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Research Article

Time Frequency Analysis of Continuous Signal using Wavelet Transform

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Abstract

In many communication systems, it is necessary to use the concepts of instantaneous amplitude and phase signals. A signal, stationary or non-stationary, needs to be processed and analyzed before it can be of any use. Signal in its raw form contains noise which is generally the high frequency component of the data. This noise needs to be eliminated for better representation of the signal. In this paper, we first demonstrate the implementation of Wavelet Transform and how it helps in efficient de-noising. Thresholding is done in the wavelet domain to de-noise the signal. A study of time-frequency analysis of the de-noised signal has been performed to achieve relevant conclusions. A case study of seismic signal is given at the end to support the theoretical analysis.

Keywords: *De-noising, thresholding, continuous wavelet transform, time-frequency analysis.*

1. Introduction

Time amplitude plot of a signal gives us the magnitude of the signal at a particular time but doesn't give us the frequency information. Application of Fourier Transform on the signal gives us a frequency domain plot. In frequency domain the time information is lost. So applying Fourier transform we transform information from time to frequency domain. In many practical applications, the frequency of a signal varies with time. Examples of such signals are speech signal, music, medical signals. In such signals Fourier transform fails as it gives the frequency spectrum of slowly growing locally interchangeable signal, and requires the complete description of the signal over all the time. To overcome this limitation of Fourier transform Short Time Fourier Transform (STFT) was developed which is also known as windowed Fourier Transform. STFT has a limitation of fixed resolution, i.e., it can either have a good frequency resolution or a good time resolution. In signal processing, time–frequency analysis comprises those techniques that study a signal in both the time and frequency domains simultaneously, using various time–frequency representations. The mathematical motivation for this study is that functions and their transform representations are often tightly connected, and they can be understood better by studying them jointly, as a two-dimensional object, rather than separately.

Dennis Gabor did a substantial work on time-frequency analysis and Gabor Transform was developed to find the time frequency analysis but it had similar limitations as STFT. The Wigner–Ville distribution was another foundational step.

Wavelet transform of a signal gives us a time scale plot of the coefficients. The time-scale plot of a signal actually gives us the coefficients at different scales at the particular time. Also there is a relation between the scale and frequency. Thus wavelet transform is capable of giving us a time-frequency analysis.

2. Materials and Methods

The seismic data are also a valuable technology used extensively by the oil and gas industry in its exploration, development and reservoir management operations. The data in the raw form, known as Time-Amplitude data of the earthquake signal is recorded every 0.02 seconds using seismic sensors which record the amplitude of the earthquake. Total 336 samples are taken for the study. The data is from database for seismic study.

For time-frequency analysis, the signal undergoes three stages. The first one is acquiring of the raw data. Raw data is the data that has not been processed for meaningful use. We make use of the available information and plot its time-amplitude graph.

It is known that when we work with real world data that signals do not exist without noise. There are many cases in which the noise corrupts the signals in a significant manner, and it must be removed from the data in order to proceed with further analysis. The process of noise removal is generally referred to as signal de-noising. Noise adds high-frequency components to the original signal which is smooth. This is a characteristic effect of noise. This paper is devoted to the recovery of digital signal that has been contaminated by evenly distributed

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noise, i.e. the additive white Gaussian noise. The next step is to perform the Continuous Wavelet Transform (CWT) (S.Mallet,1990).

Continuous wavelet transform is applied to the signal to transform the signal from spatial domain to wavelet domain. Wavelet domain is the transformation of time amplitude representation of the signal to time- scale representation which gives us the coefficients. Continuous wavelet transform is given by the following equation :

$$W(a,b)= \int_{-\infty}^{\infty} x(t)\,\frac{1}{\sqrt{|a|}}\,\Psi^{*}\left(\frac{t-b}{a}\right)dt \qquad \text{(i)}$$

where b acts to translate the function across x(t) and the variable a acts to vary the scale of the probing function, Ψ. Ψ^* is the complex conjugate of Ψ i.e. the mother wavelet. Continuous wavelet transform decomposes the signal into component wavelets which are scaled and translated which put together reconstructs the original signal.

The scale is represented in the equation by a. Scale is a useful property of signals. The smaller the scale factor, the more "compressed" the wavelet. Conversely, the larger the scale, the more stretched the wavelet.

There is clearly a relationship between scale and frequency. Higher scales correspond to the more "stretched" wavelets. The more stretched the wavelet, the longer the section of the signal with which it is being compared, and therefore the coarser the signal features measured by the wavelet coefficients.

