

## Research Article

## Dynamic Modeling, Optimal Control Design and Comparison between various control schemes of Home Refrigerator

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### Abstract

Several conventional methods present in refrigerators today consider varying actuators between its minimum and maximum values (Single speed control) or at several values between minimum and maximum values (Discrete speed control). The work done here presents an organized control design method based on appropriate modeling techniques to co-ordinate control inputs in order to achieve improvement over the conventional control schemes. A moving boundary lumped parameter model has been developed to describe the dynamics of heat exchangers. Single speed and Discrete speed control strategies are simulated with this dynamic model. A low order model has been established of the entire system integrating the compressor and expansion valve model which is in good agreement and reflects major dynamic characteristics of vapor compression cycle. LQR control has been applied by appropriate objective function formulation followed by simulation and comparison with other control strategies.

**Keywords:** Dynamic Modeling, Model Linearization, Optimal Control Design, Refrigerator system

### 1. Introduction

In a vapor compression cycle, the evaporator and condenser are two phase flow heat exchangers that interact with indoor or outdoor air respectively depending on its cooling application. There is an implicit relation between energy efficiency and the thermodynamic states of the refrigerant at various components in the system loop during its operation. The vapor compression cycle can be characterized by specific enthalpy, evaporating temperature, condensing temperature, superheat at evaporator outlet and sub-cooling at the condenser outlet. Sufficient amount of information regarding refrigerant behavior and thermodynamic properties can be obtained from it. A proper regulation of these states and dynamics can lead to energy efficient operations. For example, The compressor operation is related to superheat. Usually, the desired superheat is expected to be in between 4-8 deg. For values below this range, there are chances where the liquid refrigerant may enter the compressor and is not acceptable since it can cause damage. Contrary when the superheat is high, the compressor discharge temperature may increase due to excessively heated vapor and can result in lower energy efficiency.

Since several years cooling systems have been operated on a cyclic on-off manner for temperature regulation. It causes frequent startup and shutdown transients that affects the dynamics of the system

ultimately resulting into poor energy efficiency. The introduction of variable speed compressors to the vapor compression cycle has added great flexibility in operation of these systems (Den Braven, et al. 1993). The capacity of heat exchanger for cooling the load can continuously be adjusted by varying the compressor speed to match the actual load on the cooling system. Thus discontinuous control could be avoided and energy efficiency could be improved along with better temperature regulation and faster stability. Variable speed fans for evaporator and condenser are also used to have tighter control over the system dynamics. The fan speed variations vary the volume of air carried over the surface area of heat exchanger, in effect changes heat transfer coefficient between atmosphere and heat exchanger wall. Alterations are also caused in Coefficient of Performance (COP), superheat and quality sub-cool temperatures. Although the industry has not taken full advantage of these variable devices to gain substantial performance enhancement and improvement.

In order to exploit the behavior of the vapor compression cycle of the inputs on the system, the knowledge of the system behavior on application of these control signals must be known. For this, the model should be expressed in an explicit input-output relation. This paper shows a control oriented model design for vapor compression system used in refrigerators. A lumped parameter model based on moving boundary concept has been put forth. This non-linear model has been simulated with Single speed control and Discrete speed control

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strategies. The model has been linearized around a certain operating point and linear model is obtained. Comparison of non-linear model and linear model are shown. Optimal control design is proceeded on the basis of this linearized model with appropriate objective function formulation that tries to minimize error and control inputs.

