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Research Article

Simulation of Anomalous Zeeman Effect and Paschen–Back Effect for Positronium Atom when Electron Spins Upward and Downward with s=± 1/2

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Abstract

The research aims are for studying and calculation spin - orbit energy correction and Darwin's correction when $(\ell=0)$ *for positronium atom, in addition to that we calculate magnetic energy correction as a function of weak magnetic field (anomalous Zeeman effect) with range (10 m Tesla -100 m Tesla) and strong magnetic field with range (25 Tesla-50 Tesla) for electron spin upward and downward with s=±1/2 we found the magnetic energy correction of the positronium atom increases with a magnetic field strength (B) according to the empirical formula.*

Keywords: *Simulation, spin, Zeeman Effect, Paschen-Back, Positronium atom.*

1. Introduction

Positronium (Ps) is an unstable atom consisting of an electron and a positron. The two particles tend to annihilate each other with characteristic life times producing gamma ray photons. The set of atomic space energy levels of Ps is in a certain sense similar to that of the hydrogen atom because both are two charged particle systems governed by the same dynamical equations. However the masses of the positive charged constituents are not the same leading to different lines in the spectrum (Fabrizio Castelli, 2012). As for hydrogen-like atom, positronium exhibits two spin states which are called ortho and para for triplet and singlet, respectively (Y.C. Jean, 1983). Interactions between the particle spins split the levels of each atom; but the splitting of the positronium levels differs from those of hydrogen-like atom not only in magnitude but also in general structure since the magnetic moment of the positron is large compared to that of the proton, the spin – interaction in positronium is larger comparable to its spin orbit interaction (H. S. Leipner *et al*, 1999).

The results from the fact that the positron is unlike the proton in hydrogen-like atom, it was cannot be considered to be stationary, and the result from the fact that an electron and positron can form a virtual photon. While our goal is to obtain high amount of anti-hydrogen via a charge exchange process with positronium, there are few studies involving excited Ps in magnetic and/or electric fields, mainly for astrophysical or antimatter researches Positronium atoms are also a good test for many theories mainly because:

- The Ps atoms is very light $(2\, m_e, \approx 1\, MeV/\ c^2)$ compared to the lightest stable atom, hydrogen $(1\, m_e + 1\, m_p\,, \approx 1 GeV\ /c^2)$ and effects related to the atom velocity ,such as Doppler and motional Stark effects ,are unusually high compared with .
- The Ps atoms is composed by two elemental leptonic particles (an electron and a positron), bonded together by pure electromagnetic forces , the fine structure splitting of ground state is of about 203 GHz .
- The Ps atom spontaneously annihilate (in 125 Ps for para-positronium , 142 ns for ortho-positronium in absence of external fields) , while the hydrogen atom is stable .

These factors are combined to give the total fine structure correction for Positronium (A. Bug *et al*, 2003) :

$$\Delta E_{fs.} = \frac{\alpha^4 mc^2}{2n^3}\left[\frac{11}{32n} - \frac{1+\varepsilon/2}{2\ell+1} + \frac{\delta_{\ell 0}\delta_{s1}}{2}\right] \tag{1}$$

where α is fine-structure constant, n is known as the principal quantum number, m is represent the mass of positronium atom and ε is given by,

$$\varepsilon = -\frac{(3\ell+4)}{(\ell+1)(2\ell+3)}\,;\ J=\ell+1, s=1 \tag{2}$$

thus, the total fine structure correction for positronium ($\ell=0, s=1$) is as in the following:

$$\Delta E_{fs.} = \frac{\alpha^4 mc^2}{2n^3}\left[\frac{11}{32n} + \frac{1}{6}\right] \tag{3}$$

The contribution to the energy due to perturbation Hamiltonian H', for the atom in an external magnetic field is given by (A. Blom):

$$\Delta E_{n\ell m}(B) = \left\langle n\ell m \middle| H' \middle| n\ell m \right\rangle \tag{4}$$

