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Research Article

Optimal Replacement Policy for two Dissimilar Component Series Repairable System using two Monotone Processes

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Abstract

*This paper studies a replacement policy (N_1, N_2) for a series repairable system consisting of two non-identical components and one repair man. It is assumed that each component after repair in the system is not 'as good as new' and the successive working times form a decreasing α -series process while, the successive repair time's form an increasing geometric process and both the processes are exposing to exponential failure law. Under this assumption by using a monotone process repair model, a replacement policy (N_1, N_2) based on the number of failures of **component 1** and **component 2** respectively is considered. An explicit expression for the long run expected cost rate is derived and the corresponding optimal replacement policy (N^*_1, N^*_2) is obtained such that the long run expected cost per unit time is minimized. Finally, numerical results are provided to highlight the obtained theoretical results. Numerical results are also exhibited by the graphically.*

Keywords: *Long-run Average, Cost Rate, Geometric Process, α-Process, Renewal Theorem and Mean Time to Failure (MTTF).*

1. Introduction

In the earlier stages, much research work has been carried out in the fields of maintenance problems based on the assumption that the system after repair is 'as good as new'. This model is referred to as perfect repair model. Barlow and Hunter (1959) introduced a minimal repair model in which a minimal repair does not change the age of the system. Thereafter, Barlow and Proschan (1983) developed an imperfect repair model under which a repair with probability p as perfect repair and with probability 1-p as minimal repair. Many others worked in this direction and developed corresponding optimal replacement polices e.g. Park (1979), Kijima (1989).

However in practice, due to the ageing and accumulated wearing, many systems are deteriorating. For a deteriorating system, it is reasonable to assume that the successive working times are stochastically non-increasing while the consecutive repair times after failures are stochastically non-decreasing. Thus a monotone process model should be a natural model for a deteriorating system. Ultimately, such systems can't work any longer. Neither can it be repaired any more.

To model such simple repairable deteriorating system Lam (1988 a, 1988 b) first introduced a geometric process repair model under the assumptions that the system after repair is not 'as good as new' and the successive working times {X^n, n=1, 2,….} of a system form a non increasing geometric process while the consecutive repair times {Y^n, n=1, 2,….} form a non – decreasing geometric process. Under these assumptions, he studied two kinds of replacement polices -one based on the working age T of the system and other based on the number of failures N of the system. He derived explicit expression for the long-run- average cost per unit time and also determined corresponding optimal replacement policy N such that the long-run-expected average cost per unit time is minimized.

Zhang developed a bivariate replacement policy (T, N) to generalize Lam's work. Under this policy, the system is replaced when the working age of the system reaches T and the number of failures of the system reaches N, whichever occurs first. He derived an explicit expression for the long-run average cost per unit time and corresponding optimal replacement policy (T^*, N^*) was determined analytically or numerically. Other replacement policies under geometric process repair model were reported by Zhang (2002, 1999,2004), Leung , Stadje and Zuckerman(1990)], Stanley (1983), Lam and Zhang(1996) and Lam ,Y (2003). However, the above various researches are related to the simple repairable system. For

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multi-component systems, Zhang and Wang (2007) first studied a two-component series repairable system under the geometric process repair model. When the working time of component 1 in the system follows the exponential distribution while that of component 2 and the repair times of both components follow general distributions, they derived some reliability indices of the system. Lam and Zhang(2003) studied a similar model, more reliability indices of the system are obtained by using the Laplace-transform technique. Zhang (1999) applied the geometric process repair model to a two-component cold standby repairable system with one repairman. He also assumed that each component after repair is not ''as good as new''. Under this assumption, by using a geometric process, he studied a replacement policy N based on the number of repairs of component 1. The problem is to determine an optimal replacement policy N* such that the long-run expected reward per unit time is maximized.

The above various research works are related to one component repairable system. However on practical application, the standby techniques are usually used for improving the reliability or raising the availability of the system. Zhang and Wang (2007) developed a bivariate variable optimal replacement policy (T,N) for a two-identical component cold standby repairable system with one repairman, to generalize Zhang, Y.L (1994,1999) work. They derived an explicit expression for the long-run-average cost per unit time and determined corresponding optimal replacement policy $(T, N)^*$ such that the long-run-average cost per unit time is minimized. The geometric process has been extensively used and applied to the reliability indices and maintenance models for a deteriorating system, including one-component system and two-component series, parallel system and standby systems. In this direction much research work carried out by Wang and Zhang (2006) and Zhang et.al (2007).