To summarize, the general correspondence between scale and frequency is:

Low scale a ⇒ Compressed wavelet ⇒ Rapidly changing details ⇒ High frequency ω.
High scale a ⇒ Stretched wavelet ⇒ Slowly changing, coarse features ⇒ Low frequency ω.

The different types of wavelets are Haar wavelet, Daubechies wavelet, Morlet wavelet, Coiflet wavelet, Symlet wavelet etc. The choice of wavelet for a particular signal depends on the standard deviation of the cone of influence. Once the proper wavelet is chosen the wavelet transform can be applied.

Application of Continuous Wavelet Transform gives us the coefficients at different scale and time. This is W(a,b) in eq. 1. The signal has its energy concentrated in a small number of wavelet coefficients, its coefficient values will be relatively large compared to the noise that has its energy spread over a large number of coefficient. Thus we need to apply thresholding to the data for denoising of the signal (D.L. Donoho,1995).

There are two types of thresholding- Hard thresholding and soft thresholding. Hard thresholding is the usual process of setting to zero the coefficients whose absolute values are lower than the threshold. Soft thresholding is an extension of hard thresholding by first setting to zero coefficients whose absolute values are lower than the threshold and then shrinking the nonzero coefficients toward zero. The equation for soft and hard thresholding is given as follows: The hard thresholding operator is expressed as

$$\begin{aligned} D(U,\lambda) &= U \; for\ all\ |U > \lambda| \\ &= 0 \; otherwise \end{aligned} \qquad \text{(ii)}$$

The Soft thresholding operator is expressed as

$$\begin{aligned} D(U,\lambda) &= sgn(U)\max(O,|U|-\lambda) \; for\ all\ |U > \lambda| \\ &= 0 \; otherwise \end{aligned} \qquad \text{(iii)}$$

We will apply soft thresholding to the signal as it gives a smoother result.

We use soft thresholding using heuristic Steins Unbiased Risk estimate(SURE) principle. This thresholding ultimately removes the noisy coefficients and what we are left with are signal coefficients which is used for reconstruction.

Suppose that Y is the noisy signal, X is the noise free signal and N is the white noise in the signal. This can be represented by

$$Y = X + N \qquad \text{(iv)}$$

Suppose W is the wavelet transformed coefficient of Y then

$$W=Xw+Nw \qquad \text{(v)}$$

where Xw represents the coefficient for original signal and Nw represent the coefficients for noise. Applying the heuristic SURE algorithm we remove the Nw component thus getting a de-noised signal. Thus we are left with W=Xw .These coefficient are reconstructed using inverse wavelet transform.

The Wigner-Ville distribution Ws of a time series signal s(t) is defined as

$$W(t,w) = \int_{-\infty}^{+\infty} s\left(t+\frac{\tau}{2}\right) s*\left(t-\frac{\tau}{2}\right) e^{-iw\tau}dt \qquad \text{(vi)}$$

This distribution was first introduced by Eugene Wigner in his calculation of the quantum corrections of classical statistical mechanics. The time series function s(t) in equation can be either real or complex. This distribution helps in finding the frequency at that particular instance of time.

3. Case Study

Seismic signal processing, focuses on the processing of data to suppress noise, enhance signals and the knowledge extracted from data can be used in various fields.

Table 1: Sample raw data

Time (Sec)	Amplitude(dB)
0.02	-0.00022
0.04	-0.00024
0.06	-0.00023
0.1	-0.00022
0.12	-0.0002
0.14	-0.0018

The raw data cannot be used directly, it needs to be processed. Maximum frequency at particular instance of time can be achieved using time frequency analysis. A sample of the raw data has been given in table. 1The raw data used in the case study is shown below.

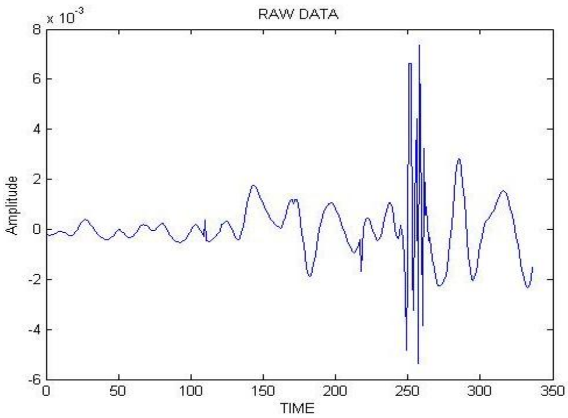

Fig. 1: This is plot of raw data. The data corresponds to the earthquake in Bhuj Gujarat in 20001