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where H'_{SO} spin-orbit operator and $H'_{mag.}$ the perturbation operator from magnetic force,

$$H' = H'_{SO} + H'_{mag.} \tag{5}$$

$$= W \sum_{\substack{m_\ell, m_s \\ m'_\ell, m'_s}} \langle n\ell m | m_\ell m_s \rangle \left\langle m_\ell m_s \left| \frac{2\vec{L}.\vec{S}}{\hbar^2} + \frac{(L_z + g_s S_z) b}{\hbar} \right| m'_\ell, m'_s \right\rangle \langle m'_\ell, m'_s | n\ell m \rangle \tag{6}$$

where b is defined as follows :

$$b = B / B_0 \ , \tag{7}$$

with $B_0 = W / \mu_B$ where

$$W = \frac{\hbar^2}{2} \langle \varepsilon(r) \rangle = \mu_B^2 \frac{z}{4\pi\varepsilon_0 c^2} \left\langle \frac{1}{r^3} \right\rangle_{n\ell} \tag{8}$$

In the last equation z is represents the atomic number for atom.

The expectation value W is not defined for $\ell = 0$, because there is no spin- orbit interaction but instead of that we can use an energy contribution named the Darwin term. One can then show that for the hydrogen-like atom eigen functions are:

$$\left\langle \frac{1}{r^3} \right\rangle_{n\ell} = \frac{z^3}{a_0^3 n^3 \ell(\ell + 1/2)(\ell + 1)} \tag{9}$$

where a_0 is known as the Bohr radius, and thus

$$W = \frac{m_e c^2 (z\alpha)^4}{4n^3 \ell(\ell + 1/2)(\ell + 1)} \tag{10}$$

The magnetic field term,

$$\left\langle m_\ell m_s \left| \frac{L_z + g_s S_z}{\hbar} \right| m'_\ell m'_s \right\rangle = (m_\ell + g_s m_s) \delta_{m_\ell m'_\ell} \delta_{m_s m'_s} \tag{11}$$

The z-component is however diagonal, since the operators L_Z and S_Z correspond to the good quantum numbers m_ℓ and m_s of the basis eigenstates, so we can write it generally as:

$$\langle m_\ell | L_z | m'_\ell \rangle \langle m_s | S_z | m'_s \rangle = m_\ell m_s \hbar^2 \delta_{m_\ell m'_\ell} \delta_{m_s m'_s} \tag{12}$$

thus the total energy shift due to the spin -orbit interaction and the magnetic field is given by:

$$\Delta E_m^{\pm} = B\mu_B (m \pm 1/2) \tag{13}$$

Hence, without approximation, the energy eigenvalues are,

$$\Delta E_m^{\pm} = B\mu_B \left(m \pm \frac{g_s - 1}{2} \right) = \pm B\mu_B g_s / 2, \ (\ell = 0) , \tag{14}$$

Now we should also add the Darwin term (P. Forman, 1970),

$$\Delta E_D = \frac{m_e c^2}{2} \frac{(z\alpha)^4}{n^3} \tag{15}$$

Thus applying the magnetic field on positronium affects only the ortho-positronium. One can find that the energy levels are quantized (L. Shiff, 1968; D.I. Griffiths, 2005).

$$|E_n| = \frac{m_e e^2}{4\hbar^2 n^2} . \tag{16}$$

where n is known as the principal quantum number.

We note that each positronium energy level is half of the corresponding hydrogen-like atom energy level; for instance, the Ps ground state energy, for n =1, is E_1 = -6.8 eV = -0.25 atomic units (a.u.), compared to -13.6 eV for hydrogen.

Results and Discussions

By using Eq.(14) , we calculate magnetic energy correction (ΔE_m^{+}) when putting positronium atom under the effect of the weak magnetic field with rang (10 m Tesla -100m Tesla) which is called anomalous zeeman effect, supposing that spin direction of electron upward and downward as in Tabl.1, atomic unit $(\hbar = m_e = c = 1)$ used in magnetic energy correction (J. Shertzer *et al*, 198).

Table (1): Magnetic energy correction for various values of weak magnetic field when spin direction of electron upward and downward.