Wang and Zhang (2007) discussed an optimal replacement policy for a two-component series system using geometric process repair. They assumed each component after repair in the system is not 'as good as new'. Under this assumption, by using geometric process repair model, a replacement policy (M, N) based on number of failure of component 1 and component 2 was studied They derived an explicit expression for the long-run-average cost per unit time and determined corresponding optimal replacement policy such that the long-run average cost per unit time is minimized. Zhang and Wang (2007) generalize the above work and study an optimal replacement policy $(M_1^*, M_2^* \ldots, M^*_N)$ for multi component series system assuming geometric process replacement model. However the geometric process is more useful model for deteriorating system, Braun et al introduced an alternative model, the α-series process ,which contributes these characteristics.

Furthermore Braun et.al (2005) explained the increasing geometric process grows at most logarithmically in time, while the decreasing geometric process is almost certain to have a time of explosion. The α-series process grows either as a polynomial in time or exponential in time. It also noted that the geometric process doesn't satisfy a central limit theorem, while the α-series process does. Braun et al (2005) also presented that both the increasing geometric process and the α-series process have a finite first moment under certain general conditions. However the decreasing geometric process usually has an infinite first moment under certain conditions. Thus the decreasing α-series process may be more appropriate for modeling system working times while the increasing geometric process is more suitable for modeling repair times of the system.

Based on this understanding, in this paper, for studying a deteriorating system, it is assumed that the successive operating times of each component form a decreasing α-process while the consecutive repair times form an increasing geometric process and each component after repair is not 'as good as new'. Under these assumptions, we study a repair replacement policy (N_1, N_2) based on the number of failures of the component 1 and component 2. An explicit expression for the long –run average cost per unit time is derived and corresponding optimal replacement policy (N_1^*, N_2^*) is determined such that the long –run average cost per unit time is minimum. Finally, numerical results are provided to highlight the theoretical results.

2. The Model

A two dissimilar components series repairable system with one repairman is studied under the following assumptions.

1. Assume that, at the beginning, two components in the system are both new and both the components in the working state as the system is a series system.
2. Assu+0me that one component fails the system break down, and the failed component will be immediately repaired.
3. It is assumed that two components shut off each other and each component after repair is not "as good as new".
4. The time interval between the completion of n^{th} repair and $(n-1)^{th}$ repair, i on component i=1,2 is called the n^{th} cycle of component i, where n=1,2 and i=1,2.
5. Assume that $\{X_n^{(i)}\}$ is the operating time for component i after $(n-1)^{th}$ repair and $\{Y^{(i)}_n\}$ in the repair time after n^{th} repair of component i, for i=1,2 = 1,2.and n=1,2,3…
6. Let $\{X_n^i\}$ be the working time after $(n-1)^{th}$ repair of component i, where i=1,2 then the sequence, $\{X_n^i\}$ n=1,2,…. form a non increasing α-process with parameters λ>0.
7. Let $\{Y_n^i\}$ be the repair time after n^{th} repair of component i, where i=1,2 then the sequence, $\{Y_n^i\}$ n=1,2,…. form an increasing geometric process with parameters μ>0.

8. Let $F_n^{(i)}(k^{\alpha_i}x)$ and $G_n^{(i)}(b_i^{n-1}y)$ be the distribution function of $X_n^{(i)}$ and $Y^{(i)}{}_n$ respectively, for i=1,2 and n=1,2,…. where $\alpha_i > 0$ and $0 < b_i < 1$.
9. $E(X_k^{(i)}) = \frac{\lambda}{k^{\alpha}}$ *and* $E(Y_n^{(i)}) = \frac{\mu}{a^{k-1}}$ *for* $i = 1,2.$ *and* $k = 1,2,3,....$ $E(X_1^{(i)}) = \lambda$ and $E(Y_1^{(i)}) = \mu$, for $i = 1,2.$
10. Let { $Y_n^{(i)}$ } be the repair time after n^{th} failure. Then sequence { $Y_n^{(i)}$, n=1,2,…..}Form a non-decreasing GP with parameters μ>0.It means that the repair time is negligible.
11. A sequence { $X_n^{(i)}$ n=1,2,….} and { $Y_n^{(i)}$ n=1,2,…..} are independent, where n=1,2,3… and i=1,2.
12. Assume that the distributions of { $X_n^{(i)}$ } and { $Y^{(i)}{}_n$ } be F ($X_n^{(i)}$) and G ($Y_n^{(i)}$) respectively and $X_n^{(i)}$ and $Y_n^{(i)}$ are exponentially distributed. Where. i=1, 2.
13. Component 1 and component 2 will be replaced by new and identical ones at the time of N_1^{th} and N_2^{th} failures respectively and both replacement times are negligible.
14. The working reward per unit time of the system is C_w , the repair cost per unit time of component i for every failure is C_i , and the replacement cost of component i at each time is Cr_i ; i=1,2.
15. The replacement policy (N_1 , N_2) is used based on the number of failure of component 1 and component 2.

