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Research Article

Prediction of dynamic stability behavior of thin square plates subjected to constant compressive and periodic including constant compressive loads on perpendicular edges

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Abstract

The dynamic stability behavior of the square plate subjected to a uniform edge in-plane periodic load, consisting of a constant compressive and periodic load in one direction and a constant compressive load in the perpendicular direction is investigated for the simply supported boundary conditions with varying uniform constant compressive loads, in this paper. The energy method, by using a single term exact trigonometric admissible function to represent the lateral deflection, is employed to obtain the dynamic instability regions. Numerical results are presented in the non-dimensional form with the varying static load parameters and the static compressive load ratios in both the digital and analogue forms. The present results show excellent agreement when compared with that available in the literature. The effect of the static load parameters and the static compressive load ratios on the dynamic instability regions is brought out in the present work.

Keywords: *Dynamic stability; simple formula; thin square plate; periodic load; constant loads*

1. Introduction

The present day lightweight structures, subjected to dynamic loads, make it essential to determine the dynamically unstable regions for different structural members. When a plate is subjected to periodic loads, it is a well known fact that for certain values of exciting frequencies the ordinary in-plane forced response will become dynamically unstable, leading to a violent vibration in the transverse direction. This is called as the dynamic instability phenomenon.

Prediction of the dynamic stability behavior of structural members subjected to periodic in-plane loads is an important input for the structural design engineers. The structural members, like plates are commonly used in many fields of engineering. Earlier studies on the dynamic stability of columns are briefly discussed (Timoshenko and Gere, 1961). The theory and application of the dynamic stability behavior of structures have been exhaustively given in the classic work of Bolotin (Bolotin, 1964). The first finite element (FE) studies on this topic for columns with various boundary conditions (Brown *et al.* 1968). It is observed in this interesting study that when the first mode shapes of the free vibration and buckling are the same or nearly the same, which is generally satisfied for the many column boundary conditions, the dynamic stability regions, more or less remain the same, by proper non-dimensionalization of the basic physical quantities in the problem involved. However, the derivation of the proper nondimensional parameters is not given in the study and is taken intuitively (Brown *et al.* 1968). The thin plate finite element model to study the dynamic instability of rectangular plates studied in this work (Hutt and Salama, 1971). Some recent studies in this topic for plates subjected to periodic loading can be seen in (Dey and Singha, 2006), (Ramachandran and Sarat Kumar, 2012). In a recent study (Rao *et al*, 2008), it is shown that these nondimensional parameters used in (Brown *et al.* 1968), can be derived rigorously and demonstrated the existence of the unique dynamic stability curves for many commonly used structural members, provided the requirement on the mode shapes (Brown *et al.* 1968) is satisfied. It may be emphasized here that, these mode shapes, though similar as mentioned earlier (Brown *et al.* 1968), differ by small extent depending on the boundary conditions and the error involved in the instability boundaries is tolerable small, for the engineering purposes, as has already been demonstrated by (Rao *et al*, 2011), based on the consideration of the Euclidean norm.

In the present study, the dynamic stability behavior of the square plate subjected to an edge uniform in-plane periodic load, consisting of a constant compressive and periodic loads in one direction and a constant compressive load in the perpendicular direction, is investigated for the plate with the simply supported boundary conditions with varying uniform constant compressive load ratios. In this

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work an energy method is used to develop a simple closed form solution to predict the dynamic instability behavior of the simply supported square plate, using exact trigonometric admissible function to represent the transverse deflection, as given by (Leissa 1965).

Elegant, simple and exact closed form analytical solution to predict the dynamic stability regions of the square plate subjected to the aforementioned loads are provided in this paper. It is to be noted here, that the prediction of dynamic stability behavior of the plate with these applied loads is a new contribution, to obtain the behavior of dynamic stability regions of square plate. In the following section, a simple formula to predict the dynamic instability behavior of the simply supported square plate subjected to the loading condition, mentioned earlier is presented.

