

Research Article

Sensitivity of Modal Parameters to Detect Damage through Theoretical and Experimental Correlation

Gomaa, F.R.^Å, Nasser A.A.^{B*}, and Ahmed, Sh. O.^B

^A Department of Production Engineering & Mechanical Design, Faculty of Engineering, Minoufiya University ^B Department of Civil Engineering, Faculty of Engineering, Minoufiya University

Accepted 15 January 2014, Available online 01 February 2014, Vol.4, No.1 (February 2014)

Abstract

Early detection and localization damage allow increased expectation of reliability; safety and reduction of the maintenance cost. Modal tests including intact and simulated damage state was proposed to investigate steel beam health monitoring. It consists mainly of piezoelectric accelerometers, impact hammer, pulse analyzer with (modal analysis consultant software) and laptop for three cases. The first case is free-free vibration. The second case is boundary condition. The third case is boundary condition with static load. Artificial crack was cut in lower beam flange at mid span. Sensitivity of modal parameters to change of steel member condition was studied for natural frequency, mode shape and damping factor. Finite Element Analysis (FEA) using (ANSYS11) was conducted to study natural frequencies, mode shape and stresses. The steel beam is modeled by three dimensional structural solid element (solid 45). Finite Element results from (ANSYS11) are used to make comparisons between numerical solutions and experimental results in terms of natural frequencies, mode shapes. Improved FE to compute damping matrices is proposed and discussed. Results show that, modal frequencies, mode shapes and damping factor may be used to identify structural damage with saving time. Sensitivity of damping ratios is more sensitive than that of natural frequency.

Keywords: Damage Detection, Structural Health Monitoring, Dynamic Properties, Finite Element, ANSYS11.

1. Introduction

Structural health monitoring is used to identify defects in civil infrastructure and aerospace applications. In order to perform health monitoring system, structural health along with sensor and actuator malfunction must be continuously checked. Structural Health Monitoring is employed to control changes in constitutive parameters of a structure, which is exposed to dangerous event as a real time monitoring devise option (M.R. Banan et al, 2007). Vibration measurements in structures under controlled excitation may be used as damage assessment by correlating changes in the dynamic system parameters such as, natural frequencies, modal shapes or damping with damage. Wavelet transform is conveniently employed for instantaneous frequencies and damping identification from free vibration structural response. Identification technique based on wavelets is useful to assess changes in the vibration characteristics due to incremental damage of nonlinear systems (R.O. Curadelli et al, 2008).

Piezoceramic sensors bonded to concrete specimens' surface initiate and receive Rayleigh waves propagating along the surface of concrete. Waves propagate through internal concrete. These waves are used to monitor the dynamic mechanical constants and cracking of concrete structures. Changes in the wave forms reflect the effects of internal micro cracking in concrete (M. Sun et al, 2008). Experimental flexural wave propagation characteristics monitor change in structural properties. Frequencydependent variation of the wave number and wave speed were measured in the frequency ranges of flexural vibration. The measured frequency was compared with undamaged members. When wave propagates through a media, the total system energy remains mostly unchanged; system forms changes between potential and kinetic energy. The various locations of damage are imposed accurately (J. Park, 2008). Structural health monitoring sensors were installed on Hong Kong's landmark Tsing Ma Bridge; the longest suspended bridge in the world. It carried both railway and regular road traffic. The sensor system offers many advantages over traditional resistive strain gauges; such as remote sensing, ease of installation, non-corrosive and lower maintenance cost (H.T. Chan et al. 2006).