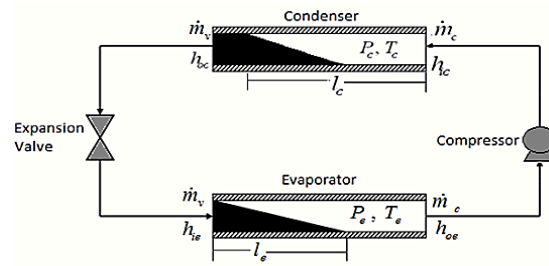
### A. Previous Work

There can be found various efforts in modeling which uses different modeling paradigms. A two phase model for predicting heat pump performance has been described in (MacArthur *et al*, 1989). It states that the modeling approach of heat exchangers can be classified in four groups namely: time constant, single node, moving boundary and multi node or spatially dependent models. The model incorporates dynamic characteristics of the system, however the complexity of the model is too high and hence cannot be used for any control design. A moving boundary approach for modeling reduces the complexity that was prevalent in spatial distribution model (DembaNdiaye *et al*, 2010). However the wall temperature was still modeled by spatial distribution approach. This again leads to complexity in terms of calculations while implementing it for control design. On-off control was introduced initially for operation of compressor in refrigerator. A dynamic model of the Vapor Compression cycle for Refrigerator systems has been developed in (Chetan Tulapurkar1 *et al*, 2010). Two separate feedback loops for control of compressor and expansion valve respectively are proposed. Based on concept of mean void fraction, transient flow problems have been simplified into lumped parameter system (Wedekind *et al*, 1978). This contribution is important as it helps describe the behavior of two-phase regions that occupy a major portion in heat exchangers. A formulation of a lumped parameter model for the vapor compression systems and development of a MIMO control for the same is described in (Xiang-Dong He *et al*).

## 2. Control Oriented System Modeling

As atomized refrigerant enters the evaporator it starts dissipating heat energy (coolness). Moving across the length of the evaporator there exists two regions which are classified on the basis of refrigerant state, two phase (or bi-phase) region and superheat region. Similarly the superheated refrigerant cools down in the condenser and as it moves across the length of the condenser in the direction of flow specific density increases and two regions namely, lumped superheat-two phase and sub-cool are present. The model of heat exchanger must be simple and yet be able to retain the dynamic characteristics of the system in order to attain the explicit goal of control system design. The lumped parameter model for evaporator and condenser has been derived on the basis of the concept of a transition point. The control oriented model is physics based lumped parameter model that characterizes individual model of each component and also interaction amongst these. Several assumptions have been made for

modeling of evaporator and condenser. The heat exchanger is considered as a long thin horizontal tube. The flow of refrigerant through the heat exchanger can be modeled as a one dimensional fluid flow.



**Fig. 1** Vapor compression cycle

The axial conduction of refrigerant is considered negligible. The refrigerant properties that characterize refrigerant behavior are obtained from a technical report[9] which calculates them using polynomial functions whose coefficients are calculated using polynomial function whose coefficients are fitted against experimental data[9]. The Fig. 1 shows a typical Vapor compression cycle (VCC) which is used for control oriented model development. For an evaporator, the energy balance for two phase and superheat regions could be given by equations (1) and (2) respectively.

$$a_{11e}\dot{P}_e + a_{12e}\dot{P}_e = \dot{m}_v(h_{ie} - h_{ge})U_{e1}l_{de}l_e(T_{fz} - T_e) \quad (1)$$

$$a_{21e}\dot{P}_e + a_{22e}\dot{P}_e + a_{23e}\dot{h}_{oe} = 0.5(\dot{m}_v + \dot{m}_c) + (h_{ge} - h_{oe}) + U_{e2}l_{de}(L_e - l_e)(T_{sh} - T_e) \quad (2)$$

The mass balance across the entire evaporator is given as:

$$a_{31e}\dot{P}_e + a_{32e}\dot{l}_e + a_{33e}\dot{h}_{oe} = (\dot{m}_v - \dot{m}_c) \quad (3)$$

The heat transfer coefficients for the two phase region in the evaporator ( $U_{ie1}$ ) are calculated using Kandlikar method [6], for superheat region ( $U_{ie2}$ ), it is calculated by Dittus-Bolter equation. The heat transfer coefficient for the air side of the evaporator ( $U_{oe}$ ) is calculated using Reynolds number, Nusselt number which vary depending upon the volume of air blown as per changes in fan speed. For condenser the energy balance for lumped superheat and sub-cool regions are given by the equations (4) and (5) respectively.

$$a_{11c}\dot{P}_c + a_{12c}\dot{l}_c = (\dot{m}_c h_{ic} - \dot{m}_c h_{lc})U_{c1}l_{dc}l_c(T_{amb} - T_c) \quad (4)$$

$$a_{31c}\dot{P}_c + a_{32c}\dot{l}_c + a_{33c}\dot{h}_{oc} = \dot{m}_v(h_{ic} - h_{oc}) + U_{c2}l_{dc}(L_c - l_c)(T_{amb} - T_{sc}) \quad (5)$$

