

## Research Article

# Performance study of Spectrum Estimation and sensing Methods in Cognitive Radios

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## Abstract

*Spectrum estimation and spectrum sensing are primary chore of cognitive radio (CR), where in dynamically it explores the radio spectrum and reliably determine portion of the frequency bands that has been used and unused by the primary users. In this paper we propose spectrum estimation using Discrete Fourier transform (DFT) which has Non parametric estimators like Periodogram, Welch and Multitaper methods. Next Discrete cosine transform (DCT) whose performance is compared with respect to fast Fourier transform (FFT) using variance as the parameter. Further spectral sensing using wavelet packet transform for various decomposition level and different types of wavelets is being compared based on its performance parameter Receiver operating characteristics (ROC). In addition this paper also discuss about spectrum sensing using orthogonal frequency division multiplexing (OFDM). Simulation results show DCT provides better spectral estimation than DFT and has less variance. Further for wavelets Daubechies wavelet has better performance in ROC with increase in depth and order of 10.*

**Keywords:** Cognitive radio, Spectrum Estimation, Spectrum Sensing, WPT, ROC, OFDM.

## 1. Introduction

Today's wireless application and services demand faster data rate and bandwidth scarcity is increasing to meet increased number of users. The irreproducible spectrum resource is precious eventually causing spectrum scarcity problem. As current wireless communication systems are provided by a fixed spectrum allocation principle, the current static frequency allocation schemes cannot adapt to the requirement of the increasing number of higher data rate services (FCC, 2002). From the observations and studies, it is learnt that most of the allocated spectrum is used inefficiently as a result; innovative techniques that can offer new ways of exploiting the available spectrum are needed. Hence a dynamic spectrum access through cognitive radio (CR) was proposed to resolve the spectrum scarcity (Mitola, 1999), where CR allows secondary user (SU) to reuse or share the same spectrum originally allocated to any primary user (PU). In cognitive radio terminology, primary users are the users with higher priority or legacy rights on the usage of a specific part of the spectrum. However, secondary users, who have lower priority, use in such a way that they do not cause interference to primary users. Therefore, secondary users need to have cognizance to sensing or estimating the spectrum reliably and check whether it is being used by a primary user, take the opportunity quickly to use spectrum at spectral nulls

where primary users have freed the spectrum and release when PU demands. Various types of spectrum estimation and spectral sensing techniques are in use. It is of interest to study performance of different methods of spectral estimation and sensing recently being used and explore their merits and demerits. This paper is organized as; in section II parametric and Nonparametric FFT based methods discussed, section III consist of spectrum estimation using discrete cosine transform (DCT) and in section IV on spectrum sensing using discrete wavelet packet transform implemented through filterbank decomposition method at different scenarios. In section V spectrum sensing using OFDM is discussed. Simulation results are discussed in section VI and finally conclusion of paper is done.

## 2. Spectrum Estimation Using DFT

The various method of spectrum estimation technique: parametric method, Non-Parametric method and subspace method. Parametric method is basically model based approach, which models the data as a sum of a few damped sinusoids and estimates their parameters. Non Parametric methods do not have any assumption about the shape of the power spectrum and in this method we try to find acceptable estimate of the power spectrum without prior knowledge about the underlying stochastic approach (Ariananda, et al, 2009).

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In this paper Periodogram, Welch and Multitaper methods have been discussed.

#### A. Periodogram Method

The basic idea of periodogram can be given as:

$$s_{xx}^p(e^{j\omega}) = \frac{1}{N} |X(e^{j\omega})|^2 = \frac{1}{N} \left| \sum_{n=0}^{N-1} x[n] \exp(-j\omega n) \right|^2 \quad (1)$$

The equation (1) is the periodogram estimate of the power spectra while  $x[n]$  and  $X(e^{j\omega})$  are the sequence whose spectrum is to be estimated and the corresponding transform in frequency domain, respectively. The main issue in periodogram is the use of rectangular windowing of waveform to obtain finite length samples. This windowing process introduces a discontinuity between the original signal and the aliased version produced by a DFT transformation. In order to mitigate the impact of rectangular window, various window functions can be applied on the data before the computation of periodogram.

#### B. Welch Method:

This method is called as averaging of Periodogram, which is also recognized as Bartlett Method can be employed to reduce the PSD variance in the periodogram estimates. The samples are divided into several segments and the periodogram of each segment is averaged. The important thing is to identify a trade-off between number of samples per segment and number of segments.

