithm is roughly 4 dB inferior to the exponential, however suppresses the original OFDM signal by 14 .7dB.

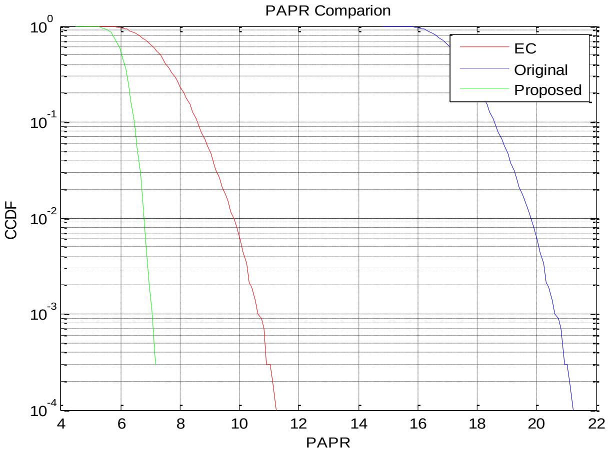

Fig. 4 The CCDF's of original OFDM Signal, EC and Proposed Signal

Table 1 BER Performance and PAPR Comparison with different companding Scheme

Schemes	E_b/N_o (dB)		PAPR (dB)
	BER=10^{-4}	BER=10^{-5}	
Original OFDM	12	14	22.3
Exponential Compandiing (EC)	11	13.5	11
Proposed Non-linear Companding	8	12	7

The simulation results of The BER vs. SNR are plotted in Fig.5. Bit Error rate of No companded OFDM signal is compared with the Exponential companded signal. To make the comparisons, some results of the companding schemes are given in Table1.

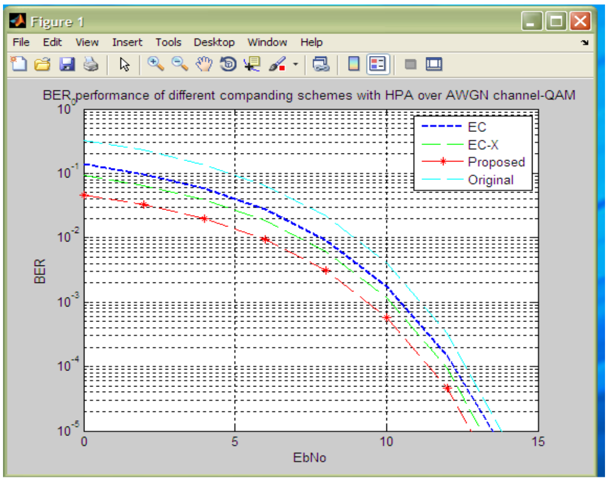

Fig. 5 BER Comparison of original OFDM Signal, EC and Proposed Signal

The required E_b/N_{os} under the proposed , exponential companding and original OFDM signals are 12dB, 13.5dB and 14dB respectively. In addition, the PAPR of the proposed scheme is 7 dB while the EC scheme is 10.7 dB.The OFDM system used in simulation consists of 16-QAM modulated data points. The size of the FFT/IFFT, the number of subcarriers, the factor of oversampling and the randomly generated input data are modulated by Quadrature Amplitude Modulation. This simulation discussed the improve Bit Error Rate in Figure 5 represents BER Vs SNR (E_b/N_o).To reach the BER of 10^{-5} ,for example the required SNR are 12dB, 13.5dB for the proposed transform and exponential companding schemes respectively, implying a 1.5dB improvement with the new algorithm. The amount of improvement increases as SNR becomes higher.

It offers the best BER performance, while it has an extremely high PAPR compared with that of companded signals. The 'x' in the figure5 denotes without de-companding operation at the receiver. In the two figures, the proposed scheme achieves more PAPR reduction and better BER performance than the EC scheme.

Conclusion

In this, the proposed non-linear companding scheme evaluated that can be effectively improve the OFDM system performance with low out-of- band distortion. In the proposed scheme ,lower amplitude components are boosted up with higher gain and higher amplitude components are compressed while maintain the average power constant by properly choosing threshold value 'c' and due to that value of PAPR could be reduces effectively of proposed scheme. But in exponential companding whose performance is found almost unchanged. Finally, both theoretical analysis and simulation show that the proposed algorithm offers improved performance in terms of BER, power spectrum while reducing PAPR effectively than EC scheme.

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