Multi-criteria method is used to identify and localize single and multiple damage in numerical models of flexural members with different boundary conditions. Multi-criteria method incorporating modal flexibility and modal strain energy is effective in multiple damage assessment in beam and plate structures (H.W. Shih et al, 2009). The measurement accuracy of natural frequency is higher than that of mode shape or modal damping. Damage is localized and sized using algorithm depending on change of natural frequencies. Natural frequency is unable to give the damage location, because the structural damage in different location causes the same frequency change. It ascertains existence of large damage (X. Wang et al, 2001). Structural damage detection using Frequency Response Functions (FRF) are obtained from nondestructive vibration tests. Change in structural stiffness matrix is reflected in changes of FRF data exemplified by the evaluation of Damage Location Vector. This evaluation requires dynamic stiffness matrix of the undamaged structure and frequency response functions change of the currently damaged structure. The dynamic stiffness matrix is obtained from a finite element model of the intact structure. Change in Frequency Response Functions (FRF) is obtained from the Finite Element Modeling (FEM) and impact hammer test. This method is applied to space truss structure and plate structure. Damage Location Vector overcomes the problems of coordinate incompatibility and noise. Dynamic Expansion method is employed to estimate the unmeasured FRFs (D. Huvnh et al, 2005). Natural frequencies of damaged beam with traversing auxiliary mass change due to change in beam inertia of auxiliary mass is traversed along the beam. Auxiliary mass enhances crack effect on beam dynamics. It facilitates identification and location of damage. The derivatives of natural frequency curve provide crack information for damage detection of beam-like structures. Experimental noise is expected to corrupt the response data and natural frequencies of beam-like structures. Deep crack increases magnitude of peak value of the derivative of natural frequency curve. The derivatives of natural frequency curves give better crack indication if the auxiliary mass increases in magnitude (SH. Zhonga et al, 2008).

Flexibility matrix is a function of mode shape and natural frequency reciprocal. The effect of high-frequency components in flexibility matrix decreases rapidly with the increase of natural frequency. Therefore, the flexibility matrix is measured with enough accuracy by several loworder modes and frequencies. Damage diagnosis technique is based on changes in dynamically measured flexibility and stiffness of structures. Damage localization is achieved by a combined assessment of changes in two measured matrices moving from the reference state to the damaged state. Stiffness matrix offers more information than that of mass matrix. The change of stiffness matrix detects damage due to stiffness change in defected structure (A.M. Yan et al, 2005). Modal strain energy is obtained by incomplete mode shape and structural stiffness matrix. Structural health monitoring is accomplished via monitoring elemental compression modal strain energy change ratio. Damaged region is identified by visual comparison of the deformation mode shapes before and after damage. The modal residue and stiffness changes are also quantified for a better representation of damage location. Measurement error of vibration mode is distinctly larger than that of natural frequency (J.T. Kim et al, 2003). System identification techniques explore the relationship between stiffness changes and changes in natural frequency and dynamic response of the structure. The identified system is assumed to be perturbation of an original spatial model that

reproduces dynamic measurements. This procedure is adopted and developed to assess damage or changes in structure parameters using dynamic data taken before and after damage occurrence. Measurements of natural frequencies are used to identify location and severity of damage (H. Sophia et al, 1997).

Rotating machinery shaft misalignment is characterized in the form of an Operational Deflection Shape (ODS) based on its dynamic behavior. An ODS derived from multiple accelerometer signals acquired at various points on the machine to diagnose shaft misalignment. Tests are performed on a machinery fault simulator under various operating conditions. Operating data is simultaneously acquired using a multi-channel data acquisition system. A new perspective of machinery fault detection is more reliable tool for determining shaft misalignment and other machine faults from operating data (N.G. Surendra et al, 2008). Operation Deflection Shape is derived from two accelerometers signals acquired at two points on steel beam structure. They are used to diagnose the crack in a structure member. Tests are performed on a steel beam under various operating conditions. Operating data is sequential acquired using two channel data acquisition system, where crack produces dominant motion. The effect of Operation Deflection Shape develop phase angle as a reliable tool for determining crack and other structural damage from operating data. A simplified approach for on-line fault detection in structures is offered (F.R. Gomaa et al, 2010).

2. Work Objective

The main objective of this work is to discuss the sensitivity of modal parameters (natural frequency, mode shape and damping ratios) to detect damage using correlation between Experimental Modal Analysis and Finite Element Analysis (FEA) (ANSYS11).

3. Experimental Work

Health monitoring of large structures is difficult due to the small number of available sensors/measurements. To detect the presence of damage in a structure, it requires a reliable model, or a good representation of the structure prior to damage. Detecting and localizing damage are based on either frequency changes or transient responses. Transient or closed-loop responses are available during operation (P. Williams, 2008). The scheme of this study was conducted on a steel beam with I-cross section.

