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Research Article

A Game Theory based Model for Cooperative Spectrum Sharing in Cognitive Radio

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Abstract

This paper throws light on the issue of spectrum sharing in cognitive radio networks where the secondary users cooperatively sense the spectrum for identifying and accessing temporarily unoccupied frequency spectrum bands. Here, it is shown that this issue can be modeled as a cooperative game in order to allocate the spectrum resources fairly to each cognitive radio. The users form coalitions to jointly sense and share the spectrum bands. The worth of each user is calculated according to the work done by the user for the corresponding coalition with respect to the information collected about the primary user activity from sensing the spectrum band. Resulting games are balanced, thus ensuring non-empty cores for allocating resources to the users. It is also found that the games are also convex, hence enabling one-point solution concepts like the Shapley value, tau value and nucleolus which lie within the core to provide stability. The simulation results for the game model applied to the case of 5 users and 8 channels are also illustrated.

Keywords: *cognitive radio; spectrum sensing; cooperative game theory; characteristic function; Shapley values*

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1. Introduction

The field of wireless communication is developing day-by-day by leaps and bounds. The current wireless networks are characterized by the policy of static spectrum allocation, where the wireless spectrum is assigned to the licensed holders for long durations. Certain bands face severe spectrum scarcity and inspite of that, a large section of the assigned spectrum is under-utilized across time and space [FCC, 2002]. Thus, spectrum scarcity results from inefficient frequency allocations rather than the actual physical shortage of the spectrum resources. So, dynamic spectrum allocation techniques are required to solve these spectrum inefficiency issues. The answer to this problem is the cognitive radio technology [Bezalel Peleg, 2007], which provides the capability for the secondary (unlicensed) users to opportunistically use the unused primary (licensed) bands in such a manner that they cause no harmful interference to the primary users or interference within limits. The secondary users sense the spectrum [H. Urkowitz, 1967], [J. Lunden, V. Koivunen, 2009] to find out the available portions of the spectrum (also known as the spectrum holes) and then select the best possible channels within this part of the spectrum to allow them to transmit or receive data efficiently. But random attempts by secondary users to access the channel may lead to collisions between multiple users. So, the secondary users cooperatively sense the channel and then share it. The user cooperation results in improved detection performance and increased network coverage as compared to the individual sensing and using [Viswanathan and P.K. Varshney, 1997]. In cooperative spectrum sensing, the secondary users form coalitions or groups to cooperately sense the channel for the presence of primary user and share the information about the state of the spectrum occupancy in terms of SNRs (signal to noise ratio) and log likelihood ratios. Out of all the users, one user in the coalition serves as the fusion center and manages the coalition's own information. However, there is a conflict of interest when it comes to sharing the benefits of cooperation while accessing the channel, which calls for the requirement of a game-theoretic model for solving these issues. The spectrum sensing and sharing can be likened to a coalitional game.

In this paper, we describe a framework for modeling the spectrum sensing and sharing problem in cognitive radios as a cooperative coalition game. The worth of individual players and also of each coalition in the game is calculated with respect to the work done by the players for its coalitions. For example, the work done can be quantified in terms of the number of primary users for which the work has been done and information obtained about each primary user's activity based on channel sensing. Resulting games have an inherent structure that provides desirable properties such as balancedness and convexity to the game. A Balanced game has non-empty cores which make allocation of resources possible while a convex game ensures that the Shapley value lies within the core of the game which makes the game stable. The

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*Corresponding author **Samkit K. Shah** is a M E student and **Dr Kiran R. Parmar** is working as Professor resource allocations [Han and K. J. Ray Liu, 2008] made from the games modeled in this manner is fair and stable which provides the users a strong incentive to cooperate in future.

This paper is organized as follows. A brief introduction to the cooperative game theory is provided in section II. In section III, the system model is overviewed. Section IV explains the game modeling. A simple example is demonstrated in section V and finally, section VI includes the conclusion of the paper.