The mass balance across the entire condenser is expressed as:

$$a_{21c}\dot{P}_c + a_{22c}\dot{l}_c = (\dot{m}_c - \dot{m}_v) \quad (6)$$

The heat transfer coefficient for the sub-cool region is given by Dittus-Bolter equation ( $U_{c2}$ ) while the lumped superheat side heat transfer coefficient ( $U_{c1}$ ) can be determined by Shah correlation. The air side heat transfer

coefficient is determined by Nusselt and Reynolds number of air that being function of rotational speed of the condenser fan. The flow rate of the refrigerant through the compressor can be determined by:

$$\dot{m}_{com} = \frac{N_c}{60} V_d \rho_{suc} \eta_v \tag{7}$$

where  $\eta_v$  is volumetric efficiency and is a function of suction ( $P_{suc}$ ) and discharge ( $P_{dis}$ ) pressures whose relation is:

$$\eta_v = 1 - \frac{1}{CR-1} \left[ \left( \frac{P_{dis}}{P_{suc}} \right)^{\frac{1}{n}} - 1 \right] \tag{8}$$

( $P_{suc}$ ) and ( $P_{dis}$ ) are expressed as functions of evaporator and condenser pressures, given as:

$$P_{suc} = P_e - \delta_1 P_e \tag{9}$$

$$P_{dis} = \delta_2 P_c \tag{10}$$

where  $\delta_1$  and  $\delta_2$  are lumped percentage drop across suction and discharge respectively. The compressor power delivered to the refrigerant is flow rate times the difference between suction and discharge enthalpies as shown in eqn(11).

$$P_{com} = \dot{m}_{com} (h_{dis} - h_{suc}) \tag{11}$$

The refrigerant mass flow rate through the expansion valve is given by:

$$\dot{m}_v = C_v a_v \sqrt{\rho_v (P_c - P_e)} \tag{12}$$

During the expansion process there is a considerable pressure drop and enthalpy change is assumed to be negligible, i.e.:

$$h_{ie} = h_{oc} \tag{13}$$

The energy balance across the freezer compartment gives the rate of change of freezer temperature.

$$Q_{leak} + Q_{food} + Q_{air} = Q_e \tag{14}$$

Also the rate of change of  $T_{food}$  can be expressed as:

$$T_{food} = \frac{q_{food}}{m_{food} c_{food}} \tag{15}$$

The total dynamic model is then formulated into non-linear state space form after simplification denoted as:

$$\dot{x} = f(x, u) \tag{16}$$

$$y = g(x, u) \tag{17}$$

where  $x = [P_e P_c l_e h_{oe} T_{fz} T_{foodfz} T_{dis}]^T$  is state vector,  $u = [N_{com} N_{evap} N_{cond} q_{load} T_{amb}]^T$  is input vector while  $y = [T_{fz} T_{sh}]$  is output vector.

### 3. Model Linearization

Linearizing the non-linear systems can be done by Taylor series expansion and on knowledge of nominal system trajectories and nominal system inputs. Consider a simple scalar first-order nonlinear dynamic system (Lebrun, J. et al) given by:

$$\dot{x} = F(x(t), f(t)) \tag{18}$$

Under usual working conditions the following differential equation is satisfied, wherein  $x_n(t)$  is nominal state trajectory and  $f(t)$  is nominal state input.

$$\dot{x}_n(t) = F(x_n(t), f_n(t)) \tag{19}$$

Assuming that the motion of the nonlinear system is in the neighborhood of the nominal system trajectory,

$$x(t) = x_n(t) + \delta x(t) \tag{20}$$

where  $\delta x$  is very small. So it is acceptable that system motion in close proximity to the nominal trajectory will be sustained by a system input which is obtained by adding a small quantity to the nominal system input.

$$f(t) = f_n(t) + \delta f(t) \tag{21}$$

$$\dot{x}_n(t) + \delta x(t) = F(x_n(t) + \delta x(t), f_n(t) + \delta f(t)) \tag{22}$$

