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Research Article

Resolution of Heat Transfer Problems by Using a New Approach: Finite Windows Methods

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Abstract

This work presents a new approach of resolution of the Partial Differential Equations (PDE), which makes it possible to solve problems defined by complex PDEs. We named this method "Finite Windows Methods" (FWM). This method is adapted to the complex problems such as the modeling of the multiphasic phenomena of transfer in porous environments (drying, flows in an oil tank, ...), which utilizes conservation PDEs (mass, energy, momentum) generally nonlinear, and of mechanical balance in the porous matrix which becomes deformed under the effect of the transfers. The first part of this work consists of presenting and describing the method in general. Then the method is tested on a simple case of transfer of heat through a wall.

Key words: *Complex, Modeling, Transfer multiphasic, Differential equations, Porous matrix*

Introduction

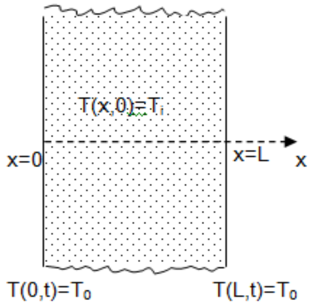
The convective drying of a food product is optimal when the drying conditions (velocity, temperature, air moisture…) are well defined (Ertekin and Yaldiz, 2004, Edoun *et al*., 2010, Takamte *et al*., 2013). However, when designing a drier, their compartments must be accurately dimensioned to meet the operational requirements (Yang *et al.*, 2009). It becomes important for designers to have a tool for numerical simulation of the heat and mass transfer phenomena which take place during such operations. But to reach that goal, it is necessary solve very complex PDE systems taking into account the characteristics of the concerned products. Two types of complexity are then faced: the non linearity and a strong coupling between the fluid and the mechanical structure.
In the literature, these equations are generally solved using the finite differences or finite volumes methods with enormous calculations and many approximations (Nougier, 1981, Risser, 2006, Leveque, 2007). Currently, one of the best methods to resolve those problems are the Finite Elements Methods (FEM). Here the concerned domain will be divided into finite elements defined by a basic geometry and defined interpolation functions for unknown quantities or geometry (Iso-parametric elements). Continuity is being ensures by the pooling of the elements, the final discrete equations to solve will then comprise a very large number of variables that usually leads to storage and numerical conditioning problems. It will then be a question of solving an enormous system of N equations with N unknowns. A judicious choice of interpolations functions or calculation points for the coefficients of equations (spectral elements) can result in improving the stability and accuracy of the solution of these equations. But generally the Finite Element Method is still quite limited.

In this work we have attempted to formulate the "Finite Windows Methods" (FWM). It will consist to relaxation loops carried out on a domain, in order to ensure a good numerical stability at each step. Throughout this document we will describe the methodology of the method.

Materiel and Method

Simple Example

The problem considered here is very simple: the heat conduction in a flat plate of length L. Its transversal dimension is supposed to be infinite, hence leading to a one dimensional heat flow.



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Given the initial and boundaries conditions shown on the figure above, the equation to be solved is:

$$\begin{cases} \dfrac{\partial T}{\partial t} - a\dfrac{\partial^2 T}{\partial x^2} = 0 \\ T(0,t) = T(L,t) = T_0, \quad t > 0 \\ T(x,0) = T_1, \quad \forall x \in [0, L] \end{cases} \qquad (1)$$

Where a the thermal diffusivity of the plate is supposed constant.

Exact analytic solution

Many methods exist for solving analytically (1). We used the separation of variables method, as it seemed to be easier and more efficient to be implemented. This method leads to the following solution:

$$T(x,t) = T_0 + 4\frac{T_i - T0}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}e^{-(2n+1)^2\pi^2\frac{at}{L^2}}\cos\left(\frac{(2n+1)\pi(x-0.5L)}{L}\right) \quad (2)$$

In this work, this exact solution has been computed with a precision of 10^{-3}, and been validated by a quick finite differences computation.

Solution using the FWM

Let consider the following grid in the Time-Space domain.