Table (1)	Mechanical	Properties of	f Test Specimen
-----------	------------	---------------	-----------------

Young's Modulus	Yield Stress (F _y)	Ultimate Stress (F _{ult})	Mass Density	Poisson Ratio	Shear Modulus
210x10 ⁴ kg/cm ²	2400 kg/cm ²	3600 kg/cm ²	7850 kg/m ³	0.3	$\frac{81 \times 10^4}{\text{kg/cm}^2}$

The beam span was one meter long. Dimensions of cross section were 103mm high, 47mm wide. Thickness of web

is 5mm. Thickness of Flange is 6.2mm. Steel beam test specimen mechanical properties are shown in Table (1).

Two steel beams were prepared. The first one is the intact beam. The second beam is cut at the mid span by a saw machine. The saw machine made artificial crack (length x width) = (16X2) mm. Crack was made in thelower flange. There are three phases for testing beam; first phase was the free-free set up. Beam is hanged by rob as shown in Figure (1). Second phase was the boundary condition (hinge-roller) with zero loads. Putting the bears on the flexural testing machines, applying no load as shown in Figure (2). Third phase was boundary condition (hinge-roller) applying load (5ton) on Flexural Testing Machine. Experimental modal analysis on the steel beam model involved the installation of a high resolution (accelerometer) and connected to pulse analyzer with software (modal analysis consultant). The selected range of frequency in free-free phase is (0-3200Hz) and (0-6000Hz) in case of boundary condition and static load. A sampling frequency of (3200Hz) was used with frequency resolution of 0.066Hz. An impact hammer was used to excite the model along both its principal axes and the resulting force recorded. The recorded accelerometer and force were then analyzed using software (model test consultant) to obtain system's Eigen values (natural frequency) and Eigen Vectors (mode shapes) for the model of vibration along the model's principal axes.

The beams are excited by impact hammer to determine the resonance frequency. The excitation signal is fed to pulse analyzer through amplifier and accelerometer. System analysis is carried out through modal analysis software using Pulse Analyzer Hardware. Accelerometers were used with wide frequency range, low noise-to-signal ratio and choice of sensitivity from 1 to 50mu/N. Impact hammer is used with sensitivity from 1 to 22mu/N to excite the beam.



Figure (1) Experimental Test Rig of Intact Beam in First Phase (Free-Free)



Figure (2) Experimental Test Rig of Mid Span Cracked Beam in Second Phase Boundary Condition (Hinge-roller)

4. Theoretical Modal Analysis

Theoretical modal analysis relies on the description of physical properties of a system to derive the modal model. Such a description usually contains the mass and stiffness of the system. Thus, it is a path from spatial data to modal model (M. Sh. El-Nagar, 2009). FEA is a computerized procedure for structure analysis. ANSYS11 was used to solve motion equation to calculate modal parameters. Steel beam is modeled by three dimensional structural solid element (solid45) in ANSYS11 program. This element is suitable for this analysis because the main objective is to determine mode shapes, natural frequencies and stresses of steel beam in case of intact beam and cracked beam. The selected element is defined by eight nodes. The element has three degree of freedom (translation in the x, y and z directions) at each node. FEM was conducted using mechanical properties of steel beam shown in Table (1). Structure (solid 45) is considered for steel beam element (Sh.O. Shokdof, 2009). Total number of steel beam meshes was around (2829) element with 3DOF for every element. Small elements were assigned near the crack where a very high deformation was expected as shown in figure (3).



Figure (3) Mesh Density of Cracked Beam in FEM Simulation of 3D Orthogonal Cutting

4.1. Analytical Damping Model

Modal damping is very complex especially in large structures. Measuring damping ratio can be used in estimating damping matrix by calculating damping coefficient. Flow chart was proposed to compute damping matrix using Reliagh damping as proportional damping approach.

$$[C] = \alpha[M] + \beta[K]. \tag{1}$$

Where α and β are unknown damping constants. Rewriting equation (1) using generalized modal properties for each model gives:

$$C_i = \alpha M_i + \beta K_i \tag{2}$$

where : C=1, k, n_i = mode number. K is damage case.

