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Research Article

A Common Fixed Point Theorem on L-Fuzzy Metric Space by Biased Mappings of Type R_m

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Abstract

As already aware of the concepts of compatible mappings, compatible mappings of type α, compatible mappings of type β and weakly compatible, we are proving a common fixed point theorem in L-fuzzy metric space on four self-mappings using the concept of Biased Mappings of Type (R_M) and the property C with a known fact that if a pair of maps is compatible or compatible of type α or compatible of type β then it is biased mapping of both type. This result could be further used to prove result on three self-mappings.

Keywords: *Fixed point, L-fuzzy metric space, Biased Mappings, Property C, Mathematics Subject Classification: 46S40, 54H25, 54E35*

1. Introduction

As it is well known to every mathematician working on fixed point theory that the notion of fuzzy sets was introduced by L.A. Zadeh (1965) .Various concepts of fuzzy metric space were introduced Z.K.Deng(1982), M.A.Erceg(1979) O. Kaleva et al.(1984), I.Kramosil et al.(1975) and fixed point theorems were proved as in S.S.Chang et al.(1997), Y.J.Cho et al.(1998), J.Goguen(1967), V.Gregori et al.(2002).Using the idea of $\mathcal{L}$-fuzzy sets J.Goguen (1967) , R. Saadati et al. (2008, 2008, 2006) introduced the notion of $\mathcal{L}$-fuzzy metric space with the help of continuous t- norm and proved fixed point theorems for φ -weakly commuting maps and weak compatible mappings. H.Adibi et al. (2006), Rajesh Shrivastava et al. (2011) also proved their results on $\mathcal{L}$-fuzzy metric space for compatible mappings of type (P) and weak compatible mappings respectively. Here we are proving the fixed point theorem on L-Fuzzy Metric Space by Biased Mappings of Type (R_M) the concept introduced by M.R.Singh at al.(2009) on four self-mappings.

2. Preliminaries

Def.2.1. Complete Lattice is a partially ordered set (L,≤) in which all subsets have both supremum (join) and infimum (meet).

Def.2.2. J. Goguen (1967) Let $\mathcal{L}$= (L,$\leq_L$) be a complete lattice and U a non-empty set called universe. An $\mathcal{L}$-fuzzy set A on U is defined as a mapping A: U→ L. For each u in U, A (u) represents the degree (in L) to which u satisfies A

Lemma 2.3. G. Deschrijver et al.(2003) Consider the set L* and operation$\leq_{L^*}$defined by L*= $\{(x_1, x_2)$; $(x_1, x_2)\in [0,1]^2$and $x_1 + x_2 \leq 1\}$
$(x_1, x_2) \leq_{L^*} (y_1, y_2)$ if and only if $x_1 \leq y_1$ *and* $x_2 \geq y_2$for every (x_1, x_2),$(y_1, y_2)\in$ L*.Then(L*,$\leq_{L^*}$)is complete Lattice.

Def.2.4. A.T. Atanassov (1986) An intuitionistic fuzzy set $A_{\zeta,\eta}$on a universe U is an object $A_{\zeta,\eta}$= $\{(\zeta_A(u), \eta_A(u); u \in U\}$ where for all u∈ U, $\zeta_A(u) \in[0, 1]$ and $\eta_A(u) \in [0, 1]$ are called the membership degree and the non-membership degree respectively of u in $A_{\zeta,\eta}$ and furthermore satisfy $\zeta_A(u) + \eta_A(u) \leq 1$.

A triangular norm Ton ([0, 1],≤) is defined as an increasing, commutative, associative mappings
T : $[0, 1]^2$ →[0, 1] satisfying T (1, x) = x ∀x∈ [0, 1].
We define first$0_{\mathcal{L}} = \inf L$.
$1_{\mathcal{L}} = \sup L$.