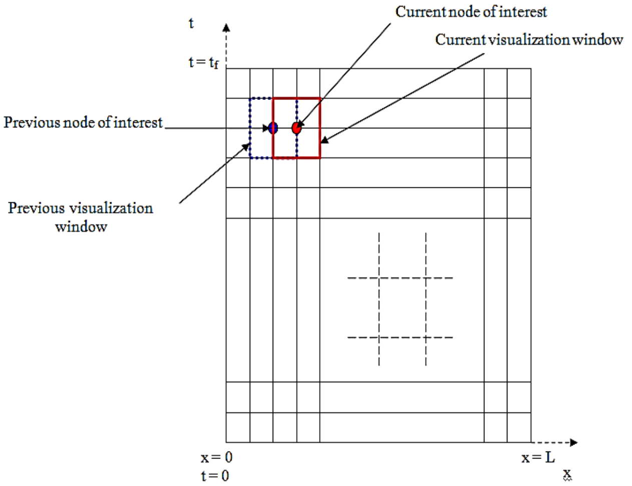

Figure 1: The time-space grid for the FWM.

The FWM principle we are going to use here is as follow. At each time, a given node will be seen through a given visualization window. Then, its Temperature will be updated according to an objective function defined in the window. After it, we move to another node and repeat the same procedure, and so on until we got a predefined convergence. The displacement scheme over the nodes will consist of moving towards increasing times, and for a given time towards increasing x coordinates. Here, we will call "a pass" a displacement between nodes that covers all the nodes of the grid. The convergence of this algorithm will be judged versus the number of passes.

Now let's make the formulation of the visualization window.

The Window Element (WE) is as shown in the figure 2. The interpolation will be made using Lagrange interpolating functions.

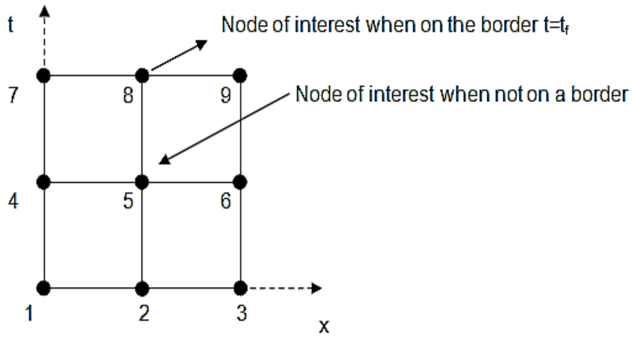

Figure 2: The time-space Window Element used.

Let call $N_j = N_j(x,t)$ the interpolation functions. Given the T_j temperatures at the nodes, the temperatures in the element can be approximated by the following polynomial function:

$$\tilde{T} = \sum_{j=1}^{9} N_j T_j \qquad (3)$$

or $\tilde{T} = N_j T_j$,

using the Einstein's notation.

Putting $\tilde{T}$ in the equation to be solved, and using the notation $\dfrac{\partial f}{\partial \alpha} = f_{,\alpha}$, we come out with the residue

$$R_0 = N_{j,t}T_j - aN_{j,xx}T_j = A_j T_j \qquad (4)$$

(with $A_j = N_{j,t} - aN_{j,xx}$)

whose absolute value has to be minimized in each point of the window. For it, we will use a least squares method. Hence, let define:

$$R_1 = \int_W R_0^2 dW \qquad (5)$$

If T_i is the temperature at the node of interest in the window, then its best value has to verify:

$$\frac{\partial R_1}{\partial T_i} = \int_W \frac{\partial}{\partial T_i}\left[(A_j T_j)^2\right] dW = 0 \qquad (6)$$

Which leads to $\int_W A_i A_j T_j dW = P_{ij} T_j = 0$, with $P_{ij} = \int_W A_i A_j dW$

Finally, a better temperature for the node of interest is:

$$T_i = -\frac{\sum_{j \neq i} P_{ij} T_j}{P_{ii}} \qquad (7)$$

Equation (7) will use for the updating process in windows. The boundaries conditions here are fixed ones. Thus they will be taken into account just by not moving on boundary nodes.

Solution using two classic FEMs

We have also solved this problem by a classic FEM approach: elementary formulation, assembling, boundary conditions, linear system solving. Two types of elementary formulations have been used using the same type of element as the above window element: least squares and Galerkine projection method. The boundaries conditions have been taken into account by the diagonal penalization technique.

Results

The space-time grid is characterized by the following parameters

- N_x, N_t the number of nodes respectively in the space axis and the time axis.
- Δ_x, Δ_t the spacing between nodes respectively in the space axis and the time axis.
- $L=(N_x-1)\Delta_x$ the total length of the plate.
- $t_f=(N_t-1)\Delta_t$ the final time of the simulation.