2. Dynamic Stability Equation

When the plate is, subjected to a uniform edge in-plane periodic load in the x- direction and constant compressive load in the y- direction, as shown in Fig.1, the periodic load $N_x(t)$ and N_y, which are defined as,

$$N_x(t) = N_s + N_t \cos\theta t = (\alpha \pm \beta \cos\theta t_t) N_{cr} \quad (1)$$

and

$$N_y = \gamma N_x \quad (2)$$

where α is the static load factor $\left(= \frac{N_s}{N_{cr}}\right)$, β is the dynamic load factor $\left(= \frac{N_t}{N_{cr}}\right)$, N_s is the constant compressive load, N_t is the time dependent load, N_{cr} is the buckling (critical) load, θ is the applied radian frequency, t_t indicates time and $\gamma = \frac{N_x}{N_y}$.

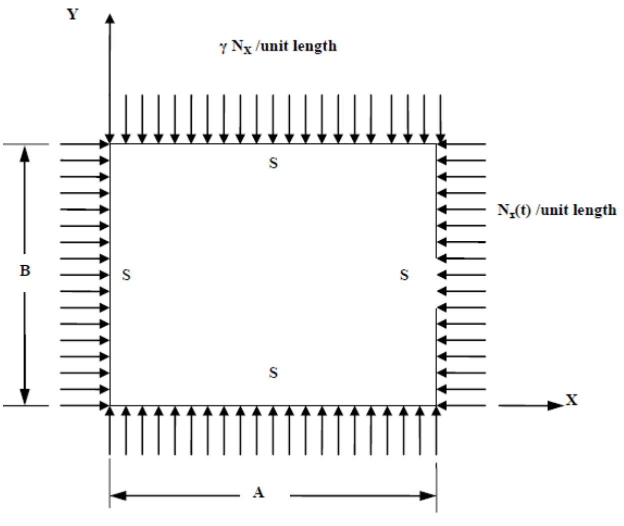

Fig.1 A simply supported plate under the applied compressive load system

For a rectangular plate of length *A* and breadth *B* with constant thickness *t*, the strain energy *U*, the work done by the external compressive edge loads *W* and the kinetic energy *T* are given by

$$U = \frac{D}{2}\int_0^B\int_0^A \left[\kappa_x^2 + \kappa_y^2 + 2\nu\kappa_x\kappa_y + \frac{1-\nu}{2}\kappa_{xy}^2\right] dxdy \quad (3)$$

$$W = \frac{N_x(t)}{2}\int_0^B\int_0^A \left(\frac{\partial w}{\partial x}\right)^2 dxdy + \frac{\gamma N_x}{2}\int_0^B\int_0^A \left(\frac{\partial w}{\partial y}\right)^2 dxdy \quad (4)$$

and

$$T = \frac{\rho t\omega^2}{2}\int_0^B\int_0^A w^2 dxdy \quad (5)$$

with

$$\kappa_x = -\frac{\partial^2 w}{\partial x^2},\ \kappa_y = -\frac{\partial^2 w}{\partial y^2},\ \kappa_{xy} = -2\frac{\partial^2 w}{\partial x \partial y} \quad (6)$$

Where *w* is the lateral displacement, ρ is the mass density, ω is the radian frequency of the plate, $N_x(t)$ and N_y are the edge uniform periodic and constant compressive loads in the Cartesian co-ordinate system, and *D* is the plate flexural rigidity given by $D = \frac{Et^3}{12(1-\nu^2)}$, E is the young's modulus and ν is the Poisson ratio.

The admissible transverse displacement *w* in terms of *m* and *n*, which are the number of half-waves in the *x* and *y* directions respectively, is assumed to be of the form,

$$w = a\sin\left(\frac{m\pi x}{A}\right)\sin\left(\frac{n\pi y}{B}\right) \quad (7)$$

Where *a* is the undetermined coefficient. The displacement distribution assumed for *w* is exact for the simply supported boundary conditions, for both the vibration and buckling problems.