#### C. Multitaper Method (MTM):

The Multitaper Spectrum Estimator (MTSE), proposed by Thomson, which uses multiple orthogonal prototype filters to improve the variance and reduce the sidelobe and leakage. Instead of using bandpass filters that are essentially rectangular windows (as in the periodogram method), the MTM method uses a bank of optimal bandpass filters to compute the estimate.

In addition, the MTM method provides a time-bandwidth parameter with which to balance the variance and resolution. This parameter is given by the time-bandwidth product,  $NW$  and it is directly related to the number of tapers used to compute the spectrum. There are always  $2*NW-1$  tapers used to form the estimate. This means that, as  $NW$  increases, there are more estimates of the power spectrum, and the variance of the estimate decreases. However, the bandwidth of each taper is also proportional to  $NW$ , so as  $NW$  increases, each estimate exhibits more spectral leakage (i.e., wider peaks) and the overall spectral estimate is more biased. For each data set, there is usually a value for  $NW$  that allows an optimal trade-off between bias and variance.

### 3. Spectrum Estimation using DCT

Use of DCT for spectral estimation in cognitive radios has been new and alternate to DFT. DCT being real transform

and has energy compaction property. Further compared to conventional DFT, leakage effect is less in DCT (Narasimhan). DCT and a conventional fast Fourier transform (FFT) are used in a scenario where five users exist to sense the presence of user at those particular frequencies. The performance index is measured using the parameter called as variance. Discrete cosine transform (DCT) is a real transform with versions known as DCT-I, DCT-II, DCT-III, DCT-IV and the most popular is the DCT-II which is also known as even symmetric DCT.

For an input sequence  $x(n)$ ,  $n=0,1,2,\dots,(N-1)$ . The DCT-II  $c_x(k)$  is given as

$$c_x(k) = \sum_{n=0}^{N-1} 2x(n) \cos \frac{\pi k(2n+1)}{2N}, 0 \leq k \leq N-1 \quad (2)$$

From equation (2) we can see that the DCT coefficients are real, the sequence has been extended so that it has even periodic symmetry about  $-1/2, (N-1/2), (2N-1/2), \dots$ . And the total number of points  $2N$  is even and hence it is called even symmetrical DCT.

To get the inverse relation (i.e.) to obtain  $x(n)$  from  $c_x(k)$  for  $n=0,1,2,\dots,(N-1)$  we use equation (3).

$$x(n) = \frac{1}{\sqrt{N}} c_x(0) + \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} c_x(k) \cos \frac{\pi k(2n+1)}{2N} \quad (3)$$

### 4. Wavelet Packet Transform

Use of Wavelet transform (WT) for spectral sensing has been discussed in (Young woo Youn, et al, 2009).

#### A. Introduction to Discrete Wavelet Packet Transform

The filterbank method is the easiest form of implementing the wavelet transform where the signal is decomposed into high and low frequency components respectively by the filtering process. The major difference between discrete wavelet transform (DWT) and discrete wavelet packet transform (DWPT) is, in DWT the next level decomposition is done only for approximation components whereas in DWPT the decomposition is done for both approximation (A) and detailed coefficients (D).

The wavelet packet decomposition of the signal for 3-levels is shown in the fig.1, where approximation coefficients indicate the lowpass components and detail coefficients indicate the highpass components of the signal and A represents approximation coefficients of level-1 and D represents the detailed coefficients of level-1 whereas AA represents approximation component of level-2, DA represents the detailed coefficients of level-1 approximation components, AD represents approximation coefficients of level-1 detailed components and DD detailed coefficients of level-1 detailed components similarly for level-3.

#### B. Power Analysis

Spectrum sensing using DWPT is done based on the received signal power. In (Weon, et al, 1999) Power analysis

is used to differentiate between the signal and the noise present in the received signal in which we perform hypothesis testing for the signal where a dynamic threshold is derived to detect the signal in presence of noise.