$B(mTesla)$	$\Delta E_m^{-} \times 10^{-8} (a.u.)$ For spin= - 1/2 downward	$\Delta E_m^{+} \times 10^{-8} (a.u.)$ For spin= 1/2 upward
10	2.1332611-	2.1332611
20	4.266522-	4.266522
30	6.3997833-	6.3997833
40	8.5330444-	8.5330444
50	10.666305-	10.666305
60	12.799567-	12.799567
70	14.932828-	14.932828
80	17.066089-	17.066089
90	19.19935-	19.19935
100	21.332611-	21.332611

Table (2): Magnetic energy correction for various of strong magnetic field when spin direction of electron upward and downward.

$B(Tesla)$	$\Delta E_m^{-} \times 10^{-5} (a.u.)$ For spin= - 1/2 downward	$\Delta E_m^{+} \times 10^{-5} (a.u.)$ For spin= 1/2 upward
25	-5.3331527	5.3331527
27.5	-5.866468	5.866468
30	-6.3997833	6.3997833
32.5	-6.9330986	6.9330986
35	-7.4664138	7.4664138
37.5	-7.999291	7.999291
40	-8.5330444	8.5330444
4205	-9.0663597	9.0663597
45	-9.5996749	9.5996749
47.5	-10.13299	10.13299
50	-10.666305	10.666305

In the same way by using Eq.(14) we calculate magnetic energy correction (ΔE_m^{+}) resulting by putting positronium atom under the effect of strong magnetic field with rarge (25 Tesla – 50 Tesla) which is called Paschen- Back effect if spin direction of electron upward and downward as in the Table (2):

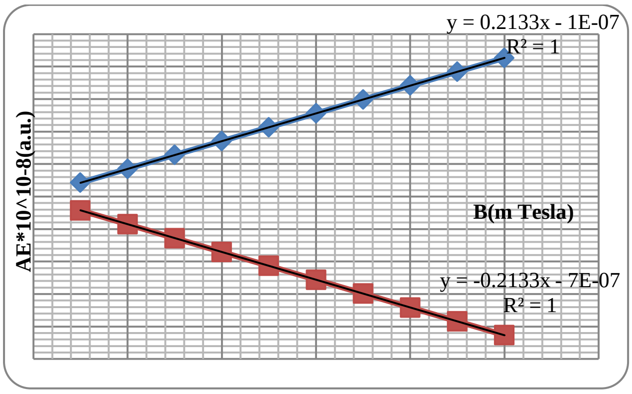
And also by using Eq.(3) we calculate energy correction that comes from spin- orbit energy which is equal to the $\Delta E_{fs.} = 1.3618673 \times 10^{-5} a.u.$ that is large as compared with

(ΔE_m^{+}) resulting from putting positronium atom in the weak magnetic field (anomalous Zeeman effect), its so because the orbiting electrons in the atom are equivalent to a classical magnetic gyroscope. The torque applied by the field causes the atomic magnetic dipole moment to precess around $\vec{B}$, an effect called Larmor precession. The external magnetic field therefore causes $\vec{J}$ to precess slowly about $\vec{B}$. Meanwhile, $\vec{L}$ and $\vec{S}$ precess more rapidly about $\vec{J}$ due to the spin-orbit interaction, where the speed of the precession about $\vec{B}$ is proportional to the field strength.

We can comparing energy correction $\Delta E_{fs.} = 1.3618673 \times 10^{-5} a.u.$, compared with (ΔE_m^{+}), that comes from putting positronium atom under the effect of the strong magnetic field (Paschen- Back effect), is small. That it is so because the criterion for observing Paschen-Back effect is that the interaction with the external magnetic field should be much stronger than the spin-orbit interaction: $\mu_B B_z >> \Delta E_{so.}$

If the satisfy this criterion, then the precession speed around the external field will be much faster than the spin-orbit precession. Also the term is Darwin correction term (ΔE_D) which represents the correction to the magnetic energy when $\ell = o$ (as in our case, positronium), and by using Eq.(15) this correction equal to $\Delta E_D = 5.3362969 \times 10^{-5} (a.u.)$.

The information in table (1) are summerise by the following figure when explains magnetic energy correction (ΔE_m^{+}) as a function for different values of the weak magnetic field (anomalous Zeeman effect) .