Def.2.5. A triangular norm (t-norm) on $\mathcal{L}$ is a mapping $\mathcal{J}$: L^2 →L satisfying the following conditions:

i. (∀x∈L) ($\mathcal{J}$(x, $1_{\mathcal{L}}$)= x); (boundary condition);
ii. (∀ (x,y) ∈ L^2) ($\mathcal{J}$ (x, y) = $\mathcal{J}$(y, x)); (commutativity);
iii. (∀ (x, y, z) ∈ L^3) ($\mathcal{J}$ (x, $\mathcal{J}$ (y, z)) = $\mathcal{J}$ ($\mathcal{J}$(x, y), z) ; (associativity);
iv. (∀ (x, x′, y, y′) ∈ L^4) (x$\leq_L$ x′ and y $\leq_L$ y′)⇒$\mathcal{J}$(x, y)$\leq_L\mathcal{J}$(x′, y′) (monotonocity);

A t- norm can also be defined recursively as an (n+1)-ary operation (n ∈N\{0}) by $\mathcal{J}^1$= $\mathcal{J}$ and

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$\mathcal{J}^n(x_{(1)}, x_{(2)}, \ldots\ldots\ldots\ldots, x_{(n+1)})=\mathcal{J}(\mathcal{J}^{n-1}(x_{(1)}, x_{(2)}, \ldots\ldots, x_{(n)}), x_{(n+1)}))$ for $n \geq 2$ and $x_{(i)} \in L$.

Def.2.6. G.Deschrijver et al.(2004)A t -norm on L^* is called t-representable iff there exists a t-norm T and t-co norm on S on [0, 1] such that $\forall$ x = (x_1, x_2), y= $(y_1, y_2) \in L^*$.

$\mathcal{J}(x, y) = \{T(x_1, y_1), S(x_2, y_2)\}$.

Def.2.7. A negation on $\mathcal{L}$ is any decreasing mapping N: L→L satisfying N $(0_{\mathcal{L}})=1_{\mathcal{L}}$and N $(1_{\mathcal{L}})=0_{\mathcal{L}}$. If N (N(x)) =x $\forall$ x ∈ L, then N is called an involutive negation.

If for all x ∈[0, 1], $N_s(x)$ = 1-x, we say that N_s is the standard negation on ([0, 1],≤).

Def.2.8. The 3-tuple (X, $\mathcal{M}$, $\mathcal{J}$) is said to be an $\mathcal{L}$-fuzzy metric space if X is an arbitrary (non-empty)set, $\mathcal{J}$ is continuous t-norm on $\mathcal{L}$ and $\mathcal{M}$ is an $\mathcal{L}$-fuzzy set on $X^2 \times(0, +\infty)$satisfying the following conditions for every x, y, z in X and t, s in (0, +∞)

i. $\mathcal{M}(x, y, t) >_L 0_{\mathcal{L}}$;
ii. $\mathcal{M}(x, y, t) = 1_{\mathcal{L}}$for all t>0 if and only if x = y;
iii. $\mathcal{M}(x, y, t)= \mathcal{M}(y, x, t)$;
iv. $\mathcal{J}(\mathcal{M}(x, y, t), \mathcal{M}(y, z, s)) \leq_L \mathcal{M}(x, z, t+s)$;
v. $\mathcal{M}(x, y, .)$:]0, ∞[→L is continuous.

In this case $\mathcal{M}$ is called an $\mathcal{L}$-fuzzy metric. If $\mathcal{M}= \mathcal{M}_{M,N}$ is an intuitionistic fuzzy set then the 3-tuple (X, $\mathcal{M}_{M,N}$, $\mathcal{J}$) is said to be an intuitionistic fuzzy metric space.

Example: Let (X, d) be a metric space. Set $\mathcal{J}(a, b) = (a_1 b_1, \min(a_2 + b_2, 1))$ for all $a=(a_1, a_2)$ and $b = (b_1, b_2)$ in $L^*$and let M and N be fuzzy sets on $X^2 \times (0, \infty)$defined as follows:

$\mathcal{M}_{M,N}(x, y, t) = (M(x, y, t), N(x, y, t)) = \left(\frac{ht^n}{ht^n+md(x,y)}, \frac{md(x,y)}{ht^n+md(x,y)}\right)$ for all t, h, m, n∈ R^+. Then (X,$\mathcal{M}_{M,N}$, $\mathcal{J}$)is an intuitionistic fuzzy metric space.

Lemma 2.9 A. George et al.(1994)Let (X, $\mathcal{M}$, $\mathcal{J}$) be an $\mathcal{L}$-fuzzy metric space .Then $\mathcal{M}$(x, y, t) is non-decreasing with respect to t for all x, y in X.