Where [C], [K], [M] are damping matrix, stiffness matrix and mass matrix respectively

Modal Damping factor is defined as:

$$\eta_{analytical} = \frac{C_i}{2\sqrt{k_i M_i}} \tag{3}$$

where $\eta_{\text{analytical}}$ = damping factor= $2\xi_i$ and ξ_i = damping ratio $\xi(\omega) = a_0 + a_1\omega + a_2\omega^2 + a_3\omega^3 + \cdots + a_n\omega^n$. (4) Modal damping constant in terms of discrete natural frequencies can be obtained by:



Figure (4) Flow Chart of Finite Element Model Updating (Improvement)



Figure (5) Stresses Distribution in Y Direction in Intact Beam in Case of Static Load (5ton)



Figure (6) Stresses Distribution in Y Direction in Cracked Beam in Case of Static Load (5ton)

Gomaa, F.R., Nasser et al

$$2\xi(\omega_i) = 2\omega(a_0 + a_1\omega + a_2\omega^2 + a_3\omega^3 + \cdots + a_n\omega^n).$$
(5)

Equation (5) is a function of ω_i and simply replaced by ${\omega_i}^2$ by $M_r^{-1}k_r$ for all frequency modes to obtain the damping matrix in higher order. In analytical solution, the proportional constants α and β are tuned, that $\eta_{analytical}$ produces the best approximation to the measured modal damping value of the undamaged structure.

To improve the efficiency of Finite Element calculations using damping factor, a routine called (Eigen value economizer) (W.Jr. Weaver et al, 1984) (Guyan reduction) is used. These routines reduce the size of the dynamical matrix:

$$[D] = [K] - \omega^2[M] + i\omega [C]$$
(6)

It condensed around selected 'master' modes having 'master' Degrees Of Freedom (DOF), for which the modal displacements are needed. These masters DOF are critical locations whose displacements participate strongly in the modes of interest for a given case. The steel beam is solved by using reduced method and gave the same results. This means a smaller [K] and faster calculations. This proposal is time consuming. It can be used in large structures. Figure (4) is a flow chart of FE Model Updating (Improvement).

4.2. Correlation of Experimental and Theoretical Data

The damage localization criteria are based on modal damping factor deviation between intact and cracked beam.

$$\delta\eta_i = \frac{(\eta_{iC} - \eta_{il})}{(\eta_{iC} + \eta_{il})} \tag{7}$$

where η_{iC} damping factor of cracked beam and η_{iI} damping factor of intact beam

A vector of modal damping deviation is defined for the experimental modes

$$\{\delta\eta\}_{exp.} = \left\{\delta\eta_{exp.1}, k, \delta\eta_{exp.n}\right\}^{T}$$
(8)

For the analytical modes of each damaged mode k:

$$\{\delta\eta\}_{theo.} = \{\delta\eta_{theo.1}, k, \delta\eta_{theo.n}\}^T$$
(9)

The experimental and analytical damping deviations are correlated throw a modal assurance criterion (MAC) (A.U. Rehman et al, 2011). The Modal Assurance Criterion (MAC) is a measure of correlation between test and analytical mode shapes. Separate frequency comparison must be used in conjunction with the MAC values to determine the correlated modes. The mode shape correlation is displayed as MAC matrix plotted versus experimental mode number on the X-axis and analytical mode number on the Y-axis. The natural frequency correspondence is usually displayed with a separate plot such as experimental natural frequency versus analytical natural frequency. Together, the two plots are used to determine the overall correlation of the modes.

5. Results and Discussion

Number of computed natural frequencies (Eigen values) are 115. It can be reduced from 115 to 80. Five mode numbers are matched with experimental as in table (2).

Numbers of computed natural frequencies are 80 from ANSYS11 in case of static load 5ton. Four modes are matched with experimental. Theoretical calculations have compound modes of torsion and bending as in figures (5, 6). Figure (5) shows stresses distribution in case of static load (5ton) in intact beam; Figure (6) shows stresses distribution in case of static load 5ton and crack at mid span. From Figure (5 and 6) stresses distribution around crack increased.

The sensitivity of natural frequencies is very low regarding structure damage as shown in table (2). It is important to focus on damping factor change than that of Eigen values (natural frequencies). The experimental data related to modal parameters of FE analysis are correlated.

The mode shapes of the updated FEM and the experimental results of the beam are compared using MAC. It is expressed as a percentage in table (2). Perfect correlation is 100%. No correlation is 0%.

Table (2) Comparison of Resonance Frequencies from Update FEM and Experimental Measurements of Steel Beam in case of boundary condition.