2. Cooperative Game Theory

A cooperative game [Martin J. Osborne, 1994], [Bezalel Peleg and Peter Sudh¨olter, 2007] is differentiated from a noncooperative game mainly by its focus on what groups of players (coalitions) in a game can achieve rather than on what individual players can do, without taking into account how groups of players function. In general, a cooperative game (N, v) consists of

- Finite set N of players.
- Characteristic function v, that maps every non-empty subset S of N, to a real number, which is denoted *v(*S), that is called the worth of the coalition S. The value *v(*S) can be thought of as the value created when the members of coalition S come together and work cooperatively which is the total payoff of the coalition that is later available for division among the all the members of the coalition S. If the set containing all the joint actions that the coalition S can take consists of all possible divisions of *v(*S) among the members of coalition S, then the game is said to have transferable utility(TU). Thus, *v(*S) is the total amount of transferable utility that the members of coalition S can achieve without the help of players outside the coalition (which are denoted by *v(*N/S)). For any characteristic function, *v(*φ) = 0 and *v(*N) determines the overall amount of value created (i.e. by grand coalition N).The division of the overall value *v (*N) among its various players is crucial in determining how a coalition emerge in such games.

For a given cooperative game (N, v),

- An allocation $(x_1, x_2, ..., x_n)$ is division of the overall value that is created, where x_i is the value that is received by a player i.
- An allocation $(x_1, x_2, ..., x_n)$ is found to be individually rational if $x_i \geq v(\{i\})$ for every player i, this means that no rational player will get an incentive to join the cooperative game as the allocation gives it lesser value than what it can get by itself alone.
- An allocation $(x_1, x_2, ..., x_n)$ is said to be efficient if $\sum_{i=1}^{n} xi = v(N)$ thus, all the value that is created is allocated.
- An allocation $(x_1, x_2, ..., x_n)$ is called to be in the core of the game if it is found to be individually rational and efficient and for each and every subset (coalition) *S* of *N*, we have $x(S) \geq v(S)$, where $x(S)$ is the *S*-allocation or sum of the values allocated to each player i in the coalition S i.e. $x(S) = \sum_{i \in S} xi$ and $v(S)$ is the worth of the coalition S.
- A game is called convex if *v(*T) + *v(*S) ≤ *v(*TUS) + *v(*T∩U) for all the coalitions T and S possible.
- It must be noted that the core is defined by a system of linear equations, so the core is convex region given that it is non-empty. Also, the exact allocation in the core is obtained by means of bargaining between the users of the game. But, when there are many players in a game, it becomes complex and tedious to solve the system of inequalities and then to find a singleton solution by bargaining. Hence, one-point solutions such as the Shapley values, Tau values, etc were developed to calculate the allocation ratio directly without the need to solve the core. The Shapyley value φ is calculated by,

$$\phi i(N,v) = \frac{1}{|\mathrm{N}|!}\sum \mu i(Si(R)) \quad \text{for each } i \in N \qquad (1)$$

Where, R is the set of all the $|N|!$ orderings of N, μi marginal contribution of user *i* and $S_i(R)$ is the set of players preceding player *i* in the ordering *R*.

3. System Model

In this section, we describe the idea of spectrum sensing in a cognitive radio context as a cooperative game. The other cognitive radio activities may also be represented using the same modeling structure. The spectrum available is subdivided into M channels and there are N secondary users who intend to cooperate with each other and make use of these spectrum channels. The primary user activity on these channels is assumed to be random. Apart from these channels, there is a common control channel through which the secondary users communicate amongst themselves. Moreover there must be a fusion center to keep record of the information collected by secondary user and also take decisions. For convenience, it is assumed that the first user to enter the network or start the coalition acts as the fusion center. For every sensing and data transmitting cycle, each secondary user has a preference for the number of channels she wants to sense after taking into account various factors such as the type of data to be transmitted or received, QoS, power constraints, data rate, etc. All these preferences are conveyed to the fusion center which forms a channel sensing map based on the preferences made by the secondary users for the number of channels it wishes to sense. The secondary users will sense the channels for primary user activity based on any one of the available channel sensing methods such as energy detection, matched filter, cyclostationarity based detection, etc. The decision metrics such as the SNR values or the log likelihood ratios are translated to the Probability of detection (Pd) of the primary user in that channel. The biggest advantage of working with the probabilities of detection over signal to noise ratios is that the detection probabilities can be directly transformed into information theoretic metrics such as entropy which is not the case with using SNR values. On the basis of the probabilities of detection of primary user activity as calculated by the secondary users, the fusion center makes a decision of whether the primary user is using the channel or not. After the spectrum holes have been identified, each and every user/coalition’s worth is calculated based on the quantity as well as the quality of the work done by it. The amount of work done by a user/coalition is calculated by the number of channels sensed by it and the power spent in sensing them, whereas the quality of work done is calculated in terms of information-theoretic metrics