Proof: Let t, s ∈ $(0, +\infty)$ be such that t<s. Then k= s-t>0 and $\mathcal{M}(x, y, t) = \mathcal{J}(M(x, y, t), 1_{\mathcal{L}})$

$= \mathcal{J}(M(x, y, t), M(y, y, k)) \leq_L M(x, y, s)$.

Def.2.10. A sequence $\{x_n\}_{n\in N}$in an $\mathcal{L}$-fuzzy metric space(X, $\mathcal{M}$, $\mathcal{J}$) is called a Cauchy sequence, if for each $\mathcal{E} \in L\backslash\{0_{\mathcal{L}}\}$and t>0, there exists n_o ∈N such that $\forall$ $m \geq n \geq n_o (n \geq m \geq n_o)$, $\mathcal{M}(x_m, x_n, t) >_L N(\mathcal{E})$.

The sequence $\{x_n\}_{n\in N}$ is said to be convergent to x ∈ X in the $\mathcal{L}$-fuzzy metric space(X, $\mathcal{M}$, $\mathcal{J}$) (denoted by $x_n \to x$ in $\mathcal{M}$) if $\mathcal{M}(x_n, x, t)= \mathcal{M}(x, x_n, t) \to 1_{\mathcal{L}}$whenever n→ +∞for every t>0. A $\mathcal{L}$-fuzzy metric space is said to be complete if and only if every Cauchy sequence is convergent.

Thus $\mathcal{J}$ is a continuous t-norm on lattice $\mathcal{L}$ such that for every $\boldsymbol{\mu} \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$, there is a $\lambda \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ such that $\mathcal{J}^{n-1}(N(\lambda), \ldots\ldots, N(\lambda)) >_L N(\boldsymbol{\mu})$.

Def.2.11. Let (X, $\mathcal{M}$, $\mathcal{J}$) be an $\mathcal{L}$-fuzzy metric space. Then $\mathcal{M}$ is said to be continuous on X× X ×]0, ∞[that is $\lim_{n\to\infty} \mathcal{M}(x_n, y_n, t_n)=\mathcal{M}(x, y, t)$ whenever a sequence $\{(x_n, y_n, t_n)\}$ in X× X ×]0, ∞[converges to a point(x, y, t) ∈X× X ×]0, ∞[that is $\lim_{n\to\infty} \mathcal{M}(x_n, x, t)=\lim_{n\to\infty} \mathcal{M}(y_n, y, t)= 1_{\mathcal{L}}and\lim_{n\to\infty} \mathcal{M}(x, y, t_n)=\mathcal{M}(x, y, t)$.

Lemma 2.12 H. Adibi, Y (2006) Let (X, $\mathcal{M}$, $\mathcal{J}$) be an $\mathcal{L}$-fuzzy metric space. Define $E_{\lambda,\mathcal{M}}:X^2 \to R^+ \cup \{0\}$ by $E_{\lambda,\mathcal{M}}(x, y) = \inf\{ t>0 : \mathcal{M}(x, y, t) >_L N(\lambda)\}$for each $\lambda \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$and x, y ∈X. Then we have

i. For any $\mu \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$there exists $\lambda \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$such that $E_{\mu,\mathcal{M}}(x_1, x_n) \leq E_{\lambda,\mathcal{M}}(x_1, x_2) + E_{\lambda,\mathcal{M}}(x_2, x_3) + \ldots\ldots\ldots E_{\lambda,\mathcal{M}}(x_{n-1}, x_n)$ for any $x_1, \ldots\ldots\ldots\ldots, x_n \in X$.
ii. The sequence $\{x_n\}_{n\in N}$is convergent w.r.t $\mathcal{L}$-fuzzy metric $\mathcal{M}$ if and only if $E_{\lambda,\mathcal{M}}(x_n, x) \to 0$.Also the sequence $\{x_n\}_{n\in N}$is Cauchy w.r.t $\mathcal{L}$-fuzzy metric $\mathcal{M}$ if and only if it is Cauchy with$E_{\lambda,\mathcal{M}}$.

Lemma 2.13. Let(X, $\mathcal{M}$, $\mathcal{J}$) be an $\mathcal{L}$-fuzzy metric space. If $\mathcal{M}(x_n, x_{n+1}, t) \geq_L \mathcal{M}(x_0, x_1, k^n t)$ for some k>1 and n ∈N .then $\{x_n\}$ is a Cauchy sequence.