The value of the four cases tested in the present work are presented in Table 1.

Let call $T_{i,j}$ the temperature at the node located i^{th} space value and j^{th} time value, and $\hat{T}_{i,j}$ the exact temperature at this node. The calculations will be conducted on four sets of parameters. Many passes will be done for each set, and the convergence at each pass will be characterized by the following vector:

$$E_{j-1}=\underset{i}{Mean}\left[Abs(\hat{T}_{i,j}-T_{i,j})\right],\quad j\geq 2$$ (fixed boundary condition at $j=1$)

which expresses the proximity of the solution to the exact one at each time step.
The following graphs ($E_j=f(j)$) show the results we have got. The results using the Galerkine projection method was quite far from the exact solution, proving that this formulation is not appropriate here at all. The following parameters have been used in all the examples:

$$\begin{cases} T_0=300\,K \\ T_i=288\,K \\ a=0.83\cdot 10^{-3}\ m^2/s \end{cases}$$

a) *Case 1 result*

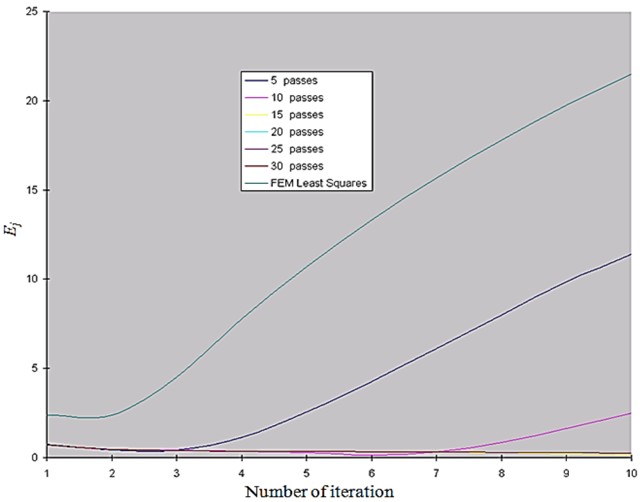

Figure 3: Convergence evolution at each time step with case 1 space-time parameters

Table 1: Parameters values of four cases

Value of space-time parameters						
	Δ_x	Δ_t	N_x	N_t	L	t_f
Case 1	0.1	120	11	11	1m	1200s
Case 2	0.1	120	11	11	1m	2400s
Case 3	0.1	60	11	11	1m	600s
Case 4	0.05	120	21	11	1m	1200s

With the given grid parameters, the classic FEM gives results further from the exact solution as the time increase. We have the same behavior with FWM for small numbers of passes. But a good precision as well as the convergence is obtained quickly by increasing the number of passes. Here 15 passes are enough for a good convergence.

b) *Case 2 result*

Here we have increase the final time, keeping the same time step of 120s. What we can notice is that more passes are needed for convergence.

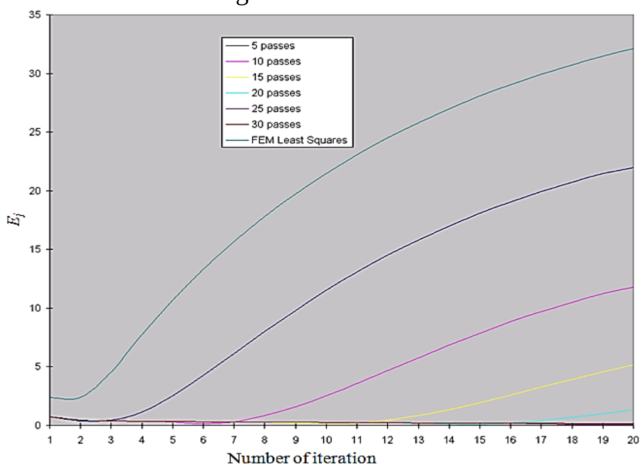

Figure 4: Convergence evolution at each time step with case 2 space-time parameters