SNR matrix		**Channels**							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Users	**1**	-12.3	-	-20.9	-	-5.9	-6.9	-10.2	-
	2	-	-6.3	-16.2	-	-5.3	-	-	-
	3	-8.2	-	-	-	-	-	-	-
	4	-	-5.2	-12.1	-10.2	-7.2	-	-9.9	-19.8
	5	-7.3	-7.1	-	-8.2	-	-6.3	-	-23.5

Fig. 2 The SNR values obtained by the secondary users from sensing the channel

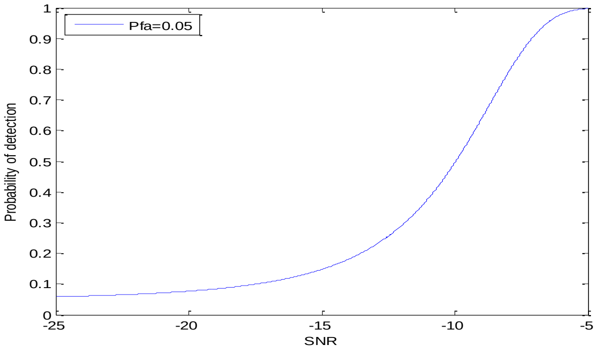

Fig. 3 Probability of detection obtained from SNR values under a constant false alarm rate of 0.05

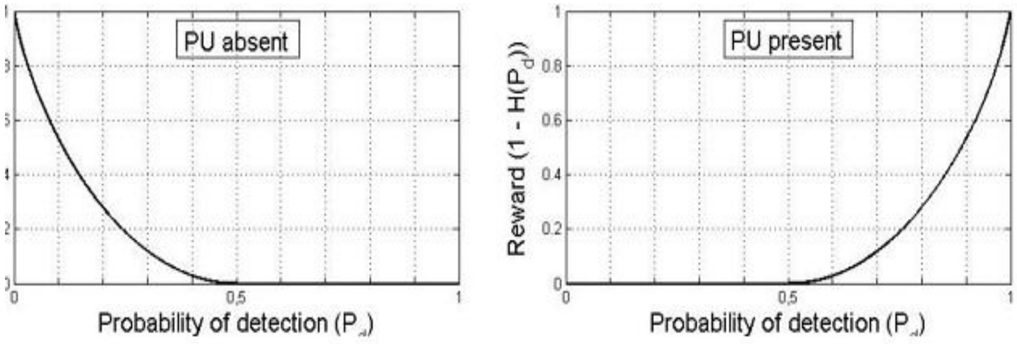

Fig. 5 The reward metric is calculated in terms of the reduction of uncertainty about the primary user activity from the detection probability values. The users whose detection probability values contradict the decision taken by the fusion center are not rewarded

Pd matrix		**Channels**							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Users	**1**	0.2685	0.5	0.0717	0.5	0.9815	0.92	0.4696	0.5
	2	0.5	0.9638	0.1197	0.5	0.9948	0.5	0.5	0.5
	3	0.7587	0.5	0.5	0.5	0.5	0.5	0.5	0.5
	4	0.5	0.996	0.2826	0.4696	0.8899	0.5	0.5086	0.0781
	5	0.8788	0.9005	0.5	0.7587	0.5	0.9638	0.5	0.0621
Decision		1	1	-1	1	1	1	-1	-1

Fig. 4 Detection of *spectrum* holes from the calculated detection probability values. In this example, a simple OR-based decision rule was employed. Here, +1 indicates the presence of primary user, whereas -1 denotes the absence of primary user.

derived from primary user detection probability i.e. to what extent is the information collected regarding primary user is accurate.