The total potential energy Π of the plate is given by

$$\Pi = U - W - T \quad (8)$$

$$\Pi = \left(\begin{array}{l}\frac{D}{2}\int_0^B\int_0^A \left[\kappa_x^2 + \kappa_y^2 + 2\nu\kappa_x\kappa_y + \frac{1-\nu}{2}\kappa_{xy}^2\right] dxdy - \\ \frac{\gamma N_x}{2}\int_0^B\int_0^A \left(\frac{\partial w}{\partial y}\right)^2 dxdy\end{array}\right) - \frac{N_x(t)}{2}\int_0^B\int_0^A \left(\frac{\partial w}{\partial x}\right)^2 dxdy \quad (9)$$

Where *U*, *T* and *W* are the strain energy, kinetic energy and potential due to the work respectively.

In the Rayleigh-Ritz method, the total potential energy is minimized with respect to the undetermined coefficient *a*, as

$$\frac{\partial \Pi}{\partial a} = \left(\frac{\partial}{\partial a}\right)(U - W - T) = 0 \quad (10)$$

Which yields, after integration and simplification of Eq.(10) by neglecting the term for *W*, an expression for the frequency parameter, λ_f $\left(= \frac{\rho t \omega^2 A^4}{D}\right)$ is obtained as

$$\lambda_f = \pi^4\left(m^2 + n^2\frac{A^2}{B^2}\right)^2 \quad (11)$$

Similarly, by neglecting the kinetic energy term *T* in Eq.(10), the expression for the buckling load parameter, λ_b

$\left(=\dfrac{N_{xcr}A^2}{\pi^2 D}\right.$, where N_{xcr} is the critical load, defined with the load in the x- direction, is obtained as

$$\lambda_b = \frac{\left(m^2 + n^2 \dfrac{A^2}{B^2}\right)^2}{\left(m^2 + \gamma n^2 \dfrac{A^2}{B^2}\right)} \qquad (12)$$

Substituting, the energies U and T and potential W in Eq. (10), after simplification, using the Eqs.(11) and Eq.(12) for, the dynamic stability equation in the non-dimensional form, as

$$1 - \frac{\left\{\left(\alpha \pm \dfrac{\beta}{2}\right)m^2 + \gamma n^2 \dfrac{A^2}{B^2}\right\}}{\left(m^2 + \gamma n^2 \dfrac{A^2}{B^2}\right)} - \frac{\theta^2}{4\omega^2} = 0 \qquad (13)$$

For a square plate (A/B =1), for the first mode of buckling and vibration ($m = n = 1$) and γ=0 for uni-axial load, Eq. (13) becomes

$$1 - \left(\alpha \pm \frac{\beta}{2}\right) - \frac{\theta^2}{4\omega^2} = 0 \qquad (14)$$

Defining, $\Omega = \dfrac{\theta}{\omega}$ and after simplification, Eq. (14) becomes

$$\Omega = \frac{\theta}{\omega} = 2\sqrt{1 - \left(\alpha \pm \frac{\beta}{2}\right)} \qquad (15)$$

Equation (15) can be treated as the dynamic stability solution containing the physically identifiable non-dimensional parameters.

Substituting, the energies U and T and potential W in Eq. (10), after simplification, using the Eqs.(11) and Eq.(12) for biaxial load, and after simplification, the dynamic stability equation in the non-dimensional form, as

$$\Omega = \frac{\theta}{\omega} = 2\sqrt{1 - \frac{1}{2}\left\{\left(\alpha \pm \frac{\beta}{2}\right) + \gamma\right\}} \qquad (16)$$

Equation (16) can be treated as the dynamic stability solution for the afore mentioned loading condition, containing the physically identifiable non-dimensional parameters.