Case 3result

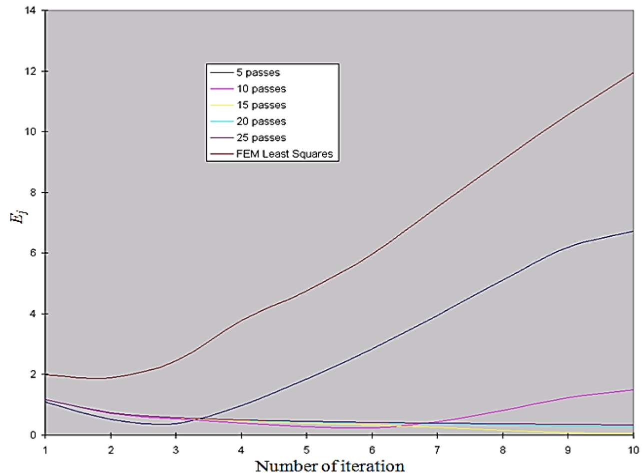

Figure 5: Convergence evolution at each time step with case 3 space-time parameters

Results of test 4

Here, the time parameters are unchanged compared to the first example. The length of the plate is still 1m, but there are twice more space nodes. It appears that more passes are needed to achieve a good convergence. It was the same result in the second example where we doubled the number of time nodes as well as the final time.

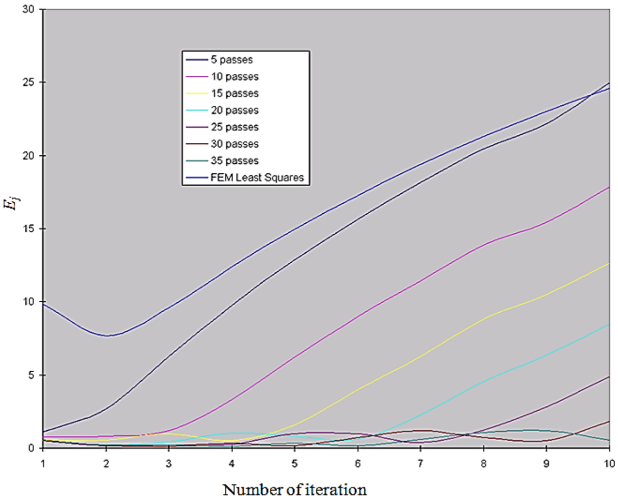

Figure 6: Convergence evolution at each time step with case 4 space-time parameters

Here we have keep the same number of nodes, reducing the time step. First, we can see that the classic FEM precision with respect with the time hasn't changed. Secondly if we consider the nodes numbers, the convergence rate is nearly the same, which means that E_j depends mainly of j than Δ_t. Obviously as the time step Δ_t has decreased, this leads to a reduced convergence rate with respect with the time.

Conclusions

The primary result here is that the FWM is an effective concept, and is even capable of solving problems in contexts were the classic FEM fails. The examples conducted above show that for this problem, contrary to what could be thought, increasing the number of nodes decreases the convergence speed for a small increase in precision. We can't not generalize this result because this fact mainly depends on the properties of the problem, the interpolation functions used and the windows geometry. The FWM also has some advantages regarding the solving process: no heavy matrix memorization or meshing technique are required and the updating equation is well scaled, as it concerns only terms defined in the window, which might be of the same order of magnitude. After this work, the next one will be the solving of a subsonic fluid flow problem leading to a set of non-linear PDEs. Also, we can start looking at some other aspects:

- What are the convergence conditions for a given problem, according to the window formulation and the domain discretisation?
- What could be the architecture of a multiple processor parallel machine capable efficiently implement the FWM?

References

Edoun M., Kuitche A., Marouzé C., Giroux F., Kapseu C. (2010). Etat de la pratique du séchage de quelques fruits et légumes à petite échelle dans le Sud Cameroun. Fruits, 66(1), 1-12

Ertekin C., Yaldiz O. (2004) Drying of eggplant and selection of a suitable thin layer drying model. *Journal of Food Engineering*, 63, 349-359

Leveque R. J., (2007). Finite Difference Methods for Ordinary and Partial Differential Equations. *SIAM, Philadelphia.*

Nougier J.P. (1981). Méthodes de Calcul Numériques. *J. Heat Transfer*, 4, 409-425.

Risser L. (2006). Différences finies pour la résolution numérique des équations de la mécanique des fluides. Support de cours

Takamte G., Edoun M., Monkam L., Kuitche A., Kamga R. (2013). Numerical Simulation of convective Drying of Mangoes (mangifera Indica L.) Under Variable Thermal Conditions , *International Journal of Thermal Technologies*, 3 (2), 48-52

Yang H., Zhou W., Lou C. (2009). Optimal design and techno-economic analysis of a hybrid solar–wind power generation system, *Applied Energy*, 86, 163-169