In the next section an optimal solution is discussed.

3. The Long-Run Average Cost Rate

According to the assumptions of the model, the explicit expression for the long-run-average cost per unit time for the series system can be obtained under the replacement policy (N_1 , N_2) in which the series system is replaced when the number of failures of **component 1** and **component 2** reaches N_1 and N_2 respectively.

Let $T_k^{(i)}$ and $S_k^{(i)}$ be respectively the operating time and the repair time for component i between (k-1)th replacement and kth replacement, k-1,2,….;i=1,2.

Let U (t) be the total working time of the series system before time t and V (t)= $S^{(1)}$ (t)+ $S^{(2)}$(t) be the total repair time of component i in (0,t) for i=1, 2 and the operating time of the system in (0,t) should be equal to the operating time of any component i=1,2.Thus

$$T(t) = T_1^{(1)} + T_2^{(1)} + \dots + T_{N1}^{(1)} + \varphi_1(t), \quad (3.1)$$

$$= T_1^{(2)} + T_2^{(2)} + \dots + T_{N2}^{(2)} + \phi_2(t), \quad (3.2)$$

$$S^{(1)}(t) = S_1^{(1)} + S_2^{(1)} + \dots + S_{N1}^{(1)} + \Psi(t), \quad (3.3)$$

$$S^{(2)}(t) = S_1^{(2)} + S_2^{(2)} + \dots + S_{N2}^{(2)} + \psi(t). \quad (3.4)$$

Where N_1 is the replacement number of component 1 before time t, and φ_1(t) is the working time of the system between the N_1^{th} replacement and time t, while N_2 is the replacement number of component before time t, and φ_2 (t) is the working time of the system between the N_2^{th} replacement and time t. And ψ_1(t) is the repair time of component 1 between N_1^{th} replacement and time t, while ψ_2 (t) is the repair time of the component 2 between the N_2^{th} replacement and time t.

According to assumption (10), component i is replaced when the number of failures of component i reaches N_i . Thus we have

$$T_m^{(i)} = \sum_{k=1}^{N_i} X_k^{(i)}, i = 1,2.; m = 1,2,3,......n^{(i)}(t), \quad (3.5)$$

$$S_m^{(i)} = \sum_{k=1}^{N_i-1} Y_k^{(i)}, i = 1,2; m = 1,2,3....., n^{(i)}(t). \quad (3.6)$$

It also also known that $T_1^{(i)}$, $T_2^{(i)}$ …….. ….are respectively the regenerative points of component i. Thus {$T_n^{(i)}$, i=1, 2, n=1, 2…….} form a renewal process. Similarly {$S_n^{(i)}$, i=1, 2, n=1, 2….} is also form a renewal process. Because $\tau_1^{(i)}$be the time interval between the installation of the system and the first replacement or two consecutive replacements of component i under policy(N_1,N_2). Let $\tau_j^{(i)}$ ($j \geq 2$) be the time between the $(j-1)^{th}$ replacement and the j^{th} replacement of the component i-under policy(N_1 , N_2). Clearly {$\tau_1^{(i)}$, $\tau_2^{(i)}$ …….} form a renewal process.

Thus $\tau_1^{(i)} = T_1^{(i)} + S_1^{(i)}$; $\tau_j^{(i)} = T_j^{(i)} + S_j^{(i)}$.