Mada	Frequ	ency (Hz)	Difference	MAC	
Mode	FEA	Experimental	Difference		
1st Bending	224.62	224	0.28%	90%	
2 nd Bending	464.37	468	0.78%	94%	
1 st Torsion	563.17	560	0.56%	98%	
2 nd Torsion	927	916	1.19%	84%	
	1282.2	1284	0.14%	85%	

5.1.	Variations	between	Measured	and	Calculated	Natural
Free	quencies					

There are four modes match between experiments and ANSYS11. Table (3) shows comparison between experimental and theoretical model analysis. Experimental results were compared with these obtained from 3D FEA in this work. Both sets of the results show very good agreement (87.2-106.8%). In case of Boundary condition, experimental results were compared with these obtained from 3D FEA in the work. Results were (97.63-106.85%) matching. In case of 5ton Static load, experimental results were compared with these obtained from 3D FEA in the work.

5.2. Sensitivity of Frequency changes from intact to cracked

Figures (7 a, b) show frequency response and coherence in intact beam in case of boundary condition. Figures (8 a, b) show frequency response and coherence in cracked beam in case of boundary condition. Natural frequencies have visible change as in Figures (7 and 8) split of natural frequency at frequency (2660Hz) occurred. The figure has a shift to the left. Natural frequencies are directly proportional to stiffness. Number of peaks increase in cracked beam. Number of peaks in intact beam is (10). Number of peaks in cracked beam is (12). Crack has significant effect on number of mode in the same range of frequency.

			Free-	Free Phase					
Mada	Experimental		Theoretical		Theoretical/Experimental (%)				
Mode	Intact Cracked		Intact	Cracked	Intact	Cracked			
1.	16	12	13.953	13.517	87.2	112.6			
2.	184	180	179.919	179.54	97.78	99.74			
3.	358	356	382.389	379.4	106.8	106.5			
4.	664	664	664	666.767	100	100.4			
Boundary Condition (hinge- roller)									
Mada	Experimental		Theoretical		Theoretical/Experimental (%)				
Mode	Intact	Cracked	Intact	Cracked	Intact	Cracked			
1.	224	222	224.62	216.75	100.27	97.63			
2.	468	440	464.37	455.79	99.22	103.58			
3.	560	560	563.17	568.6	100.56	101.53			
4.	916	878	927	921.64	101.2	104.9			
5.	1284	1200	1282.2 1282.3		99.85	106.85			
		Bounda	ry Condition (h	inge- roller with 5	oton Load)				
M.J.	Experimental		Theoretical		Theoretical/Experimental (%)				
Mode	Intact	Cracked	Intact	Cracked	Intact	Cracked			
1.	244	240	246.2	246.26	100.9	102.6			
2.	712	700	712.07	707.19	100.01	101.02			
3.	732	728	733.81	732.69	100.2	100.6			
4.	892	892	897.08	898.58	100.5	100.7			

Table (3) Comparison of Natural Frequencies between Experimental and Theoretical Modal Analysis



Figure (7. a) Frequency Response in Intact Beam in Case of Boundary Condition



Figure (7. b) Coherence in Intact Beam in Case of Boundary Condition



Figure (8. a) Frequency Response in Case of Cracked Beam (Boundary Condition)







Figure (9) Response of Intact Beam in Case of Static Load (5ton) and Frequency Range (0-6000 Hz) from ANSYS11







Experimental (ω=150 Hz) (Bending) **Figure (11)** Mode Shape of Intact Beam in Case of Free-Free Beam

Figures (9, 10) show frequency response of intact and cracked beam in case of static load (5ton)and wide frequency range (0-6000 Hz) using analytical analysis. The first mode is the dominant mode. The amplitude increased according to crack location.