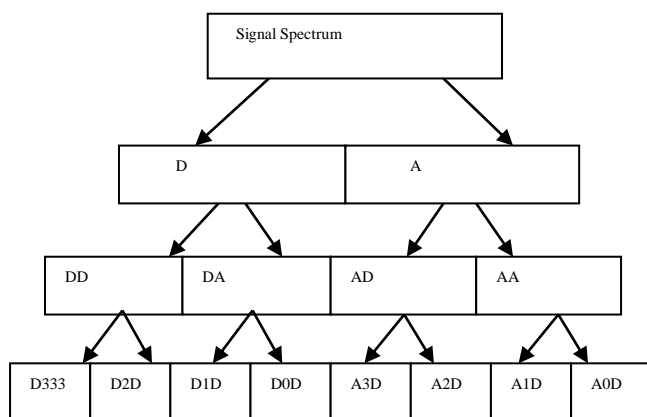


Fig 1 Wavelet packet decomposition for level-3.

C. Hypothesis Testing

In hypothesis testing the decisions is made on received data which is random in nature. The significance of its the predicted data is according to a pre-determined threshold probability. These concepts are sometimes referred to as the Neyman-Pearson thresholding. In hypothesis testing one of two hypotheses can be assumed to be true (i.e.)

1. If the measured signal power results in interference only then it is denoted as *null hypothesis H0*.
2. If the measured signal power results in interference and signal then it is denoted as *alternative hypothesis H1*.

The problem of signal detection in additive white noise can be formulated as a binary hypothesis testing problem with the following hypotheses

$$H_0: y(n) = w(n) \quad n = 1, 2, \dots, N.$$

$$H_1: y(n) = x(n) + w(n) \quad n = 1, 2, \dots, N.$$

Where  $y(n)$ ,  $x(n)$  and  $w(n)$  are the received signal samples, transmitted signal samples and white noise samples at CR respectively. Since the signals are described statistically therefore the decision between the two hypotheses and its analysis starts with a statistical description of the *probability density function* (pdf) that describes the measurement to be tested under each of the two hypotheses is given in equation (4) and equation (5).

$$P_{y(y|H_0)} = \prod_{n=0}^{N-1} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}(y_n / \sigma)^2\right) \quad (4)$$

$$P_{y(y|H_1)} = \prod_{n=0}^{N-1} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}(y_n - \mu / \sigma)^2\right) \quad (5)$$

If  $H_0$  is true, then  $P_{y(y|H_0)} < \lambda$ . Suppose if  $P_{y(y|H_0)} > \lambda$  which is of a type 1 error for this distribution.

Similarly, if  $H_1$  is true, then  $P_{y(y|H_1)} < \lambda$ , if  $P_{y(y|H_1)} > \lambda$  which is of a type 2 error for this distribution.

Where  $\lambda$  is the threshold, the likelihood ratio test is performed on the pdfs and simplified further to obtain the relation between the probability of detection ( $P_D$ ) and probability of false alarm ( $P_{FA}$ ) as shown in equation (6).

$$P_D = Q\left(Q^{-1}(P_{FA}) - \sqrt{\frac{n\mu^2}{\sigma^2}}\right) \quad (6)$$

Q represents the Q-function whereas  $Q^{-1}$  represents inverse Q-function and  $\mu$  represents the mean value of the signal and  $\sigma^2$  represents the variance. This relation between the  $P_D$  and  $P_{FA}$  leads to the performance plot called as Receiver Operating Characteristics (ROC).

5. OFDM for Spectral Sensing

Multicarrier communication systems has been suggested for Cognitive Radios as it offers advantage in terms of its flexibility to allocate resources between the different SUs. Orthogonal frequency division multiplexing (OFDM) one such candidate that can be used for CR system. Inherent FFT basis of OFDM eases spectrum sensing in frequency domain. Waveform can easily be shaped by simply turning off some subcarriers where primary users exist. OFDM systems can be adapted to different transmission environments and available resources. While employing CR, Secondary Users (SUs) should not interfere with other licensed users or primary users (PUs) using the spectrum, so to guarantee an interference-free communication between SUs and licensed users we use primary user based OFDM, and using OFDM the transmission parameters can be changed based on the spectrum awareness which includes bandwidth, FFT size, filters, windows, modulation, transmit power, and active subcarriers used for transmission. Some of the advantages of OFDM include high spectral efficiency, robustness against narrowband interference (NBI), scalability, and easy implementation using fast Fourier transform (FFT). Hence OFDM is a good fit for CR to achieve flexibility and adaptability in the transmission (Farhang et al, 2008).