Proof: For every $\lambda \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ and $x_n \in X$, we have

$E_{\lambda,\mathcal{M}}(x_{n+1}, x_n)= \inf\{t> 0: \mathcal{M}(x_{n+1}, x_n, t) >_L N(\lambda)\}$

$\leq \inf\{t>0: \mathcal{M}(x_0, x_1, k^n t) >_L N(\lambda)\}$

$= \inf\{\frac{t}{k^n}: \mathcal{M}(x_0, x_1, t) >_L N(\lambda)\}$

$=\frac{1}{k^n} \inf\{t>0: \mathcal{M}(x_0, x_1, t) >_L N(\lambda)\}$

$=\frac{1}{k^n} E_{\lambda,\mathcal{M}}(x_0, x_1)$.

From lemma 2.12, for every $\mu \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ there exists $\lambda \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$, such that

$E_{\mu,\mathcal{M}}(x_n, x_m) \leq E_{\lambda,\mathcal{M}}(x_n, x_{n+1}) + E_{\lambda,\mathcal{M}}(x_{n+1}, x_{n+2}) + \cdots + E_{\lambda,\mathcal{M}}(x_{m-1}, x_m)$

$\leq \frac{1}{k^n} E_{\lambda,\mathcal{M}}(x_0, x_1) + \frac{1}{k^{n+1}} E_{\lambda,\mathcal{M}}(x_0, x_1) + \cdots + \frac{1}{k^{m-1}} E_{\lambda,\mathcal{M}}(x_0, x_1)$

$= E_{\lambda,\mathcal{M}}(x_0, x_1) \sum_{j=n}^{m-1} \frac{1}{kj} \to 0$.

Hence the sequence $\{x_n\}$ is a Cauchy sequence.

Lemma 2.13. Let (X, $\mathcal{M}$, $\mathcal{J}$) be an $\mathcal{L}$-fuzzy metric space. Then $\mathcal{M}$ is continuous function on X× X ×]0, ∞[.

Def.2.14. Let A and S be two mappings from $\mathcal{L}$-fuzzy metric space(X, $\mathcal{M}$, $\mathcal{J}$) into itself and $\{x_n\}$be a sequence in X such that $\lim_{n\to\infty} Ax_n=\lim_{n\to\infty} Sx_n=u$ for some u ∈ X.Then the mapping A and S are said to be

i. Weakly commuting if $\mathcal{M}(ASx, SAx, t) \geq_L \mathcal{M}(Ax, Sx, t)$ for all x in X.
ii. Compatible if $\lim_{n\to\infty} \mathcal{M}(ASx_n, SAx_n, t)=1_{\mathcal{L}}$.
iii. Compatible of type (A) $\lim_{n\to\infty} \mathcal{M}(ASx_n, SSx_n, t)= 1_{\mathcal{L}}, \lim_{n\to\infty} \mathcal{M}(SAx_n, AAx_n, t)= 1_{\mathcal{L}}$.
iv. Compatible of type (P)if $\lim_{n\to\infty} \mathcal{M}(AAx_n, SSx_n, t)= 1_{\mathcal{L}}$.

Def.2.15. Two self-maps A and S of $\mathcal{L}$-fuzzy metric space(X, $\mathcal{M}$, $\mathcal{J}$) are said to be S-biased of type (R_M) if $\lim_{n\to\infty} \mathcal{M}(SAx_n, Sx_n, t) \geq_L \lim_{n\to\infty} \mathcal{M}(ASx_n, Ax_n, t)$for t>0 whenever $\{x_n\}$is a sequence in X such that Ax_n, $Sx_n \to p \in X$.

Two self-maps A and S of $\mathcal{L}$-fuzzy metric space(X, $\mathcal{M}$, $\mathcal{J}$) are said to be A-biased of type (R_M) if $\lim_{n\to\infty} \mathcal{M}(ASx_n, Ax_n, t) \geq_L \lim_{n\to\infty} \mathcal{M}(SAx_n, Sx_n, t)$for t>0 whenever $\{x_n\}$is a sequence in X such that Ax_n, $Sx_n \to p \in X$.

