Research Article

Study of Rate of Convergence of different types of Grids in CFD Analysis

Bharath Kumar Komarabathini [A*], Deekshitha Kancharla [A], Kumari Monika [A], Praveen Kumar Chikoti [A] and Krupakar Pasala [A]

[A] Department of Aeronautical Engineering, Vardhaman College of Engineering, Shamshabad, R.R dist, A.P, INDIA.

Abstract

This paper deals with nonlinear FVM approach based on different kinds of meshing (using grids-C,O,H).Theory is utilized to obtain the convergence rate, CPU time, coefficient of drag, coefficient of lift, lift/drag of the airfoil(NACA-0012). The airfoil considered is studied with three different types of grids viz. O-grid, C-grid and H-grid and accordingly obtain the values of pressure, velocity and temperature. Airfoil is studied with two different flow regimes (i.e., 0.8M & 1.2M). A Finite discretization is applied throughout the process in case of gridding. The computational method does not require any tuning of parameters. The solution obtained shows the good resolution of all flow phenomenon and are obtained by computational analysis.

Keywords: Convergence, Airfoil, Descretization, Continuum, Flow separation, Flow regimes.

Introduction

Taking the NACA-0012 airfoil into consideration which has: Maximum thickness 12% at 30% chord, maximum camber 0% at 0% chord.

NACA-0012

Grid generation of NACA-0012

C-grid: has line of points in one direction roughly like the letter 'C' and one of these line will typically bend back to meet up with itself at some point.
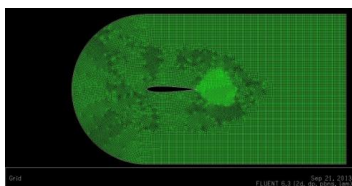

Fig.1.C-grid meshing of an airfoil NACA-0012

The C-mesh generated in fig.1 consists
 -40845 quadrilateral cells.
-786 2D Pressure far field faces
-104 2D wall faces

-81245 2D interior faces
-41290 nodes.

O-grid: has lines of points where the last point wraps around and meets the first point, thus you will have some grid lines that look likes letter 'O'.
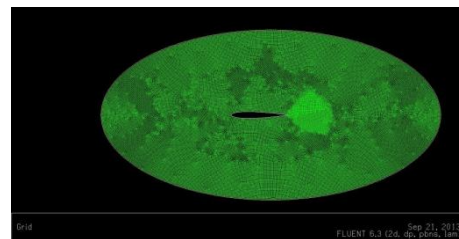

Fig.2.O-grid meshing of an airfoil NACA-0012

The O-mesh generated in fig.2 consists
-37958 quadrilateral cells
-728 2D pressure far field faces
-104 2D wall faces
-75500 2D interior faces
-38374 nodes.

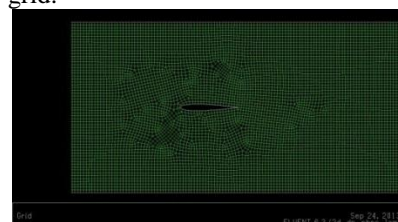**H-grid**: is pretty much any structured grid.That is not an c grid and o grid.

Fig.3 H-grid meshing of an airfoil NACA-0012

*Corresponding author: **Bharath Kumar Komarabathini**

The H-mesh generated in fig.3 consists
-7203 quadrilateral cells
-360 2D pressure far field faces
-42 2D wall faces
-14205 2D interior faces
-7404 nodes.

Discretization of airfoil by considering a continuum around it for the study of properties such as coefficient of lift, coefficient of drag, convergence rate, CPU time by meshing it with different grids.A grid is an discrete representation which designates the cells or elements on which the flow is solved.The nodes formed during the meshing process help us in analysing the P,V,T properties. In finite meshing we get accurate data as the nodes are closely placed compared to the course mesh.

Analysis of airfoil

Formulating the airfoil NACA-0012 using Gambit 2.26 and griding it in Ansys-Fluent with c-mesh,o-mesh and h-mesh respectively the following results are obtained.

The research area is restricted to:
Model: Viscous laminar
Boundary circumstances Pressure far turf
Materials: fluids-ideal gas
Pressure velocity coupling simple

Dissertation

Pressure-satndard
Density-first order upwind
Momentum-first order upwind
Energy-first order upwind

Mathematical Models

The flow around the airfoil has been analysed by solving the equations for conservation of mass and momentum. Finite volume method has been used to convert the prevailing equations of flow in to algebraic equations that are solved computationally.Pressure-velocity coupling has been done by SIMPLE algorithm. Turbulence of the flow has not been modeled. The computational details viz. foremost equations that are solved, computationally the details of geometry and geometrical modeling,and grid generation for the wing under study, periphery conditions that are enforced are discussed and presented in this paper. The air flow around the airfoil is regarded as steady and compressible.