3. Results and Discussion

By using Eq. (15), the dynamic stability of square plate with uni axial load and using Eq.(16) the effect of the in-plane loads considered, in the present study, as shown in Fig.1, is brought out. The dynamic instability boundaries Ω_1 and Ω_2, between which it is dynamically unstable are given with varying β and for α = 0.0, 0.25 and 0.5 given in Table 1. In Table.1, the variation of Ω_1 and Ω_2 for the value of β (dynamic load factor) varying from 0 to 1.0 for the simply supported square plate with uniform uniaxial in-plane periodic load i.e. (γ = 0.0) for α = 0.0, 0.25 and 0.5 are presented. The dynamic stability regions of the square plate are given in (Ramachandra and Sarat Kumar, 2012) and the present results for γ = 0.0 (Uniform uniaxial periodic load) are deduced from the dynamic stability formulae Eq.(15), derived here, for α = 0.0 and 0.6 as is given in Tables 2 and 3 respectively. The excellent agreement between the two results strongly indicates the usefulness of the simple dynamic stability formula, developed in the present work.

Table 1: Variation of Ω_1 and Ω_2 for square plate subjected to uniaxial compression without static compressive load (γ= 0.0)

β	α = 0.0		α = 0.25		α = 0.5	
	Ω_1	Ω_2	Ω_1	Ω_2	Ω_1	Ω_2
0.0	2.0000	2.0000	1.7320	1.7320	1.4142	1.4142
0.1	1.9493	2.0493	1.6733	1.7888	1.3416	1.4832
0.2	1.8973	2.0976	1.6124	1.8439	1.2649	1.5491
0.3	1.8473	2.1447	1.5491	1.8973	1.1832	1.6124
0.4	1.7888	2.1908	1.4832	1.9493	1.0954	1.6733
0.5	1.7320	2.2360	1.4142	2.0000	1.0000	1.7320
0.6	1.6733	2.2803	1.3416	2.0493	0.8944	1.7888
0.7	1.6124	2.3237	1.2649	2.0976	0.7745	1.8439
0.8	1.5491	2.3664	1.1832	2.1447	0.6324	1.8973
0.9	1.4832	2.4083	1.0954	2.1908	0.4472	1.9493
1.0	1.4142	2.4493	1.0000	2.2360	0.0000	2.0000

Table 2: Variation of Ω_1 and Ω_2 for square plate Subjected to uniaxial periodic load for α= 0.0

β	Present study		Ramachandra & Sarat Kumar*	
	Ω_1	Ω_2	Ω_1	Ω_2
0.0	39.4801	39.4801	39.09	39.09
0.1	38.4790	40.4531	38.63	40.00
0.2	37.4527	41.4066	37.27	40.91
0.3	36.4657	42.3363	36.59	42.04
0.4	35.3109	43.2463	35.22	42.73
0.5	34.1890	44.1386	34.54	43.64
0.6	33.0309	45.0131	32.72	44.54
0.7	31.8287	45.8698	31.81	45.45
0.8	30.5792	46.7127	30.68	46.36
0.9	29.2783	47.5398	29.45	46.81
1.0	27.9163	48.3491	27.27	47.72

*Values are read from the graph

Table 3: Variation of Ω_1 and Ω_2 for square plate subjected to uniaxial periodic load for α = 0.6

β	Present Formula		Ramachandra & Sarat Kumar*	
	Ω_1	Ω_2	Ω_1	Ω_2
0.0	24.9693	24.9693	25.03	25.03
0.05	24.1755	25.7370	24.18	25.64
0.1	23.3563	26.4831	23.45	26.36
0.15	22.5055	27.2096	22.48	27.09
0.2	21.6231	27.9163	21.51	27.93
0.25	20.7033	28.6052	20.79	28.54
0.3	19.7393	29.2783	19.81	29.27

*Values are read from the graph

Table 4: Variation of Ω_1 and Ω_2 for square plate subjected to applied compressive load system with static compressive load ratio (γ = 0.25)