Def.2.16. We say that $\mathcal{L}$-fuzzy metric space$(X, \mathcal{M}, \mathcal{I})$ has a property C, if it satisfies the following condition $\mathcal{M}(x, y, t) = C$ for all $t > 0$ implies $C = 1_{\mathcal{L}}$.

1. *Main Theorem*

Theorem 3.1. Let A, B, S and T be self-maps of a $\mathcal{L}$-fuzzy metric space$(X, \mathcal{M}, \mathcal{I})$ with a property C into itself and for $\alpha \in (0, 1)$satisfing

i. $A(X) \cup B(X) \subseteq S(X) \cap T(X)$;
ii. $\mathcal{M}(Ax, By, t) \geq_L \mathcal{M}(Sx, Ty, {}^{t}/_{\alpha})$;
iii. One of each pair {A,S} and {B, T}is continuous;
iv. The pair {A, S} is S-biased of type (R_M) and {B, T} is T-biased of type (R_M).

If one of the range space of A, B, S and T is complete subspace of X then A, B, S and T have common fixed point in X.

Proof: Since from (i) $A(X) \subseteq T(X)$ thus we choose any arbitrary point $x_0 \in X$ such that there exists a point $x_1 \in X$ so that $Ax_0 = T x_1$.

Similarly as $B(X) \subseteq S(X)$, thus for any arbitrary point $x_1 \in X$ there exists a point $x_2 \in X$ so that $Bx_1 = Sx_2$.

Therefore by induction we construct a sequence $\{y_n\}$ such that $y_{2n} = Tx_{2n+1} = Ax_{2n}$ and$y_{2n+1} = Sx_{2n+2} = Bx_{2n+1}$ for $n \in N \cup \{0\}$.

We first prove $\{y_n\}$ is a Cauchy sequence in $(X, \mathcal{M}, \mathcal{I})$.

$\mathcal{M}(y_{2n}, y_{2n+1}, t) =$
$\mathcal{M}(Ax_{2n}, Bx_{2n+1}, t) \geq_L \mathcal{M}(Sx_{2n}, Tx_{2n+1}, {}^{t}/_{\alpha}) =$
$\mathcal{M}(y_{2n-1}, y_{2n}, {}^{t}/_{\alpha})$.

$\Rightarrow \mathcal{M}$ is non-decreasing in $(X, \mathcal{M}, \mathcal{I})$.

$\Rightarrow \mathcal{M}(y_n, y_{n+1}, t) \geq_L \mathcal{M}(y_{n-1}, y_n, {}^{t}/_{\alpha}) \geq_L \mathcal{M}(y_{n-2}, y_{n-1}, {}^{t}/_{\alpha^2})$ ……..$\geq_L \mathcal{M}(y_0, y_1, {}^{t}/_{\alpha^n})$.

This implies $E_{\lambda,\mathcal{M}}(y_n, y_{n+1}) \leq \alpha^n E_{\lambda,\mathcal{M}}(y_0, y_1)$.

Therefore for every $\mu \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$there exists $\gamma \in L\backslash\{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$such that

$E_{\mu,\mathcal{M}}(y_n, y_m) \leq$
$E_{\gamma,\mathcal{M}}(y_n, y_{n+1}) + E_{\gamma,\mathcal{M}}(y_{n+1}, y_{n+2}) + \ldots\ldots\ldots + E_{\gamma,\mathcal{M}}(y_{m-1}, y_m)$

$\leq$
$\alpha^n E_{\gamma,\mathcal{M}}(y_0, y_1) + \alpha^{n+1} E_{\gamma,\mathcal{M}}(y_0, y_1) + \ldots. + \alpha^{m-1} E_{\gamma,\mathcal{M}}(y_0, y_1)$
$\leq E_{\gamma,\mathcal{M}}(y_0, y_1) \sum_{i=n}^{m-1} \alpha^i \to 0$.

Thus by lemma (2.11), $\{y_n\}$ is a Cauchy sequence.Now suppose that S(X) is complete subspace of X then the $\{y_{2n+1}\} \subseteq S(X)$ converges to a point $u \in S(X)$.Since $\{y_{2n}\}$ is also a subsequence of $\{y_n\}$which converges to $u \in S(X)$.Therefore the sequence $\{y_n\}$converges to a point $u \in S(X)$.Thus$Ax_{2n}, Bx_{2n+1}, Sx_{2n+2}andTx_{2n+1}$converges to $u \in S(X)$.As S(X) is complete subspace of X, thus there exist a point $v \in X$ such that $Sv = u$.

