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Research Article

# Vibration Analysis of Convergent-Divergent Shaft-Rotor System

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### Abstract

*Cantilevered shaft-rotor systems consisting of disk and profiled shafts are considered. The procedures for the determination of the deflection, slope, shear force and bending moment at the extremities of the shaft are used. Transfer Matrix Method is used for the computation of the resonance, critical speed or whirling frequency conditions. Here shaft-rotor has three portions namely convergent, neck and divergent portion. For particular profiles, shaft length and rotor speeds, the response of the system are determined for the establishment of the dynamic characteristics. A built-in shaft-rotor system with one disk and 1N force on the disk is investigated for illustration purposes. Hence the analysis of the shaft whose radius is the function of length has been done.*


***Keywords:*** *Convergent, Shaft, Rotor, Frequency, Whirling, Response.*

## 1. Introduction

Rotating Shafts are employed in industrial machines like steam and gas turbines, internal combustion engines, compressors and pumps for power transmission. With increasing demand for power and high speed transportation, study of vibratory motion becomes essential. Rotor Dynamics deal with dynamics of rotating machinery. It is different from structural vibrations analysis because of the gyroscopic moments. Basic idea of Transfer Matrix Method was first put forth by Holzer for finding natural frequencies of torsional systems. Later adapted by Myklestad for computing natural frequencies of airplane wing. Prohl applied it to rotor-bearing systems and included gyroscopic moments in his computations. Lund [4] showed how system damping could be accounted for including self-exciting influences, such as oil whip and internal frictions. The above developments leads to the method came to be known as "The Transfer Matrix Method" (TMM). The design of shaft-rotor systems includes computation of the critical speed. An improved method for calculating critical speeds and rotor stability is done by Murphy and Vance. Whalley and Abdul Ameer used frequency response analysis for profiled shafts to study dynamic response of distributed-lumped shaft rotor system. They studied the system behaviour for the shafts with diameters which are functions of their lengths. They derived an analytical method which uses Euler-Bernoulli beam theory in combination with transfer matrix method (TMM). Here, profiled shaft with two disks have been considered with the method TMM for Frequency Response Calculation and then is validated with Whalley and Ameer for a single disk.

## 2. Mathematical modeling

*2.1 Shaft Model*

Input and output relationship for the distributed parameter shaft model (Whalley and Ameer, 2009) is given by,

$$(y_2, \theta_2, M_{y2}, Q_{y2})^{T} = F(s)\,(y_1, \theta_1, M_{y1}, Q_{y1})^{T}$$

$$F(s)=\begin{bmatrix} f_1 & \frac{f_2}{2\gamma(0)} & \frac{-c(0)l^2 f_3}{2\gamma^2(0)} & \frac{-c(0)l^3 f_4}{2\gamma^3(0)} \\ \frac{\gamma(0)f_4}{2l} & f_1 & \frac{-c(0)lf_2}{2\gamma(0)} & \frac{-c(0)l^2 f_3}{2\gamma^2(0)} \\ \frac{-\gamma^2(0)f_3}{2l^2c(0)} & \frac{-\gamma(0)f_4}{2c(0)l} & f_1 & \frac{f_2}{2\gamma(0)} \\ \frac{-\gamma^3(0)f_2}{2c(0)l^3} & \frac{-\gamma^2(0)f_3}{2l^2c(0)} & \frac{\gamma(0)f_4}{2l} & f_1 \end{bmatrix}$$

The elements of F(s) are-

$f_1 = (\cosh\gamma(l)+\cos\gamma(l))/2$
$f_2 = (\sinh\gamma(l)+\sin\gamma(l))/2$
$f_3 = (\cosh\gamma(l)-\cos\gamma(l))$ &
$f_4 = (\sinh\gamma(l)-\sin\gamma(l))$

Where $\gamma(x)=l\sqrt{\Gamma(x)}$, l = length of the distributed parameter shaft,
$\Gamma^{\frac{1}{2}}(x)=s^{\frac{1}{2}}(L(x)c(x))^{\frac{1}{4}}$ and