On expansion by Taylor series about the nominal system trajectory and input, we get a simplified form after ignoring the negligible higher order terms.

$$\delta \dot{x}(t) = \frac{\partial F}{\partial x}(x_n, f_n) \delta x(t) + \frac{\partial F}{\partial f}(x_n, f_n) \delta f(t) \tag{23}$$

The linearized system then can be represented as:

$$\delta \dot{x}(t) + a_0(t) \delta x(t) = b_0(t) \delta f(t) \tag{24}$$

A more generalized expression for the same is,

$$\dot{x} = Ax(t) + Bu(t) \tag{25}$$

This method has been implemented numerically where the system is linearized around the steady state by giving perturbations of 1% of their maximum values. Thus for finding matrices A and C the model is evaluated at  $x_0$  (steady state) to obtain  $\dot{x}$  and then the model is evaluated at perturbed condition  $x_0 + dx$  to obtain  $\dot{x}_p$  keeping  $u_0$  constant.

$$A = \frac{\dot{x}_p - \dot{x}}{dx} \ \& \ C = \frac{y_p - y}{dx} \tag{26}$$

Similarly for B and D model is evaluated at  $u_0$  and  $u_0 + du$  to obtain  $\dot{x}$  and  $\dot{x}_p$  respectively at constant  $x_0$ .

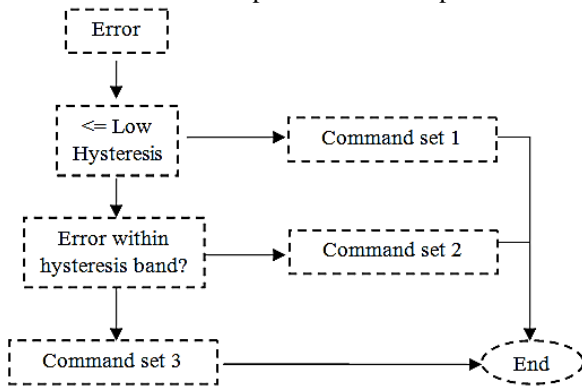
$$B = \frac{\dot{x}_p - \dot{x}}{du} \ \& \ D = \frac{y_p - y}{du} \tag{27}$$

Comparison of non-linear model and linearized model simulation is shown in Fig. 6. Comparison results show that dynamic characteristics of non-linear model are also reflected in linearized model and are in good agreement with each other.

**4. Control Schemes**

*A. Single Speed Control*

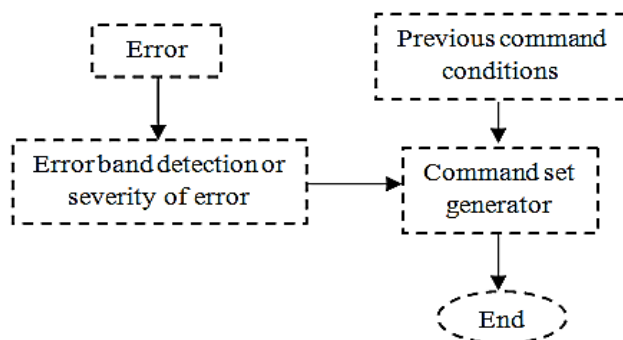
The actuators that act as control inputs to the system are speeds of compressor, condenser fan and evaporator fan. In Single speed control strategy, the actuators are in cyclic on-off mode. The actuators can either be on (running at rated speed) or off. As shown in the Fig. 2, the command set generation takes place on the basis of hysteresis band defined. If the error is within the hysteresis band, the command set latches and hence previous conditions are maintained while for error outside the hysteresis band command sets are generated respectively. Single speed control is evaluated for performance comparison.



**Fig. 2** Single speed control

*B. Discrete Speed Control*

This control scheme can regulate the compressor and fans at various speeds in a discrete manner according to the thermal load. This is implemented by dividing the entire error range in several bands. These error bands define severity of error. The larger the deviation of output from set-point, the more severe is the error. However the control command generated is not only on the basis of severity of error but also on previous command conditions.