Let D (t) is the cost function of the system at time t, according to the assumptions of the model, it can be expressed by

$$D(t) = C_1 S^{(1)}(t) + C_2 S^{(2)}(t) + r_1 n^{(i)}(t) + r_2 n^{(i)}(t) - C_w T(t) \quad (3.7)$$

Then time t can be expressed by

$$t = T(t) + S^{(1)}(t) + S^{(2)}(t). \quad (3.8)$$

Let C (N_1 , N_2) be the long-run- expected cost per unit time of the system under the replacement policy (N_1 , N_2). We have: [see Ross, S.M (1970)].

$$C(N_1, N_2) = \lim_{t \to \infty} \frac{E(D(t))}{E(t)} \quad (3.9)$$

$$= \lim_{t \to \infty} \frac{E\left[C_1 S^{(1)}(t) + C_2 S^{(2)}(t) + r_1 n^{(i)}{}_1(t) + r_2 n_2{}^{(i)}(t) - C_w T(t)\right]}{E\left[T(t) + S^{(1)}(t) + S^{(2)}(t)\right]}. \quad (3.10)$$

$$C(N_1, N_2) = \frac{\left(C_1 \frac{E(S^{(1)}(t)}{E(T\ (t))} + C_2 \frac{E(S^{(2)}(t)}{E(T\ (t))} + r_1 \frac{E(n_1^{(i)}(t)}{E(T\ (t))} + r_2 \frac{E(n_2^{(i)}(t)}{E(T\ (t))} - C_w\right)}{\left(1 + \frac{E(S^{(1)}(t)}{E(T\ (t))} + \frac{E(S^{(2)}(t)}{E(T\ (t))}\right)} \quad (3.11)$$

Since{ $\tau_1^{(i)}$, $\tau_2^{(i)}$ …….} , {$T_n^{(i)}$, i=1, 2, n=1, 2…….}and {$S_n^{(i)}$, i=1, 2, n=1, 2….} are respectively renewal processes. Let W_i be the length of a renewal cycle of component i-under policy (N_1 , N_2).Then according to the renewal reward theorem Ross .

Table 4.1: The values of long-run average cost per unit time

		N_1						
		2	3	4	5	6	7	8
N2	2	23.95666	21.11015	20.10158	19.64537	19.41679	19.29882	19.24029
	3	21.76829	19.65539	18.95846	18.68573	18.5831	**18.55943**	18.57545
	4	20.87405	19.08431	18.51433	18.31234	**18.25672**	18.26769	18.31093
	5	20.4639	18.85254	18.34319	18.17075	18.13282	18.15592	18.20832
	6	20.28548	18.78646	**18.30745**	**18.14549**	**18.11149**	18.13603	18.18887
	7	**20.23444**	**18.81323**	18.3489	18.1875	18.15012	18.1703	18.21881
	8	20.25973	18.89678	18.43857	18.27222	18.22733	18.23982	18.28116
	9	20.33337	19.01715	18.56026	18.3859	18.33114	18.33396	18.3664
	10	20.43875	19.16228	18.70419	18.5202	18.45426	18.44623	18.46872
	11	20.56531	19.32443	18.86397	18.66965	18.59191	18.57243	18.58434
	12	20.70604	19.49835	19.03524	18.83049	18.74079	18.70962	18.71066
	13	20.85606	19.68029	19.21484	18.99999	18.8985	18.85567	18.84576
	14	21.01189	19.86749	19.4004	19.17607	19.0632	19.00895	18.98819
	15	21.17095	20.05785	19.59007	19.35708	19.23344	19.16815	19.13679
	16	21.33131	20.2497	19.78235	19.54168	19.40799	19.3322	19.29059
	17	21.49148	20.44172	19.97602	19.72873	19.58582	19.50016	19.44875
	18	21.6503	20.63282	20.17002	19.91726	19.76603	19.67119	19.61052
	19	21.80687	20.8221	20.36347	20.10638	19.94781	19.84454	19.77522
	20	21.96047	21.00882	20.55558	20.29535	20.13041	20.01953	19.94219

We have $\lim_{t \to \infty} \frac{E(S^i(t))}{E(U(t))}$; i=1, 2.

$$\lim_{t \to \infty} \frac{\frac{E(S^i(t))}{t}}{\frac{E(T^i(t))}{t}} = \frac{\frac{E(S^{(i)})}{E(w)}}{\frac{E(T^{(i)})}{E(w)}} = \frac{E(S^{(i)})}{E(T^{(i)})}, i=1,2. \quad (3.12)$$

Similarly $$\lim_{t \to \infty} \frac{E(N^{(i)}(t))}{E(T(t))} = \frac{1}{E(T^{(i)})}, i = 1,2. \quad (3.13)$$

According to results in equations (3.12) and (3.13), equation (3.11) becomes

$$C(N_1, N_2) = \frac{\left(C_1 \frac{E(S^{(1)})}{E(T^{(1)})} + C_2 \frac{E(S^{(2)})}{E(T^{(2)})} + \frac{r_1}{E(T^{(1)})} + \frac{r_2}{E(T^{(2)})} - C_w\right)}{\left(1 + \frac{E(S^{(1)})}{E(T^{(1)})} + \frac{E(S^{(2)})}{E(T^{(2)})}\right)}. \quad (3.14)$$