4. Game Modeling

In this section, we will describe the details of how the characteristic function of a game is modeled, or in other words, how the worth of the users and the coalitions are calculated at the fusion center. As the primary user activity is assumed to be random, we can consider it as a binary random variable and hence, the average probability of finding a primary user on a given channel before sensing a channel without any apriori information can be taken as 0.5. Hence, the uncertainty associated with the primary user activity as measured by the binary entropy function is at its ma*xi* mum value. Upon sensing the channel, the signal to noise ratios are translated to the probability of detection of the primary user occupying the channel. The probabilities of detection help to reduce the uncertainty regarding the primary user activity. Hence, when the detection probabilities are close to either 1 or 0, we have the most valuable information in terms of presence or absence of the primary user on that channel. The amount of reduction in uncertainty serves as a good metric for quantifying the quality of work done by the user in sensing the channel. Thus, the worth of each individual user is calculated based on the reduction in uncertainty that she brings by sensing the channels. But this information cannot be rewarded directly, but will have to be weighted by the total number of entities which are sensing the channel because, the fusion center gets information about primary user activity in a particular channel from many entities and information from all the entities are equally combined at the fusion center to take a decision regarding the primary user activity. It is decision method used that is irrespective of OR rule, AND rule, majority rule, etc. Finding the worth of a coalition is slightly more cumbersome as compared to calculating worth of a player. The coalition is considered as if it were a single player.

The values of probability of detection for the coalition are selected from the players within the coalition in such a manner that the best values of probability of detection among the users are selected for that coalition. Now, the best probabilities of detection are the ones that agree as closely as possible with the decision of the primary user activity taken by the fusion center. The information obtained from a coalition is far more reliable than the information obtained from a single player. The total reduction in uncertainty about the primary user activity brought in by the coalition is appropriately weighted by the number of users in that coalition. A characteristic function v, associates every nonempty subset S of N, to a real number, which is denoted *v(*S), which can be thought of as the worth of the coalition S or the value created by the coalition S when all its members come together and interact which forms the total payoff which is later available to the members of S for division amongst them. The equation for characteristic function is given by,

$$v(S) = |S| \sum_{j=1}^{M} \left(\frac{1 - H\left(\left| \max_{\forall i \in S} (pij, Pj) \right| \right)}{c(j)} \right) \tag{2}$$

where, S can be any coalition in {1, 2, ...,N}, M is the number of channels, |.| represents the cardinality of the set, H(.) returns the binary entropy value , pij is the probability of detection of the primary user as calculated by the user i on channel j, while Pj is the spectrum decision taken by fusion centre on channel j and c(j) is the total number of entities sensing channel j where +1 indicates that the primary user is present and -1 indicates that the primary user is absent.

5. Example

In order to demonstrate the proposed game theoretical approach for spectrum sharing as an example, we translated the proposed model into a MATLAB code and carried out simulations.

Channel sensing map		**Channels**							
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Users	**1**	X	-	X	-	X	X	X	-
	2	-	X	X	-	X	-	-	-
	3	X	-	-	-	-	-	-	-
	4	-	X	X	X	X	-	X	X
	5	X	X	-	X	-	X	-	X

Fig. 1 The channel sensing map created by the fusion center based on the preferences given by the secondary users for the number of channels to be sensed.

As an example, here we analyze the situation in which there are 5 secondary users sensing 8 channels. Here, it is assumed that any one user acts as the fusion center of this network. Initially, the secondary users randomly choose the number of channels that they prefer to sense. Then, the fusion center creates a channel sensing map based on the preferences given by the users as shown in Figure 1.

Here in Figure 1, the ‘X’ in the table represents that the channel is being sensed by that user. The values of SNRs as obtained by the users on the channels being sensed by it are collected. The values of SNRs are kept between -25 and -5 db. This is shown in Figure 2.

Then a cyclostationarity based detector is used which translates the SNR values to probability of detection (Pd) of primary user under a fixed false alarm rate (Pf) constraint. Here, we have taken value of Pf as 0.05. The mapping of SNR to Pd is shown in Figure 3 using a ROC curve. Based upon this mapping, the resulting probability of detection values are obtained and these are used to locate the spectrum holes. This is shown in Figure 4.