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$$c(x)=\frac{1}{EI(x)}$$

*2.2 Rigid disk model*

The output vector from the shaft will become the input for the rigid rotor model. i.e., for single disk and shaft model, we have

$$y_3(s)=y_2(s)$$
$$\Theta_3(s)=\Theta_2(s)$$
$$M_{Y3}(S)=-J\Omega s\Theta_2(s)+M_{y2}(s)$$
$$Q_{y3}(s)=ms^2Y_2(s)+Q_{y2}(s)$$

Hence

$$(Y_3(s),\theta 3(s),My3(s),Qy3(s))^T=R(s)\,(Y_2(s),\theta 2(s),My2(s),Qy2(s))$$

Where,

$$R(s)=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&-J\Omega s&1&0\\ms^2&0&0&1\end{bmatrix}$$

## 3. Numerical Results

*3.1 Cantilever Shaft-rotor system with a disk*

A cantilever rotor shaft system with one disc at free end is shown for illustration purposes in Fig. 1. Effects of bearings are neglected. As we proceeded in above sections, in the same way for the system illustrated in Fig.3 can be formulated as-

$H(s) = R(s).F(s)$

$(Y_1(s),Y_3(s))$,$(\Theta_1(s),\Theta_3(s))$, $\left(M_{y1}(s),M_{y3}(s)\right)$, and $(Q_{y1}(s),Q_{y3}(s))$ are the deflections, slopes, bending moments and shear forces at the fixed and free end respectively.

$$H(s)=\begin{bmatrix}H_{11}&H_{12}\\H_{21}&H_{22}\end{bmatrix}$$

Input-output vectors relationship is given by:

$$\begin{bmatrix}y_3\\\theta_3\\M_{y3}\\Q_{y3}\end{bmatrix}=H(s)\begin{bmatrix}y_1\\\theta_1\\M_{y1}\\Q_{y1}\end{bmatrix}$$

After applying the boundary conditions, we getdeflection $y_3$ at the free end of the system, so we will ultimately get the Transfer Function. For example, TF will be obtained for NN=20, 40 and 60 respectively.

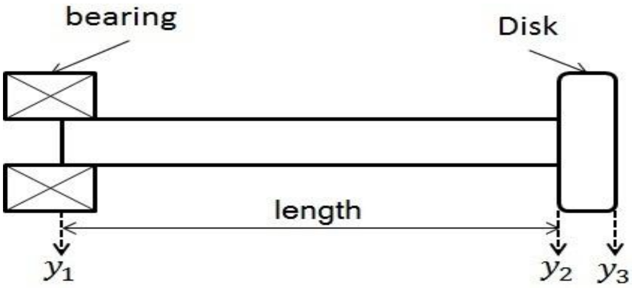


**Fig. 1** Cantilever rotor with a disc

**Validation**: The profile equation for the shaft-rotor (Whalley and Ameer, 2009) is given by-

$$r(x)=r0(1-NN(x^2))$$

For NN=25, r0=0.005 and 10,000 rpm, for single disk we get approximately the same results as in (Whalley and Ameer, 2009).