Governing equations

The principal navier stokes equations for the flow considered int his work are written in vector form as:

$$\frac{\partial G_1}{\partial x} + \frac{\partial G_2}{\partial y} + \frac{\partial G_3}{\partial z} = \frac{\partial G_1 y}{\partial x} + \frac{\partial G_2 y}{\partial y} + \frac{\partial G_3 y}{\partial z}$$

Where G_1, G_2, G_3 are the inviscid flux vector given by:

$$G_1 = \begin{bmatrix} \rho u_1 \\ p + \rho u_1^2 \\ \rho u_1 u_2 \\ \rho u_1 u_3 \end{bmatrix}; \quad G_2 = \begin{bmatrix} \rho u_2 \\ \rho u_2 u_1 \\ P + \rho u_2^2 \\ \rho u_2 u_3 \end{bmatrix}; \quad G_3 = \begin{bmatrix} \rho u_3 \\ \rho u_3 u_1 \\ \rho u_3 u_2 \\ p + \rho u_3^2 \end{bmatrix}$$

$G_1 V$, $G_2 V$, $G_3 V$ are the viscous flux velocity, and is given by:

$$G_1 y = \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{xz} \end{bmatrix}; \quad G_2 y = \begin{bmatrix} 0 \\ \tau_{yx} \\ \tau_{yy} \\ \tau_{yz} \end{bmatrix}; \quad G_3 y = \begin{bmatrix} 0 \\ \tau_{zx} \\ \tau_{zy} \\ \tau_{zz} \end{bmatrix};$$

In the above given equation density is denoted by ρ, the velocities are given by u_1, u_2, u_3 and P is the pressure. Noramal stress is given by $\tau_{xx}, \tau_{yy}, \tau_{zz}$:

$$\tau_{xx} = 2\mu.\frac{\partial u_1}{\partial x} + \mu_{bulk}\left[\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial z}\right]$$

$$\tau_{yy} = 2\mu.\frac{\partial u_2}{\partial y} + \mu_{bulk}\left[\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial z}\right]$$

$$\tau_{zz} = 2\mu.\frac{\partial u_3}{\partial z} + \mu_{bulk}\left[\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial z}\right]$$

- The formula for the shape of a NACA 0012 foil, with "12" being replaced by the percentage of thickness to chord, is

- $$y_{t=0.2}\,c\sqrt{\frac{x}{c}} - 0.1260\left(\frac{x}{c}\right) - 0.3516\left(\frac{x}{c}\right)^2 + 0.2843\left(\frac{x}{c}\right)^3 - 0.1015\left(\frac{x}{c}\right)^4$$

where:
- c - chord length,
- x - position along the chord from 0 to c,
- y - half thickness at a given value of x (centerline to surface), and
- t - maximum thickness as a fraction of the chord (so 100 t gives the last two digits in the NACA 4-digit denomination).

Note that in this equation, at (x/c) = 1 (the trailing edge of the airfoil),thickness ids negligible and not equall to zero. If zero-thickness trailing edge is considered, for instance for computational effort, any of the coefficients is to be modified such that they sum to zero. Modifying the last coefficient (i.e. to -0.1036) will result in the smallest change to the overall shape of the airfoil.

Results and discussion

In case of c-grid

Airfoil NACA-0012 is meshed with C-grid by FVM. Concentration of nodes are higher at leading edge and trailing edge in order to study the flow separation at different flow regimes.
    Boundary conditions are applied and flow separations are studied at 0.8M. Leading edge experience high static pressure(approx. 4.21e+04 pascals)and it tends to be negative as it is flowing over the airfoil towards trailing edge. Again there is sudden hike of pressure at trailing

edge . Compared to leading edge ,pressure is low at trailing edge..
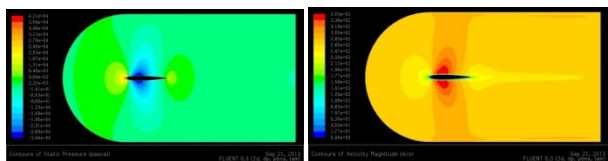

Fig.4.Pressure and Velocity analysis of airfoil in C-grid meshing at 0.8M.

Velocity is almost constant at leading edge and trailing edge mainting it at 1.59e+02m/s. There is formation of boundary on airfoil (1.41e+02m/s).