**Fig. 3** Discrete speed control

The simplest form of block diagram for the same is shown in Fig. 3. Discrete speed control is simulated further in the study and is the baseline.

*C. Optimal Control Design*

A control scheme based on LQR (Linear Quadratic Regulator) design is proposed that improves the performance over the existing control method. LQR is based on linear model of the system. The model has been linearized as explained in previous section around the operating point  $T_{fz}=0$  deg F and  $T_{sh}=3$  deg F. The outputs to be controlled are  $y = [T_{fz} T_{sh}]^T$ . Both of these should track the desired temperature setpoints  $y_{sp} = [T_{fzsp} T_{shsp}]^T$ . An integrator is added to the system which eliminates the steady state error of the closed loop system. Augmenting the integrator to the linear system expressed in (25) we get,

$$\dot{x}_a = A_a x_a + B_a \tilde{u} \tag{28}$$

$$y_a = C_a x_a \tag{29}$$

Where  $x_a = \begin{bmatrix} \tilde{x} \\ \zeta \end{bmatrix}$ ,  $y_a = \begin{bmatrix} \tilde{y} \\ \zeta \end{bmatrix}$

$$A_a = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}, C_a = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}$$

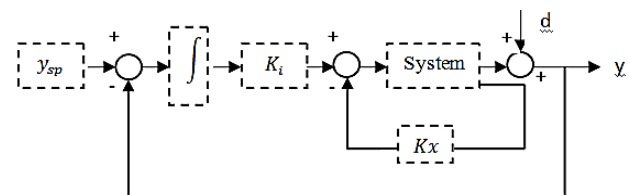
The performance index is,

$$J = \int (y_a^T Q y_a + \tilde{u}^T R \tilde{u}) \tag{30}$$

A control law  $\tilde{u} = -Kx_a$  that minimizes the performance index (30) is obtained. The optimal control command could be denoted as the sum of state and integral error feedbacks multiplied by their respective gains.

$$u = -K_x \tilde{x} - K_i \zeta \tag{31}$$

Fig. 4 shows the block diagram of LQR control for the augmented system that involves a combination of state and output feedback control.



**Fig. 4** Block diagram of LQR control for the system

**5. Results**

The results have been normalized and show a comparison between linear and non linear model from which is evident that the linear model retains the dynamics of the vapor compression system. The Single speed control takes into

account the maximum compressor speed while the control command is 'on' and zero speed when the command is 'off'. Due to such nature of control command, we find that the results are oscillating.