An estimation of four fundamental mode shapes of a beam element was carried out, through curve fitting method (software) of a shock tests based on FRF_s . The motion can be described by 3DOF (X-displacement, Y-displacement and rotation) with respect to geometrical centre. Results are reported in the following (monographic) as in Figures (11, 12), between intact and cracked beam. At the same

5.3. Variation in Mode Shape

Gomaa, F.R., Nasser et al

Sensitivity of Modal Parameters to Detect Damage through Theoretical and Experimental Correlation





Experimental at (ω =150Hz)

OHz) Theoretical (ω =180Hz) (Bending – Torsion) Figure (12) Mode Shape of Cracked Beam in Case of Free-Free Beam





Hz) Theoretical (ω =382Hz) (Bending – Torsion) **Figure (13)** Mode Shape of Intact Beam in Case of Free-Free Beam



Experimental (ω=354Hz) Theoretical (ω=380Hz) Figure (14) Mode Shape of Cracked Beam in Case of Free-Free Beam



Experimental (ω = 660HZ)

 $\label{eq:constraint} \begin{array}{c} Theoretical \ (\omega=664 \ Hz) \\ \mbox{Figure (15) Mode Shape of Intact Beam in Case of Free-Free} \end{array}$

frequency, Damage case (crack) show variation in mode shapes according to crack.

Figures (14, 16) show the effect of crack in the mode shape at the same frequency. The deflection increased near crack. Figures (11, 12, 13, 15) show good agreement between experimental and theoretical results. Localized anomalies are visible. Changes in mode shape appear quite insignificant. There are differences in mode shape from Experimental Modal Analysis and FEA (ANSYS) because bending modes are measured. Mode shapes from FEA (ANSYS) are measured in bending and torsion. Curve fitting of FRF are provided to model damping value using peak method. Damping ratio curve fitting is shown in figure (17) for intact and cracked beam. Best curve fitting

179 | International Journal of Current Engineering and Technology, Vol.4, No.1 (Feb 2014)

Sensitivity of Modal Parameters to Detect Damage through Theoretical and Experimental Correlation

Gomaa, F.R., Nasser et al



Experimental (ω = 660Hz) Theoretical (ω = 570Hz) Figure (16) Mode Shape of Cracked Beam Boundary Condition (Hinge-Roller)



Figure (17) Variation of Damping Ratios according to Mode Number in Case of Intact and Cracked Beam

has nonlinear behavior as shown in figure (17). Figure (17) Variation of Damping Ratios according to Mode Number in Case of Intact and Cracked Beam

5.4. Results of Correlation between Numerical Model and Experimental Results

Verification using experimental data: in order to assess the localization capability of the proposed method, the correlation of damping deviation, natural frequency, mode shape, or resonance between analytical and test data are observed. The best correlation between a numerical model and experimental data produces the highest value of correlation indicated that the crack in model k lies closest to the test structure's actual crack. The numerical damping factors are computed from equation (2) and related to the corresponding data of healthy model. The vectors of damping deviations are then compared to the experimental damping values obtained from the response data of the intact and cracked beam.

The number of available Eigen values (natural frequencies) for comparison between test and analysis is limited by the correlation and the quality of analytical and experimental mode shapes and by suitability of the individual measured resonance for curve fitting (frequency spacing and peak height). Four modes have been selected to investigate the influence of incomplete model data as shown in table (4). The analytical modes chosen for correlation are marked in table (4).

1 able (4)	Data Set	for Ex	periment	al verifica	ition

Mode	1	2	3	4	5	6	7	8	9	10	11	12	13	14
6			x			x	x	x	x		x			
4					x	x	x	x						
5					x	x	x		x		x			
3					x	x			x					

6. Conclusion

The occurrence of damage in a structure produces changes in its global dynamic characteristics such as its natural frequency, mode shape, modal participation factors, frequency response functions (FRFs), etc. An understanding of these changes leads to the detection and the characterization of the extent of damage.

The main objective of this research is to determine damage existence in the structure by observing its dynamics systems by performing experimental work of damaged steel beams to measure dynamic properties changes. Comparison between experimental results of intact and cracked beam and the results from ANSYS program to validate the Finite Element Method (ANSYS). Then, updating the Finite Element Method was employed to perform the numerical analysis

• Update finite element model using measured damping ratio and calculating damping matrix in high order,

save time consuming. This is a start point for large structures.

- Correlation between numerical analysis and test data gives good agreement in all the studied case.
- Crack has significant effect on number of modes, amplitudes and natural frequency.
- Damping ratios are more sensitive to crack localization. So damping is a promising damage indicator in Structural Health Monitoring (SHM).
- The influence of boundary condition has a significant effect on measured vibration frequencies and mode shape. There are variations in ω_n according to the effect of boundary condition.
- Results of an experimental study on beam element of structure show that this method can detect damage at element without any knowledge of exciting force or (surrounding forces). This is very useful for civil structure, where it is difficult to measure exciting force.