According to the assumption of the model and definition (2), we have

$$E\left(U^{(i)}\right) = E\left(\sum_{k=1}^{N_i} X_k^{(1)}\right) = \sum_{k=1}^{N_i}\left[\frac{\lambda_i}{k^{\alpha}}\right], i=1,2. \quad (3.15)$$

$$E\left(S^{(1)}\right) = E\left(\sum_{k=1}^{N_1-1} Y_k^{(1)}\right) = \sum_{k=1}^{N_1-1}\left[\frac{\mu_1}{b_1^{(k-1)}}\right] \quad (3.16)$$

$$E\left(S^{(2)}\right) = E\left(\sum_{k=1}^{N_2-1} Y^{(2)}{}_j\right) = \sum_{j=1}^{N_2-1}\left[\frac{\mu_2}{b_2^{(k-1)}}\right] \quad (3.17)$$

Substituting the results of equations (3.15) to (3.17) in (3.14), we have

$$C(N_1, N_2) = \frac{\left(C_1 \frac{(m_1)}{(l_1)} + C_2 \frac{(m_2)}{(l_2)} + \frac{Cr_1}{(l_1)} + \frac{Cr_2}{(l_2)} - C_w\right)}{\left(1 + \frac{(m_1)}{(l_1)} + \frac{(m_2)}{(l_2)}\right)} \quad (3.18)$$

This is an expression for the long run average cost rate under policy N.

Where

$$l_1 = \sum_{k=1}^{N_1} \frac{\lambda_1}{k^{\alpha_1}},$$

$$l_2 = \sum_{k=1}^{N_2} \frac{\lambda_2}{k^{\alpha_2}},$$

$$m_1 = \sum_{k=1}^{N_1-1} \frac{\mu_1}{b_1^{(k-1)}},$$

$$m_2 = \sum_{k=1}^{N_2-1} \frac{\mu_2}{b_2^{(k-1)}}.$$

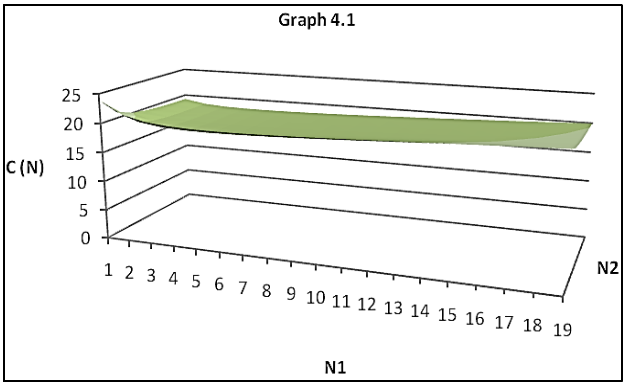
For fixed l, we can find N_2 (l) such that C_2 (N_2^*(l)) is minimized, namely N_1 =1,2,……,l, we can find N_2^*(1), N_2^*(2), N_2^*(3)…….. N_2^*(l)…….respectively such that the corresponding C_2 (1,N_2^*(l)), C_2 (2,N_2^*(l)), ………. C_2 (l,N_2^*(l))…..are minimized.

According to the definition of AGP, the successive working times of component i-after repair will be shorter and shorter, while the consecutive repair times of component i-i=1,2 after failure will become longer and longer. Ultimately, it can't work any longer neither can it be repaired. So, the total life time of the system is limited. The minimum of C (N_1, N_2 exists, we can find (N_1^*, N_1^*,) such that C (N_1^*, N_2^*) is minimum.

The next section provides numerical results to highlight the theoretical results.

4. Numerical Results and Conclusions

For the given hypothetical values of the parameters of α_1=0.95, α_2=0.62, b_1=0.95, b_2=0.92, λ_1=3, λ_2=4, μ1=8, μ2=4,
C_1=20, C_2=25, r_1=200, r_2=240, C_w=50



Conclusions

From the table 4.1 and graph 4.1, it is examined that the average cost rate is minimum i.e., **18.11149** when the failures of component 1 and component 2 reaches 6 and 6 respectively. Thus the system should be replaced when the failures of component 1(N_1) and component 2(N_2) reaches 6 and 6 respectively.

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