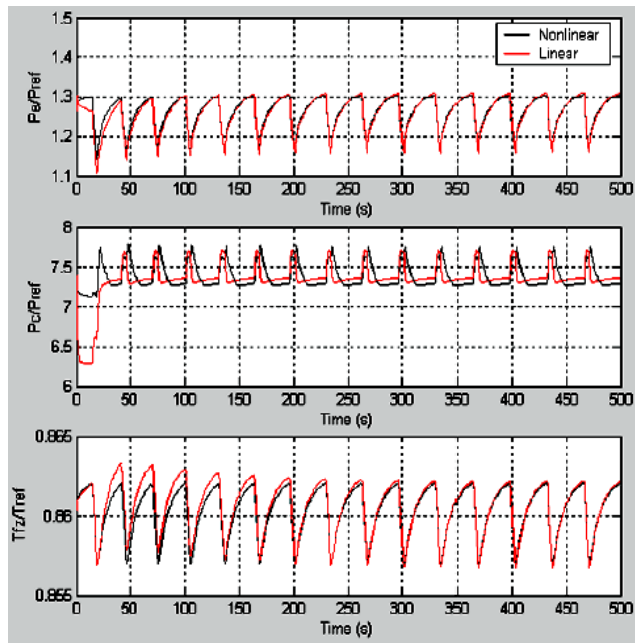


Fig. 6 Non linear vs. Linear model

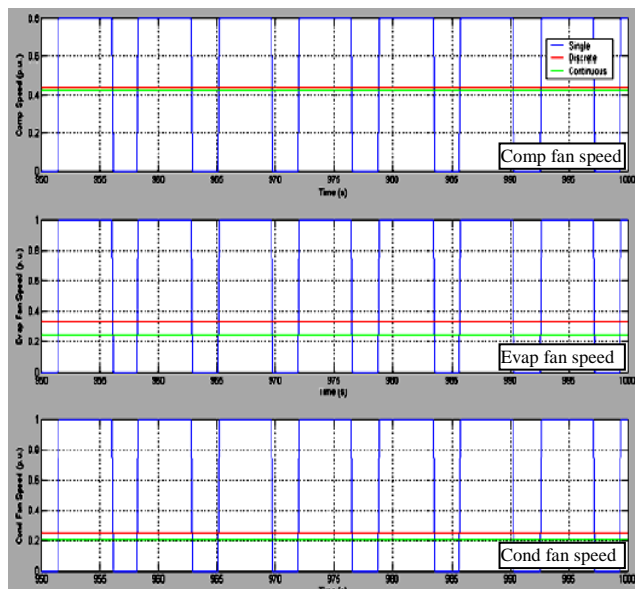


Fig. 7 Fan speed results (p.u.)

In Discrete speed control the speeds of evaporator fan, condenser fan and compressor fan take up values in steps of 33%, 66%, 100% of maximum speed in accordance with the algorithm. The simulation of LQR is done by tuning values for matrices Q and R taken as  $Q = \text{diag}([1.01 \ 1 \ 1])$   $R = \text{diag}([6.08 \ 1 \ 1])$ . Improved results are expected by extensive tuning of Q and R matrices. It is a difficult task, however the best simulation results so achieved during the course of work are shown in this section.

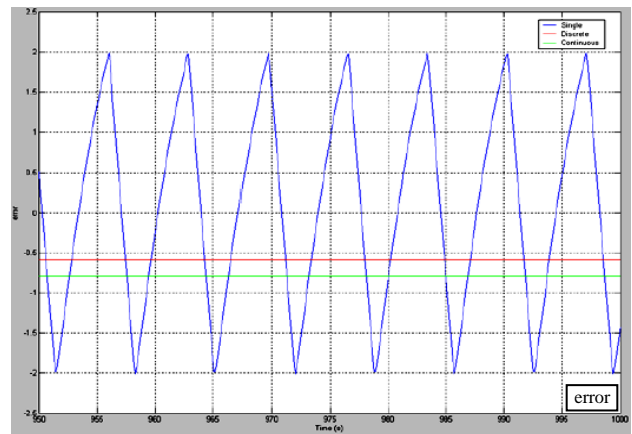


Fig. 8 Freezer temperature error

It has the greatest capability to manoeuvre the system to steady state. The compressor, evaporator fan and condenser fan speed values are normalized. Hence the graph is per unit speed vs time. The evaporator and condenser fan speeds are weighted equally while compressor speed is weighted more. From the graphs we can see that compressor speed in Continuous control has lower speed than Discrete control. Since compressor consumes most of power during the operation of the system, a minor improvement of speed can result in much larger energy savings. The compressor consumes about 90% of the total energy in vapor compression cycle.

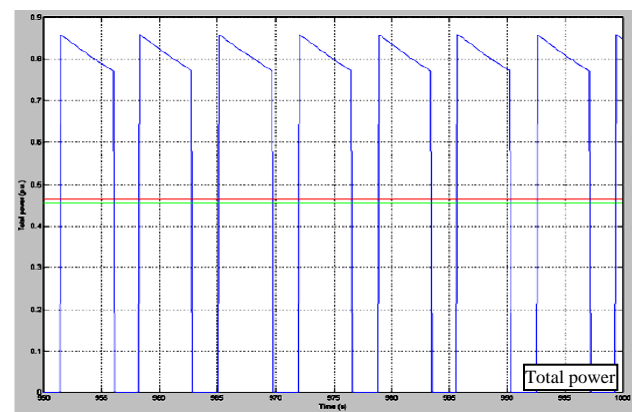


Fig. 9 Total Power (p.u.)