References

- M.R. Banan and Y. M. Pour, (2007), Detection and Assessment of Damage in 2D Structures Using Measured Modal Response, *Journal of Sound and Vibration*: Vol.306, pp. 803–817.
- R.O. Curadelli, J.D. Riera, D. Ambrosini and M.G. Amani, (2008), Damage Detection by Means of Structural Damping Identification, *Engineering Structures*: Vol.30, pp. 3497-3504.
- M. Sun, W.J. Staszewski, R.N. Swamy, and Z. Li, (2008), Application of Low-Profile Piezoceramic Transducers for Health Monitoring of Concrete Structures, *NDT& E International*: Vol.41, pp.589–595.
- J. Park, (2008), Identification of Damage in Beam Structures Using Flexural Wave Propagation Characteristics, *Journal of Sound and Vibration*: Vol.318, pp. 820–829.
- H.T. Chan, L. Yu, H.Y. Tam, Y.Q. Ni, S.Y. Liu, W.H. Chung and L.K. Cheng, (2006), Fiber Bragg Grating Sensors for Structural Health Monitoring of Tsing Ma bridge: Background and Experimental Observation, *Engineering Structures*: Vol.28, pp. 648–659.
- H.W. Shih, D.P. Thambiratnam and T.H.T. Chan, (2009), Vibration Based Structural Damage Detection in Flexural Members Using Multi-Criteria Approach, *Journal of Sound and Vibration*, pp.1-17.

- X. Wang, N. Hu, H. Fukunaga, and Z.H. Yao, (2001), Structural Damage Identification Using Static Test Data and Changes in Frequencies, *Engineering Structures*: Vol.23, pp. 610–621.
- D. Huynh, J. He, and D. Tran, (2005), Damage Location Vector: A Non-Destructive Structural Damage Detection Technique, *Computers and Structures*: Vol.83, pp. 2353–2367.
- SH. Zhonga, S.O. Oyadijia, and K. Dingb, (2008), Response Method for Damage Detection of Beam-Like Structures Using High Accuracy Frequencies with Auxiliary Mass Spatial Probing, *Journal of Sound and Vibration*: Vol.311, pp. 1075– 1099.
- A.M. Yan and J.C. Golinval, (2005), Structural Damage Localization by Combining Flexibility and Stiffness Methods, *Engineering Structures*: Vol.27, No.12, pp. 1752–1761.
- J.T. Kim and N. Stubbs, (2003), Crack Detection in Beam-Type Structure Using Frequency Data, *Journal of Sound and Vibration*: Vol.259, No.1, pp. 145–160.
- H. Sophia and G. Karolos, (1997), Damage Detection Using Impulse Response, *Elsevier Science*: Vol.30, No.8, pp. 4757-4764.
- N.G. Surendra, Li. Zhuang and H.R. Mark, (2008), Using Operating Deflection Shapes to Detect Misalignment in Rotating Equipment, *Conference & Exposition on Structural Dynamics, IMAC XXVI*, February (4-7).
- F.R. Gomaa, A.A. Nasser and Sh.O. Ahmed, (2010), Damage Detection in Structures Using Operation Deflection Shape, *The Seventh Alexandria International Conference of Structural and Geotechnical Engineering (AICSGE7)*, 27-29 December, MS13- MS23.
- P. Williams, (2008), Structural Damage Detection from Transient Responses Using Square-Root Unscented Filtering, *Acta Astronautica*: Vol.63, pp. 1259 – 1272.
- M.SH. El-Nagar, (2009), Static and Dynamic Analysis of Cellular Raft Foundation, *Ph.D. Thesis*, Faculty of Engineering, Menofia University.
- Sh.O. Shokdof, (2009), Advanced Techniques for Damage Detection of Structural Members, *M.Sc. Thesis*, Faculty of Engineering, Menofia University.
- W.Jr. Weaver and P.R. Johnston, (1984), Finite Element for Structure Analysis, Prentice-Hall, Inc., Englewood Cliffs, New Jersey.
- A.U. Rehman, K. Worden and J.A. Rongong, (2011), Experimental Investigation of the MAC Based Crack Detection Technique for Repeating Structures, *IOMAC'11 – 4th International Operational Modal Analysis Conference*, pp.1-8.