As the flow of velocity is directioned from leading edge to trailing edge velocity reaches it peak point over the surface of airfoil. And ascending on to trailing edge velocity successively subsidies. Continuum conduces static velocity.

From figures 4 and 5:

Table.1C-Mesh computational analysis results

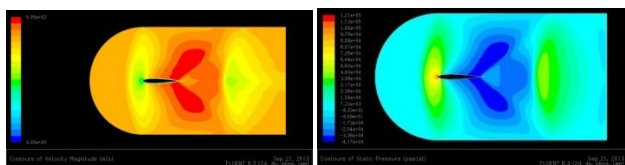|	PRESSURE AT 0.8M	VELOCITY AT 0.8M	PRESSURE AT 1.2M	VELOCITY AT 1.2M
AT L.E	4.21e+04	1.77e+02	1.21e+05	2.53e+01
OVER AIRFOIL	-3.04e+04	3.53e+02	-9.33e+02	4.29e+02
AT T.E	2.22e+04	1.59e+02	-3.36e+02	3.54e+02

Fig.5.Pressure and Velocity analysis of airfoil in C-grid meshing at 1.2 M.

In case of o-grid

NACA-0012 is meshed with o-grid by FVM. Concentration of nodes are higher at leading and trailling edge, to study the parameters varying over it at different flow regimes.
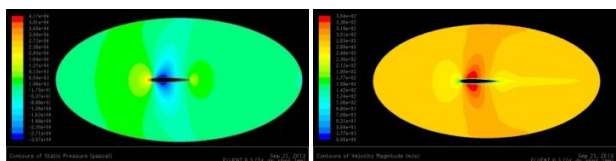

Fig.6. Pressure and Velocity analysis of an airfoil in O-grid meshing at 0.8M.

Applying boundary conditions and studying under 0.8M. The pressure on the surface of the airfoil varies from 0-1 co-ordinates. The tip of the leading edge experiences high static pressure(4.17e+04) and vary from LE to TE.

Moving over the surface of airfoil pressure gradually drops out and experiences negativity(-3.07e+04). Moving on to the TE the pressure seems to be constantly moderate(5.50e+04).

Velocity parameter seems to be vaying over the outward from TE to LE .The velocity of fluid at the tip of TE is moderate(1.77e+02 m/s). Moving over the surface velocity gradually increases and reaches its peak point at (3.54e+02 m/s) and decreases over the rest of surface.O-type grid results are comparitively similar to c type grid .

From figures 6 and 7

Table.2 O-mesh computational analysis results (approx)

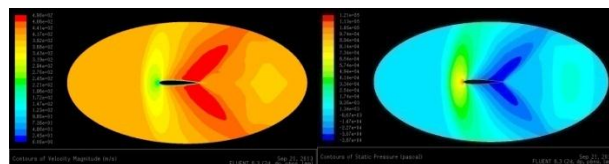|	PRESSURE AT 0.8M	VELOCITY AT 0.8M	PRESSURE AT 1.2M	VELOCITY AT 1.2M
AT L.E	3.81e+04	1.77e+02	1.13e+05	7.35e+01
OVER AIRFOIL	-3.07e+04	1.77e+02	1.34e+04	3.92e+02
AT T.E	1.27e+04	3.99e+02	-3.87e+04	3.43e+02

Fig.7.Pressure and Velocity analysis of an airfoil in 0-grid meshing at 1.2M.

In case of h-grid

H-grid gives a clear picture of flow separation. Separation of flow over the airfoil, at the tip of leading edge and trailing edge is studied and is analysed. Continum maintains constant air flow. H-grid shows contradict results compared to C-grid and O-grid

Static pressure is taken into consideration and pressure in continuum is obtained. Pressure at leading edge is very high compared to that off trailing edge. Static pressure at LE is 3.14e+04 and that of TE is 1.11e+04. Over the surface of airfoil pressure drops out to its negativity. cotinum doesn't show any pressure change it is static at 2.47e+03Pa.
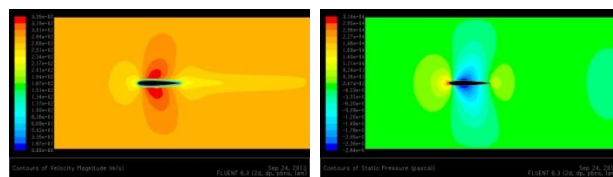

Fig.8.Pressure and Velocity analysis of an airfoil in H-grid meshing at 0.8M.

Velocity parameter figures out velocity in continuum. Constant velocity is maintained throughout the continuum at 3.01e+02m/s. Airfoil is bounded by various flow separation layers. On the airfoil velocity is almost

negligible, and suddenly its hikes to the highest velocity. At the tip of LE and TE velocity is same (i.e 2.51e+02).