The freezer temperature error is plotted in which the error oscillates within the hysteresis band in Single Speed control. However the error is marginally less in discrete speed control rather than continuous control. The improvement are expected by fine tuning of weighting matrices. The total power consumption as shown below of continuous control is lesser than discrete control. The power consumed by continuous control is lesser by around 2 watts at every time instant as compared to baseline. The simulation was carried out with a constant load of 30 watts. A larger energy saving is expected for varying loads. To simulate that several linearized models around different operating points are required.



## Conclusions

A dynamic model for a Vapor compression cycle has been established. The model is linearized at a certain operating point and a linear model is obtained. As we can see from the results the linearized model retains the dynamics of the non-linear model and is in good agreement. The model formulated is evaluated for three different control strategies viz. Single speed control, Discrete Speed control and the proposed optimal control strategy. The Optimal control strategy offers tighter control over the system dynamics by offering flexibility for speed control over the entire range. It can be seen from the results that the overall energy consumption with continuous control is lesser by 2 watts than the baseline discrete control. Better results are expected by rigorous tuning of Q and R matrices. Besides that, for dynamic load conditions and with the availability of linear models at different operating points, the Optimal control strategy can prove to be even more advantageous.

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## Nomenclature

- $P_s$  Saturation pressure  
 $T_s$  Saturation temperature  
 $\rho_g$  refrigerant mass density at saturated gas  
 $\rho_l$  refrigerant mass density at saturated liquid  
 $\gamma$  MVF  
 $h_{ie}$  specific enthalpy at evaporator inlet  
 $\dot{m}_v$  mass flow rate across expansion valve/at inlet of evaporator  
 $h_{oe}$  specific enthalpy at evaporator outlet  
 $\dot{m}_{com}$  mass flow rate of compressor/evaporator outlet  
 $\dot{m}_i$  mass flow rate at the end of two phase region  
 $h_{ge}$  specific enthalpy of gas/at the end of two phase region in evaporator  
 $l_e$  length of two phase region in evaporator  
 $L_e$  Total length of evaporator  
 $U_{e1}$  Heat transfer coefficient of two phase region  
 $U_{e2}$  Heat transfer coefficient of superheat region  
 $\rho_{le}$  specific density of liquid refrigerant in evaporator  
 $\rho_{ge}$  specific density of gaseous refrigerant in evaporator  
 $z_1; z_2$  Time varying limits  
 $a_{ij}, b_{ij}$  Coefficients obtained by modeling  
 $D_{ie}$  Inner diameter of the hollow pipe of evaporator  
 $D_{oe}$  Outer diameter of the hollow pipe of evaporator  
 $T_{fz}$  Freezer temperature  
 $T_{she}$  Superheat temperature  
 $T_e$  Evaporator temperature  
 $P_e$  Evaporator pressure  
 $A_c$  cross sectional area of condenser  
 $h_{ic}$  specific enthalpy at condenser inlet  
 $h_{oc}$  specific enthalpy at condenser outlet  
 $h_{ic}$  specific enthalpy at the end of lumped superheat-two phase region  
 $l_c$  length of two phase region in condenser  
 $L_c$  Total length of condenser  
 $U_{c1}$  Heat transfer coefficient of lumped superheat-two phase region  
 $U_{c2}$  Heat transfer coefficient of subcool region  
 $U_c$  Overall heat transfer coefficient of condenser  
 $D_{ic}$  Inner diameter of the hollow pipe of condenser  
 $D_{oc}$  Outer diameter of the hollow pipe of condenser  
 $T_c$  Condenser temperature  
 $T_{amb}$  Ambient temperature  
 $P_c$  Condenser pressure  
 $m_e$  Mass of refrigerant inside the evaporator  
 $m_c$  Mass of refrigerant inside the condenser  
 $m_{ref}$  Total refrigerant mass  
 $x$  States of system  
 $N_{evap}$  Evaporator fan speed  
 $N_{cond}$  Condenser fan speed  
 $N_{com}$  Compressor speed  
 $T_{fzsp}$  Freezer temperature setpoint  
 $T_{shsp}$  Superheat temperature setpoint  
 $K_x$  state feedback gain  
 $K_i$  integral gain

## Abbreviations

- COP Coefficient of Performance  
MVF Mean Void Fraction