Fig(1): Magnetic energy correction as a function of weak magnetic field when s=±1/2

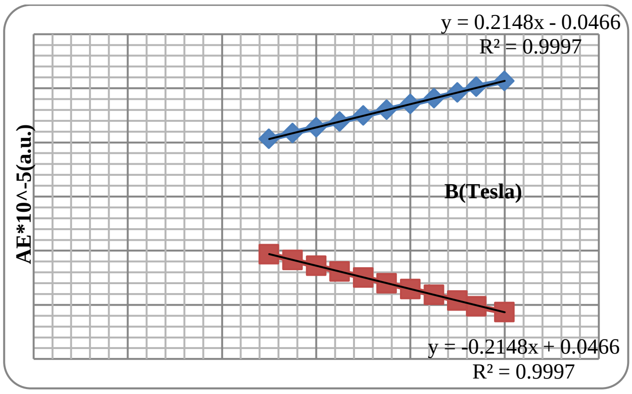
In this research and from **Fig (1)** we obtain the fit equation, which is representing empirical equation, gives the relation for magnetic energy correction (ΔE_m^{+}) as a function for values of the weak magnetic field as follows:

$$\Delta E_m^{+}(a.u.) = \left(0.2133B(mTesla) - 10^{-7}\right) \times 10^{-8}$$

$$\Delta E_m^{-}(a.u.) = \left(-0.2133B(mTesla) - 7 \times 10^{-7}\right) \times 10^{-8}$$

for anomalous Zeeman effect when electron spin upward and downward with $s = \pm 1/2$

By using table (2) we also draw the following **Fig.(2)** that explain magnetic energy correction (ΔE_m^{+}) as a function for values of the strong magnetic field (Paschen-Back effect) when $s = \pm 1/2$.



Fig(2): Magnetic energy correction as a function of weak magnetic field when $s = \pm 1/2$.

By using Fig.(2), we conclude empirical equation that gives the relation for magnetic energy correction (ΔE_m^{+}) as a function for values of strong magnetic field as shows:

$$\Delta E_m^{+}(a.u) = \left(0.2148B(Tesla) - 0.0466\right) \times 10^{-5}$$

$$\Delta E_m^{-}(a.u.) = \left(0.2133B(Tesla) + 0.0466\right) \times 10^{-5}$$

For Pachen-Back effect when electron spin upward and downward with $s = \pm 1/2$

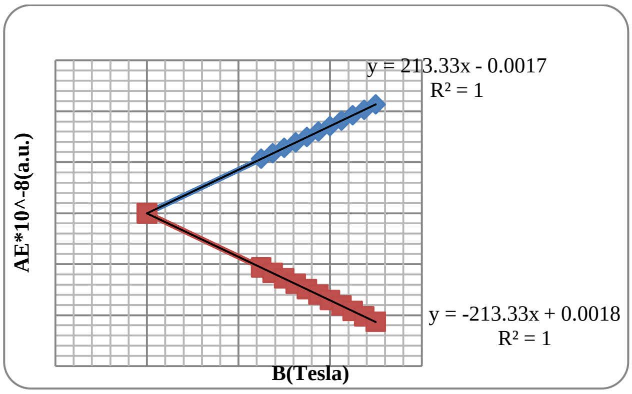


Fig (3): Explain together all magnetic fields in this research

We can also notes that magnetic energy values (ΔE_m^{+}) in Fig.(2), are large as compared with values (ΔE_m^{+}) in Fig.(1), that is resulted from the distance among energy levels increases. That comes from putting positronium atom under the effect of the strong magnetic field. Therefore, we can make use of from this research to estimate magnetic energy values as a function of the strong magnetic field in theoretical method, that we van not calculate it in laboratory because there is no sufficient of strong magnetic fields i.e. (B=10 Tesla) while there is sufficient of weak magnetic field as (10m Tesla -100m Tesla) when study behavior magnetic energy correction

($\Delta E_m^{\mp}$) of positronium atom for weak magnetic fields (anomalous Zeeman effect) and strong magnetic fields (Pashen-Back effect) together and in the same diagram (XY) as in **Fig.(3)**, thus the magnetic energy correction for strong fields larger than weak fields , this comparison explain the magnetic energy correction for weak fields very small as represent original point in level (XY) as **Fig.(3)** ,thus the magnetic energy correction for strong fields very important in calculation .

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