Put $y = x_{2n+1}$ and $x = v$ in (ii), we get

$\mathcal{M}(Av, Bx_{2n+1}, t) \geq_L \mathcal{M}(Sv, Tx_{2n+1}, {}^{t}/_{\alpha})$.

Take the limit as $n \to \infty$, we get $\mathcal{M}(Av, u, t) \geq_L \mathcal{M}(Sv, u, {}^{t}/_{\alpha}) = \mathcal{M}(u, u, t/\alpha) = 1_{\mathcal{L}}$.

This implies Av= u= Sv.

Now suppose that S is continuous then $SSx_{2n} \to Su$.

Also {A, S} is S-biased of type (R_M).

$\Rightarrow \lim_{n\to\infty} M(SAx_n, Sx_n, t) \geq_L \lim_{n\to\infty} M(ASx_n, Ax_n, t)$for $t>0$.

Now to prove Su=u, for this put $x = Sx_{2n}$, $y = x_{2n+1}$in (ii).

$\mathcal{M}(ASx_{2n}, Bx_{2n+1}, t) \geq_L \mathcal{M}(SSx_{2n}, Tx_{2n+1}, t/\alpha)$.

Now take the limit as $n \to \infty$

$$\lim_{n\to\infty} \mathcal{M}(ASx_{2n}, Ax_{2n}, t)$$
$$= \lim_{n\to\infty} \mathcal{M}(ASx_{2n}, Bx_{2n+1}, t) \geq_L \lim_{n\to\infty} \mathcal{M}\left(SSx_{2n}, Tx_{2n+1}, \frac{t}{\alpha}\right).$$

$\Rightarrow$

$$\lim_{n\to\infty} \mathcal{M}(SAx_{2n}, Sx_{2n}, t) \geq_L \lim_{n\to\infty} \mathcal{M}(ASx_{2n}, Ax_{2n}, t) \geq_L \lim_{n\to\infty} \mathcal{M}\left(SSx_{2n}, Tx_{2n+1}, \frac{t}{\alpha}\right).$$

$\Rightarrow \mathcal{M}(Su, u, t) \geq_L \mathcal{M}(Su, u, t/\alpha)$.

Therefore we have $\mathcal{M}(Su, u, t/\alpha) \geq_L \mathcal{M}(Su, u, t/\alpha^2) \geq_L \ldots\ldots\ldots. \mathcal{M}(Su, u, t/\alpha^n)$.

But by lemma (2.9) we know that $\mathcal{M}(x, y, t)$ is non-decreasing with respect to t for all x, y in X.

$\Rightarrow \mathcal{M}(Su, u, t/\alpha) \leq_L \mathcal{M}(Su, u, t/\alpha^n)$.

Therefore $\mathcal{M}(Su, u, t/\alpha) = C$ for all $t>0$.Since $\mathcal{M}(Su, u, t/\alpha)$ has the property (C), it follows that $C = 1_{\mathcal{L}}$that is Su=u.

Now to prove Au=Su.

Put x = u and y = x_{2n+1}in (ii), we get

$\mathcal{M}(Au, Su, t) = \mathcal{M}(Au, u, t) = \mathcal{M}(Au, Bx_{2n+1}, t) \geq_L \mathcal{M}(Su, Tx_{2n+1}, {}^{t}/_{\alpha})$.

$\Rightarrow \mathcal{M}(Au, Su, t) \geq_L \mathcal{M}(u, u, {}^{t}/_{\alpha}) = 1_{\mathcal{L}}$.

Thus Au = Su.

Therefore Au = u= Su.

Now to prove Bu=u=Tu.

Suppose that T is continuous, thus $TTx_{2n+1} \to Tu$.As {B, T} is T-biased type of (R_M)

$\Rightarrow \lim_{n\to\infty} M(TBx_n, Tx_n, t) \geq_L \lim_{n\to\infty} M(BTx_n, Bx_n, t)$for $t>0$.

To prove Tu=u, for this take $x = x_{2n}$, $y = Tx_{2n+1}$in (ii).