The reduction in the uncertainty about primary user activity is used as the metric to reward the user for the interference information that it senses from the channel. It must be noted here that this reward is based on the spectrum hole decision which is made by the fusion center. This concept is clearly illustrated in Figure 5. Using Equation (2), the characteristic function for this game is calculated to be

V = {*v*(1), *v*(2), *v*(3), *v*(4), *v*(5), *v*(12), *v*(13), *v*(14), *v*(15), *v*(23), *v*(24), *v*(25), *v*(34), *v*(35), *v*(45), *v*(123), *v*(124), *v*(125), *v*(134), *v*(135), *v*(145), *v*(234), *v*(235), *v*(245), *v*(345), *v*(1234), *v*(1235), *v*(1245), *v*(1345), *v*(2345), *v*(12345)}
={0.8516,0.7333,0.1049, 0.8381, 1.1545, 2.6974,1.9092, 3.3415,4.2385,1.6765,2.9915,3.6791,1.9205, 2.5394,

4.2106,4.5155,7.9936,7.8647,5.6483, 7.2238, 9.3181, 4.8580, 5.8642, 9.2555, 6.7175, 11.5063, 11.6410, 17.6825, 13.8011, 12.8761, 23.8243}

Moreover, using equation (1), the Shapley values for the 5 users are calculated and the values obtained are:

Users	**1**	**2**	**3**	**4**	**5**
Shapley values	21.306	19.038	11.324	23.114	25.216

Based upon these Shapley values calculated, the allocations can be made either on the number of channels that are available or on the total data rates that can be obtained in the free channels. Here, it can be observed that user 5 obtains maximum allocation (around 25%) because it sensed many channels and that too with high accuracy as compared to others. Similarly, user 4 got almost 21% allocation as it sensed good number of channels but with lesser accuracy. In similar fashion, the allocations for other users can be justified. This procedure is now continuously repeated over time for all the sensing and transmission cycles.

Conclusion

In this paper, we presented a new framework for modeling the spectrum sensing as well as sharing problem in cognitive radio network as a transferable utility cooperative game. Here, the characteristic function of the cooperative game is based upon the worths of the users/coalitions which are calculated according to the work done by the users for the coalitions. Also, the amount of work done by the users is quantified in terms of the information that is obtained regarding the probability of detecting a primary user. The games are found to be totally balanced and convex in nature. Moreover, the one-point allocation solution such as the Shapley value lies in the center of gravity of the core. Thus we can say that the allocation of spectrum resources is fair and stable ensuring that the users get a strong incentive to cooperatively sense and access the spectrum rather than working individually.

References

J. Lunden, V. Koivunen, A. Huttunen, and H.V. Poor (November 2009) Collaborative cyclostationary spectrum sensing for cognitive radio systems, Signal Processing, IEEE Transactions on, vol. 57, no. 11, pp. 4182 –4195,

Zhu Han and H.V. Poor (January 2009), Coalition games with cooperative transmission: a cure for the curse of boundary nodes in selfish packet-forwarding wireless networks, Communications, IEEE Transactions on, vol. 57, no. 1, pp. 203 –213

M. Mir´as and E. S´anchez (2008) Juegos Cooperativos con utilidad transferable usando MATLAB: TUGlab, Servicio de Publicaciones da Universidade de Vigo.

Zhu Han and K. J. Ray Liu (2008), Resource Allocation for Wireless Networks: Basics, Techniques, and Applications, Cambridge University Press, New York, NY, USA.

Bezalel Peleg and Peter Sudh¨olter, Introduction to the Theory of Cooperative Games (Theory and Decision Library C), Springer, 2nd ed. edition, October 2007.

S. Haykin, Cognitive radio: brain-empowered wireless communications, Selected Areas in Communications, IEEE Journal on, vol. 23, no. 2, pp. 201 – 220, February 2005

FCC (Nov 2002), Spectrum policy tak force report (et docket no. 02- 135).

Viswanathan and P.K. Varshney, Distributed detection with multiple sensors i. fundamentals, Proceedings of the IEEE, vol. 85, no. 1, pp. 54 –63, January 1997.

Martin J. Osborne and Ariel Rubinstein, A Course in Game Theory, The MIT Press, October 1994.

H. Urkowitz (April 1967), Energy detection of unknown deterministic signals, Proceedings of the IEEE, vol. 55, no. 4, pp. 523 –531.