From figures 8 and 9

Table.3 H-mesh computational analysis results (approx.)

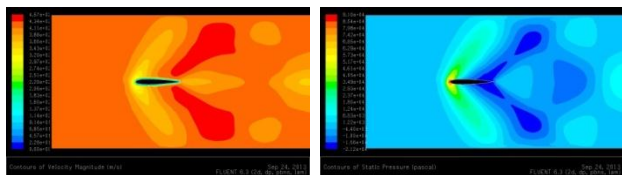|	PRESSURE AT 0.8M	VELOCITY AT 0.8M	PRESSURE AT 1.2M	VELOCITY AT 1.2M
AT L.E	3.14e+04	6.68e+01	7.42e+04	6.85e+04
OVER AIRFOIL	-2.64e+0	3.35e+02	1.24e+04	3.88e+02
AT L.E	-2.64e+0	3.35e+02	-1.56e+04	9.14e+01

Fig.9. Pressure and velocity analysis of an airfoil in H-grid meshing at 1.2M.
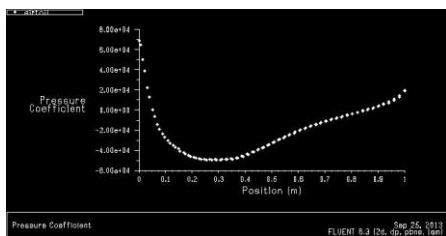
(1)Coefficient of Presuure in C-mesh:

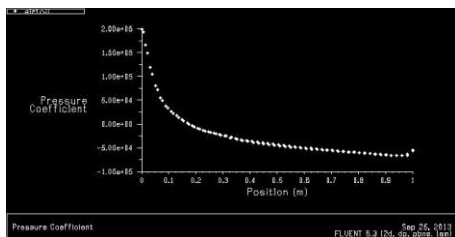**Fig.10.** Coefficient of pressure at 0.8M

Fig.11.Coefficient of pressure at 1.2M

Fig.10 and ,Fig.11 represents the variation of pressure coefficient over NACA-0012   surface at different flow regimes(i.e 0.8M,1.2M) which undegoes the c-grid meshing. Pressure is invariantly fluctuating over LE to TE .At initial stage attack over surface is high, it gradually decreases to the minimum point of the value at the co-ordinates(0.3 to 0.4).The pressure force later increased suddenly to reach its optimum level at 1.

Airfoil under 1.2M experience its maximum pressure at the prominent edges, considerably decreasing the force of attack over its surface and minimizes it effects on reaching the TE. Bernoulli's principle shows pressure varying inversely such that co-ordinates of velocity changing according to the pressure co-efficient.