β	α = 0.0		α = 0.25		α = 0.5	
	Ω_1	Ω_2	Ω_1	Ω_2	Ω_1	Ω_2

0.0	1.8708	1.8708	1.7320	1.7320	1.5811	1.5811
0.1	1.8439	1.8973	1.7029	1.7608	1.5491	1.6124
0.2	1.8165	1.9235	1.6733	1.7888	1.5165	1.6431
0.3	1.7829	1.94938	1.6431	1.8165	1.4832	1.6733
0.4	1.7606	1.9748	1.6124	1.8439	1.4491	1.7029
0.5	1.732	2.0000	1.5811	1.8708	1.4142	1.732
0.6	1.7029	2.0248	1.5491	1.8973	1.3784	1.7606
0.7	1.6733	2.0493	1.5165	1.9235	1.3416	1.7888
0.8	1.6431	2.0736	1.4832	1.9493	1.3038	1.8165
0.9	1.6124	2.0976	1.4491	1.9748	1.2649	1.8439
1.0	1.5811	2.1213	1.4142	2.0000	1.2247	1.8708

Table 5: Variation of Ω_1 and Ω_2 for square plate subjected to applied compressive load system with static compressive load ratio (γ = 0.5)

β	α = 0.0		α = 0.25		α = 0.5	
	Ω_1	Ω_2	Ω_1	Ω_2	Ω_1	Ω_2
0.0	1.4142	1.4142	1.2247	1.2247	1.0000	1.0000
0.1	1.3784	1.4491	1.1832	1.2649	0.9486	1.0487
0.2	1.3416	1.4832	1.1401	1.3038	0.8944	1.0954
0.3	1.3038	1.5165	1.0954	1.3416	0.8366	1.1401
0.4	1.2649	1.5491	1.0488	1.3784	0.7745	1.1832
0.5	1.2247	1.5811	1.0000	1.4142	0.7071	1.2247
0.6	1.1832	1.6124	0.9486	1.4491	0.6324	1.2648
0.7	1.1401	1.6431	0.8944	1.4832	0.5477	1.3038
0.8	1.0954	1.6733	0.8366	1.5165	0.4472	1.3416
0.9	1.0488	1.7029	0.7745	1.5491	0.3162	1.3784
1.0	1.0000	1.732	0.7071	1.5811	0.0000	1.4142

Table 6: Variation of Ω_1 and Ω_2 for square plate subjected to applied compressive load system with static compressive load ratio (γ = 1.0)

β	α = 0.0		α = 0.25		α = 0.5	
	Ω_1	Ω_2	Ω_1	Ω_2	Ω_1	Ω_2
0.0	1.8708	1.8708	1.7320	1.7320	1.5811	1.5811
0.1	1.8439	1.8973	1.7029	1.7608	1.5491	1.6124
0.2	1.8165	1.9235	1.6733	1.7888	1.5165	1.6431
0.3	1.7829	1.94938	1.6431	1.8165	1.4832	1.6733
0.4	1.7606	1.9748	1.6124	1.8439	1.4491	1.7029
0.5	1.732	2.0000	1.5811	1.8708	1.4142	1.732
0.6	1.7029	2.0248	1.5491	1.8973	1.3784	1.7606
0.7	1.6733	2.0493	1.5165	1.9235	1.3416	1.7888
0.8	1.6431	2.0736	1.4832	1.9493	1.3038	1.8165
0.9	1.6124	2.0976	1.4491	1.9748	1.2649	1.8439
1.0	1.5811	2.1213	1.4142	2.0000	1.2247	1.8708

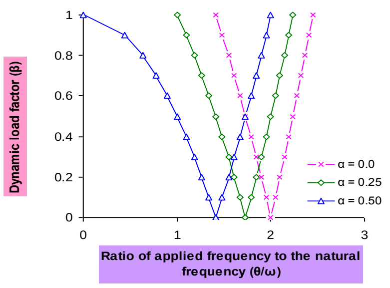

Fig. 2 Dynamic stability regions for square plate subjected to uniaxial compression without static compressive load (γ= 0.0)