$\mathcal{M}(Ax_{2n}, BTx_{2n+1}, t) \geq_L \mathcal{M}(Sx_{2n}, TTx_{2n+1}, t/\alpha)$.

$\mathcal{M}(BTx_{2n+1}, Ax_{2n}, t) \geq_L \mathcal{M}(TTx_{2n+1}, Sx_{2n}, t/\alpha)$.

Now take the limit as $n \to \infty$

$$\lim_{n\to\infty} \mathcal{M}(BTx_{2n+1}, Bx_{2n+1}, t)$$
$$= \lim_{n\to\infty} \mathcal{M}(BTx_{2n+1}, Ax_{2n}, t) \geq_L \lim_{n\to\infty} \mathcal{M}\left(TTx_{2n+1}, Sx_{2n}, \frac{t}{\alpha}\right).$$

$\Rightarrow$

$\lim_{n\to\infty} \mathcal{M}(TBx_{2n+1}, Tx_{2n+1}, t) \geq_L \lim_{n\to\infty} \mathcal{M}(BTx_{2n+1}, Bx_{2n+1}, t)$

$\geq_L \lim_{n\to\infty} \mathcal{M}\left(TTx_{2n+1}, Sx_{2n}, \frac{t}{\alpha}\right)$.

$\Rightarrow \mathcal{M}(Tu, u, t) \geq_L \mathcal{M}(Tu, u, t/\alpha)$.

Then as proved above and using the lemma (2.9) that $\mathcal{M}(x, y, t)$ is non-decreasing with respect to t for all x, y in X, we have

$\Rightarrow \mathcal{M}(Tu, u, t/\alpha) \leq_L \mathcal{M}\left(Tu, u, \frac{t}{\alpha^n}\right)$.

Therefore $\mathcal{M}(Tu, u, t/\alpha) = C$ for all $t>0$.Since $\mathcal{M}(Tu, u, t/\alpha)$ has the property (C), it follows that $C = 1_{\mathcal{L}}$that is Tu=u.

Thus Tu = u.

To prove Bu = Tu, put x =x_{2n}, y = u in (ii), we get

$\mathcal{M}(Ax_{2n}, Bu, t) \geq_L \mathcal{M}(Sx_{2n}, Tu, {}^{t}/_{\alpha})$.

Take limit as n tends to ∞ we get

$\mathcal{M}(Tu, Bu, t) = \mathcal{M}(u, Bu, t) \geq_L \mathcal{M}(u, u, {}^{t}/_{\alpha}) = 1_{\mathcal{L}}$.

Thus Bu=Tu

Therefore Tu=u=Bu.

This implies A, B, S and T have a common fixed point u $\in$X.

Corollary 3.2. Let A, S and T be self-maps of a $\mathcal{L}$-fuzzy metric space(X, $\mathcal{M}, \mathcal{I}$) with a property C into itself and for $\alpha \in (0, 1)$satisfing

i. A(X)$\subseteq$S(X)$\cap$T(X);
ii. $\mathcal{M}$(Ax, Ay, t)$\geq_L \mathcal{M}$(Sx, Ty,$t/_\alpha$);
iii. One of each pair {A,S} and {A, T}is continuous;
iv. The pair {A, S} is S-biased of type (R_M) and {A, T} is T-biased of type (R_M).

If one of the range space of A, S and T is complete subspace of X then A, S and T have common fixed point in X.

Proof: To prove the result take B = A in the theorem 3.1.

Corollary 3.3. Let A, B and T be self-maps of a $\mathcal{L}$-fuzzy metric space(X, $\mathcal{M}, \mathcal{I}$) with a property C into itself and for $\alpha \in (0, 1)$satisfing

i. A(X)$\cup$B(X)$\subseteq$T(X);
ii. $\mathcal{M}$(Ax, By, t)$\geq_L \mathcal{M}$(Tx, Ty,$t/_\alpha$);
iii. One of each pair {A,T} and {B, T}is continuous;
iv. The pair {A, T} is T-biased of type (R_M) and {B, T} is T-biased of type (R_M).

If one of the range space of A, B and T is complete subspace of X then A, B and T have common fixed point in X.

Proof: To prove the result take S = T in the theorem 3.1.

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