(2)Coefficient of Pressure in O-mesh:
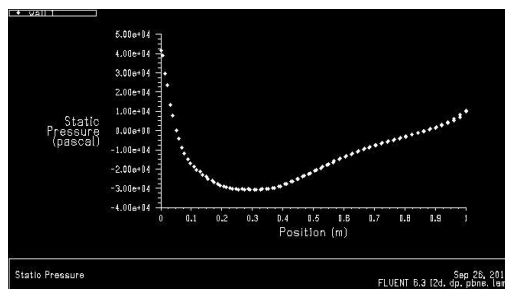

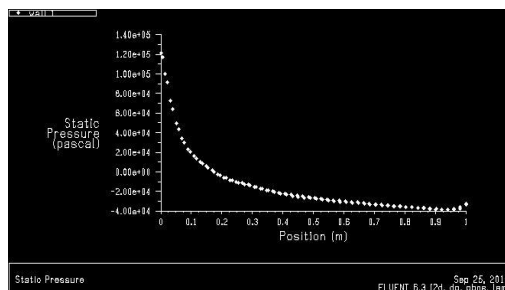
Fig.12. Coefficient of pressure at 0.8M

Fig.13.Coefficient of pressure at 1.2M

As shown in fig.12. pressure is high at the L.E (i.e., 3.81e+04) and dereases over airfoil and again a sudden increase is noticed at the T.E(1.27e+04). In case of 1.2M flow regime pressure decreses from L.E(1.13e+05) to T.E(-3.87e+04) as represented in fig.13. So at 1.2 M the airfoil experiences low pressure.
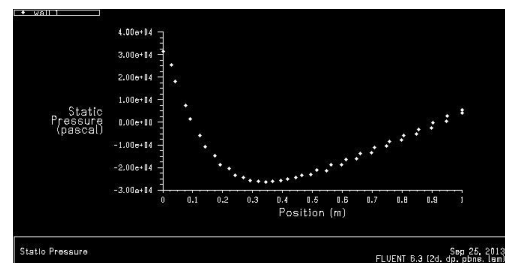
(3)Coefficient of Pressure in H-mesh
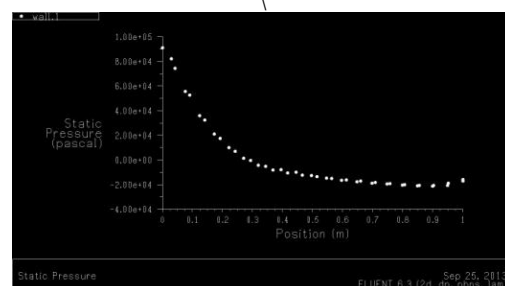

Fig.14.Coefficient of pressure at 0.8M

Fig.15.Coefficient of pressure at 1.2M

In fig.14,fig.15, it is observed that ,the variation over h-grid pressure coefficient is fluctuating. Leading limits experiences the high static pressure and reaches to its minimum attack over the region 0.3-0.4 , pressure seems to be increasing gradually over the surface and reaching its ultimate force at the TE. 1.2M flow regime has varying pressure. Attacking over leading edge is very high and exponentially decreases,   its minimum attack force reaches to the co ordinate at 1. The variation here is linked with the nodes formed in gridding, since the formed nodes are very low at concentration points the accurate parameters cannot be evaluated.

At 0.8M:

The problem got converged at 485 iterations with a CPU time of 4.32 min for C-mesh, whereas in a O-mesh the iterations noticed are 287 and CPU time resulted to be 3.2 min. However the iterations in H-mesh are 180 with a CPU time of 2.9 min.

	AT 0.8 MACH	
	No.of Iterations	CPU Time
C-MESH	485	4.32 min
O-MESH	287	3.21 min
H-MESH	180	2.9 min

At 1.2M :

The iterations undergone and the CPU time noticed are very high. Probably ,when C-mesh & O-mesh will get iterated at a point ,but it will take infinite time for converging in case of H-mesh. The case is not getting converged for even at very high CPU time.

Conclusion

It can be concluded from the above numerical modeling and analysis that the grid generation using C-mesh and O-mesh are reasonably accurate when analysing the airflow over an airfoil. It can also be concluded that the convergence of the problem at 1.2M is really consuming very high time and memory.

Nomenclature

NACA-National advisory committee for aeronautics.
L.E –Leading edge
T.E –Trailing edge
CPU Time –Convergence per unit time
FVM –Finite volume method
ρ -Density
P-Pressure
τ-Noramal stress
u-Velocity
G_nV_n-Viscous flux velocity
G_n-Inviscid flux vector

References

Alcouffe, R.E.; Brandt, A.; Dendy, J.E. (Jr.);   Painter, J.W (1981):*The multi-grid methods for the diffusion equation with strongly discontinuous coefficients*. Siam J. Sci. Stat. Comput., 2, pp. 430–454.

Dinar, N(1978) :*Fast methods for the numerical solution of boundary-value problems*. PH.D. Thesis, Dept. of Applied Mathematics, Weizmann Institute of Science, Rehovot.

Nicolaides, R.A(1978): *On multi-grid convergence in the indefinite case*. Math. Comp., 32, pp. 1082–1086.

Wesseling, P(1977): *Numerical solution of stationary Navier-Stokes equation by means of a multiple grid method and Newton iteration*. Report NA-18, Dept. of Mathematics, Delft University of Technology, Delft.

Beyn, W.J.; Lorenz, J(1982):*Spurious solution for discrete superlinear boundary value problems*. Computing, 28, pp. 43–51

Fedorenko, R.P(1964): *The speed of convergence of an iterative process*. U.S.S.R. Computational Math. and Math. Phys., 4 no. 3, pp. 227–235.

Chorin, Alexandre Joel(1967): *"A Numerical Method for Solving Incompressible Viscous Flow Problems," Journal of Computational Physics*, Vol. 2, No. 1,.

A Rizzi and H. Viviand,(1981) :*Numerical methods for the computation of inviscid transonic flows with shock waves*.(Viewey, Verlay, Bruncshweig)

B.korean center for mathematics and computer science *report N.M-R8601*,(1986):in proceedings, V. Aerodynamic Seminar , Aachen.

P.W Hemker and S.P spekijse(1986):*appl. Num math.*2, 475.

S oscher and F.Solomon(1982*): Math. Comput* 38, 339.