Applying Continuous Wavelet Transform(CWT) we get the coefficients in wavelet domain. Thresholding using heuristic SURE principle is applied on the coefficient to de-noise the signal. This gives us the coefficient of de-noised signal. The coefficients before and after applying the thresholding are given below:

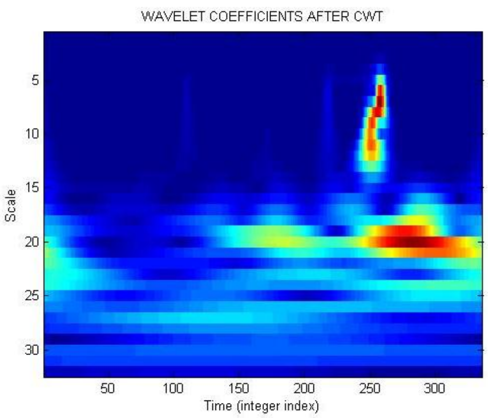

Fig. 2: The coefficients obtained after applying CWT on signal

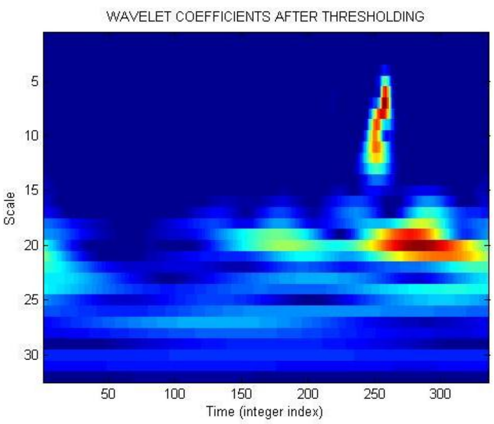

Fig. 3: The coefficients obtained after applying soft thresholding on the signal

Thus on applying inverse CWT these coefficients we get a smooth signal which is free from noise.

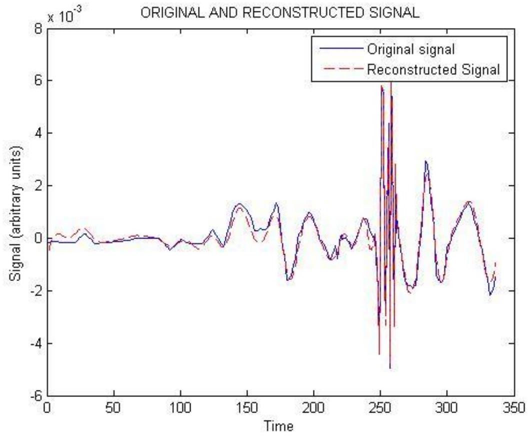

Fig 4: Original signal and the de-noised signal

The next step is applying Wigner-Ville Distribution to get the Time-Frequency analysis. The Wigner-Ville uses a variation of the autocorrelation function where time remains in the result, called instantaneous autocorrelation function. Thus by appropriate analysis of these signal we can understand at what particular instance of time the frequency is maximum. This analysis can be used in various fields.

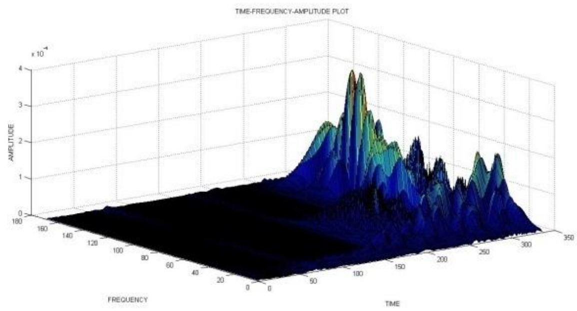

Fig. 5: Time-Frequency-Amplitude plot of the denoised signal

Conclusions

The study in this paper demonstrates the application of continuous wavelet transform on the signal that transforms the signal from time domain to wavelet domain. Thresholding is then applied on the coefficients to denoise the signal.The required value of the threshold is found using heuristic SURE principle. The coefficients after thresholding are reconstructed to get a denoised signal. The time-frequency analysis obtained using the Wigner-Ville Distribution is sensitive to noise. So the denoised signal is used. In the support of this study, for the sesmic data signal , the required maximum frequency is obtained.

Thus the proposed methodology of using Wavelet Transform can be used to obtain the time-frequency analysis of the continuous signal efficiently. In the future this application can be extended to various other types of continuous signals in the field of signal processing.

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