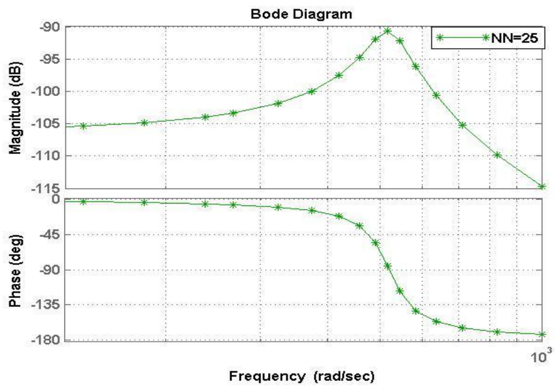


**Fig. 2** The Bode plot for profiled shaft with single disk

*3.2 Convergent-divergent shaft rotor system*

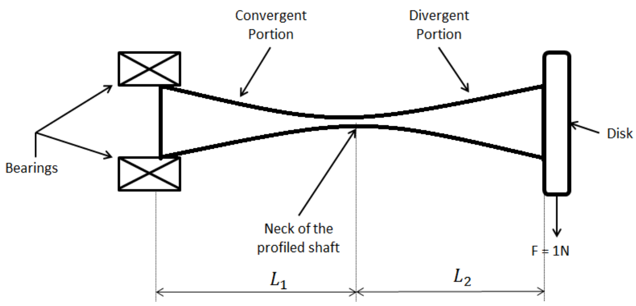


**Fig. 3** Convergent divergent shaft rotor system with 1 N force at the tip

Fig. 3 shows the convergent-divergent shaft-rotor system for the illustration purpose. The profile equation for the shaft-rotor is given by-

$$r(x)=r0(1\mp NN(x^2))$$

The '$\mp$' sign indicates the decrement and then increment in the cross-sectional area of the shafts. A unit impulse is applied on the disk at the free end to produce excitations in the shaft-rotor system.

For convergent part the diameter of the shaft is continuously decreasing while after the neck portion, i.e., for the divergent part the diameter is continuously increasing till the complete length of the shaft-rotor.Values of different parameters of the convergent-divergent shaft-rotor system are given in Table 1.

**Table 1** Various parameters of convergent-divergent shaft-rotor system as shown in Fig. 3.

| Parameters | Values |
|---|---|
| Length of the shaft-rotor, $l_1, l_2$ (m) | 0.1, 0.1 |
| Mass of the disk, m (Kg) | 0.75 |
| Diameter of the disk, D (m) | 0.09 |
| Young's Modulus of Elasticity, E (GPa) | 209 |
| Density of the material, $\rho$ ( $Kg/m^3$ ) | 7800 |
| Rotational Speed, N (rpm) | 10000 |
| Profile Value, NN (Constant) | 15 |

The values for the 'neck-radius' for different values of NN is given in Table 2.

**Table 2** Neck radius for various profile values (NN) for convergent-shaft length of 0.1 m

| Profile Value NN | Neck radius (m) of the shaft-rotor shown in Fig 4 |
|---|---|
| 15 | 0.0043 |
| 25 | 0.0037 |
| 40 | 0.0030 |

The transfer function obtained for the system shown in Fig. 3, for the values given in Table 1, is given as,

$$\frac{1.333\ s + 7090}{s^3 + 5317\ s^2 + 7.673\times10^5\ s + 1.044\times10^9}$$