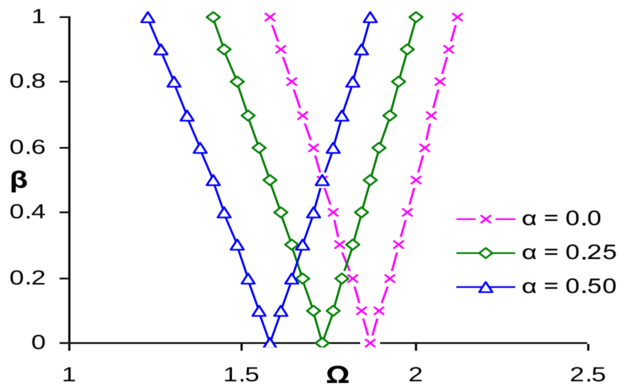

Fig. 3 Dynamic stability curves for square plate subjected to the applied compressive load system with static compressive load ratio (γ=0.25)

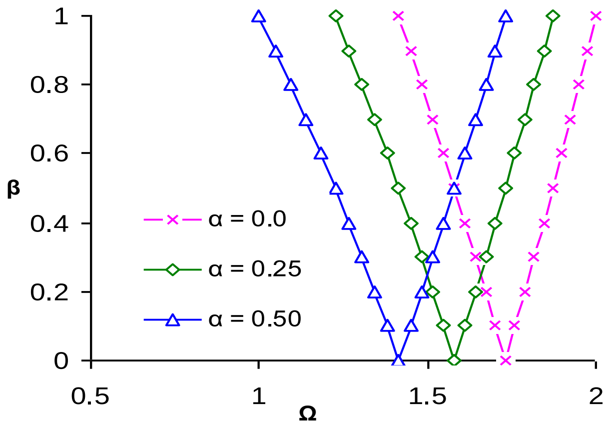

Fig. 4 Dynamic stability curves for square plate subjected to the applied compressive load system with static compressive load rati (γ= 0.5)

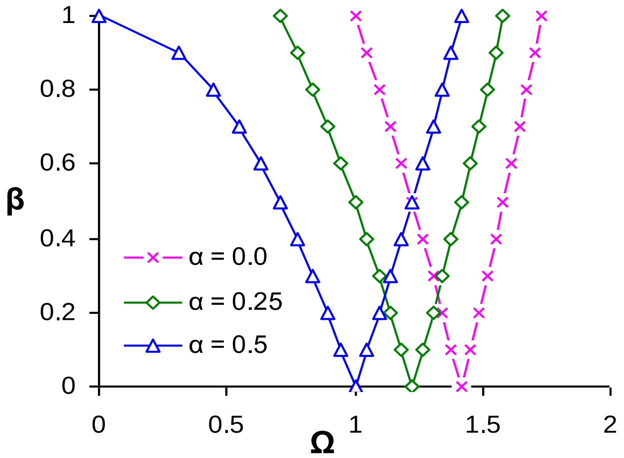

Fig. **5** Dynamic stability curves for square plate subjected to the applied compressive load system with static compressive load ratio (γ= 1.0)

In Tables 4 to 6, the dynamic stability regions for the uniform biaxial load for γ = 0.25, 0.5 and 1.0 for α = 0.0, 0.25 and 0.5 are presented. Figures 3 to 5 shows the stability boundaries given in tables 4 to 6 respectively. One can observe that, by increasing of the static compressive load parameter α, the width of the dynamic instability regions increase and by increasing the static compressive load ratio (γ) the regions of the dynamic instability increases and shifts towards the vertical axis.

4. Conclusions

Accurate closed form solutions are obtained to predict the dynamic stability regions of the simply supported square plate subjected to uniform in-plane compressive periodic load with a constant compressive load component in one direction and a static compressive load in the perpendicular direction. Simple one term standard trigonometric admissible function that satisfies all the boundary conditions is used to obtain the solution employing the energy method. It is noted here, by increasing the static load factor α, the regions of instability increase and by increasing the static compressive load ratio (γ) the dynamic instability regions increases and shift towards the vertical axis using physically recognizable dynamic load factor (β).

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