*3.2.1 Varying Profiles*

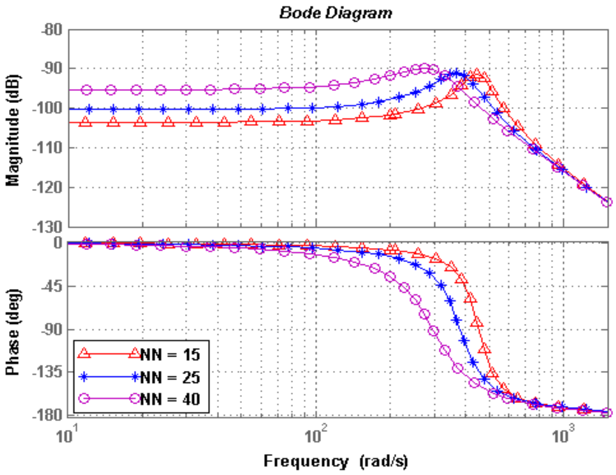


**Fig. 4** Bode plot for varying profiles

*3.2.2 Different lengths*

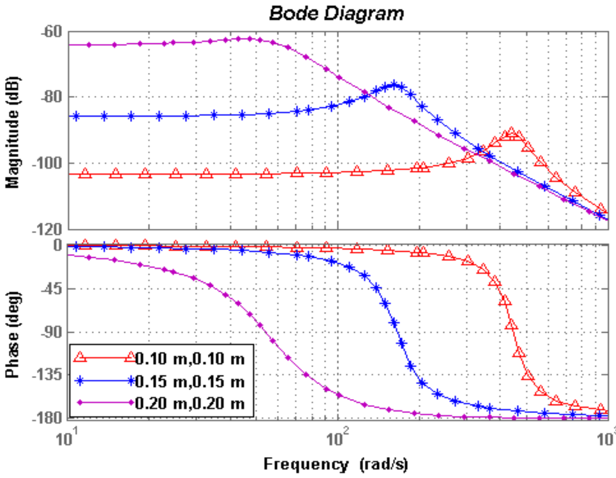


**Fig. 5** Bode plot for varying length with profile value N=15

*3.2.3 Changing rotor speed*

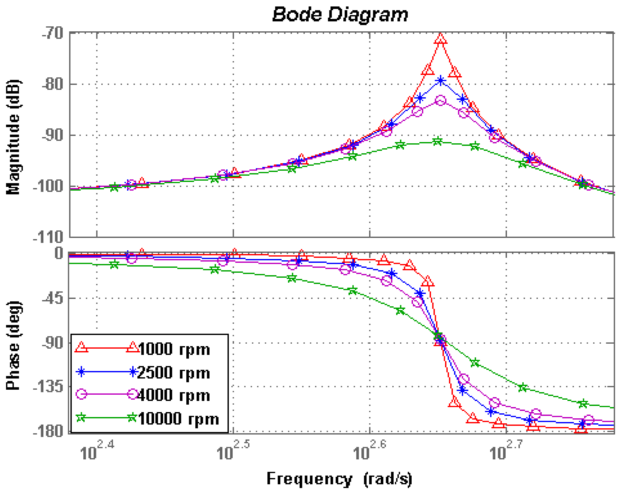


**Fig. 6** Bode plot for varying rotor speeds

3.2.4 Summary

Bode plots for various profile values, shaft lengths and rotor speed has been plotted in Figs. 4, 5 and 6 respectively. The results obtained from the Bode plots are tabulated in Table 3.

**Table 3** Results obtained from Bode plots for convergent-divergent shaft-rotor system

| $l_1$ (m) | $l_2$ (m) | Speed (rpm) | Value of NN | Critical Frequency (rad/sec) | Amplitude (dB) |
|---|---|---|---|---|---|
| 0.10 | 0.10 | 10000 | 15 | 447 | -91.3 |
| 0.10 | 0.10 | 10000 | 25 | 370 | -91.0 |
| 0.10 | 0.10 | 10000 | 40 | 273 | -89.9 |
| 0.15 | 0.15 | 10000 | 15 | 161 | -76.4 |
| 0.20 | 0.20 | 10000 | 15 | 47 | -62.4 |
| 0.10 | 0.10 | 1000 | 15 | 449 | -71.3 |
| 0.10 | 0.10 | 2500 | 15 | 449 | -79.3 |
| 0.10 | 0.10 | 4000 | 15 | 448 | -83.4 |

## Conclusion

The establishment of the vibrational characteristics of profiled shaft elements present challenging problems. The vibration analysis via bode plot has been done for the convergent-divergent rotor system. Here, disk-rotor system is shown for the illustration purposes. For a particular speed but value of NN is varied, as shown in Fig. 4, the resonant frequency decreases with the increase in the value of NN while amplitude increases for values of NN from 15 to 40. For different lengths, we obtain a bode plot as shown in Fig. 5, and it depicts that with slight increase in length of convergent and divergent part, the resonance frequency decreases drastically while the amplitude increases. For changing rotor speeds, we notice that the critical frequency remains the same while the amplitude of vibration decreases due to increasing effect of gyroscopic couple with increasing speed. Hence, such type of complicated systems can be analysed with simplicity using the approach discussed here in.

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