However OFDM suffers from high interference due to large side lobes DFT. In addition use of the cyclic prefix (CP) in each OFDM symbol decreases the bandwidth efficiency. The leakage among the frequency subbands has a serious impact on the performance of DFT-based spectrum sensing. In order to overcome leakage effect of OFDM, it demands hard synchronization. Interference among subcarriers that originate from different SUs and between PUs and SUs results in a leakage problem. In order to overcome this wavelet method by decomposition is alternate to OFDM whose filters has good stop band attenuation and avoids interference without use of CP.

In this paper how SU senses the unused part of the spectrum using the OFDM systems is discussed. The OFDM converts the data in time domain to frequency domain by using FFT. Therefore with FFT all the points in the time-frequency grid obtained are scanned to detect the presence of the user. The receiver tries to detect the existence of primary user in the band based on the simple hypothesis testing which uses Neyman-Pearson threshold where the Neyman-Pearson threshold  $T$  is expressed in equation (7).

$$T = \sqrt{2N\sigma^2} \operatorname{erf}^{-1}(1 - 2P_{FA}) \tag{7}$$

While the threshold  $T$  is computed with from  $N$  number of samples, the variance of noise  $\sigma^2$  which is assumed to be known and the desired probability of false alarm  $P_{FA}$ .

Algorithm for spectral sensing using OFDM is:

- Step 1: The input data bits are generated randomly of length  $N$  and modulate it using binary phase shift keying (BPSK).
- Step 2: The modulated serial bit stream is converted to parallel bit streams based on the number of subcarriers.
- Step 3: The presence and the absence of the user are indicated based on the assignment of the subcarriers.
- Step 4: The data obtained from step 3 is converted to time domain using inverse Fourier transform (IFFT).
- Step 5: Add cyclic prefix to the data obtained from step 4.
- Step 6: Generate additive white Gaussian noise (AWGN) to the data obtained from step 5.
- Step 7: Remove the cyclic prefix.
- Step 8: Take FFT to the data obtained from step 7.
- Step 9: Obtain the power spectral density of each user.
- Step 10: The presence or absence of the user is detected based on the Neyman-Pearson threshold testing.

## 6. Simulation results

### Simulation Environment

In simulation the presence and absence of the signal is assumed randomly which is then modulated using a sinusoidal carrier signal and it is transmitted in additive white Gaussian noise (AWGN) as shown in fig.3. Simulation is carried for a. DFT based parametric and Nonparametric estimates

- b. DFT and DCT basis for estimation,
- c. Detection based on Wavelet for different order and depth
- d. OFDM based sensing.

Estimation performance is analysed by spectrum plot and variance.

The detection performance can be determined by varying the probability of false alarm from 0.1 to 0.9 and finding the probability of detection. Performance is also analysed with the ROC plot.

### A. Non parametric Estimation methods-FFT based

The tabular column of variance of different spectrum estimation methods for Nonparametric methods, the Table shows that MTM method as less variance compare to other methods, hence it is more preferable than other methods.

**Table I.** Variance of different Non-parametric methods

Type of spectrum estimation	Variance
Periodogram	3.86E+04
Welch	82.12(Blackman)
Multitaper	52.16

The choice of window will play important role as well in the spectrum estimation. Table II shows the variance of the estimated spectrum for different windows introduce different window Kernels in frequency domain.

**Table II.** Variance of Welch method using different windows

Different windows	variance
Rectangular	75.04
Hamming	60.47
Hanning	55.12
Blackmann	46.6
Kaiser	75.77
Triangular	61.36

**Table III.** Variance of Multitaper method for different values of time-bandwidth product.

Time-bandwidth product	Variance
2	25.83
5/2	18.63
3	14.92
7/2	11.79
4	9.82

From the Table III it is seen that as there is increase the time-bandwidth product the variance is reduced but in case of the signal with least guard band then there is a trade-off of variance and time-bandwidth product.

### B. Comparison of spectrum estimation using FFT and DCT through simulation.

The simulation environment for spectrum estimation is set as follows. We assume the signals are transmitted at frequencies 1 kHz, 1.2 kHz, 1.4 kHz, 1.6 kHz, and 1.8kHz the spectral plot of the transmitted signal and their estimation using FFT and DCT is shown in fig 2, the measure of variance for these estimators are tabulated in

table IV from which it indicates that DCT is the best estimation method compared to FFT.

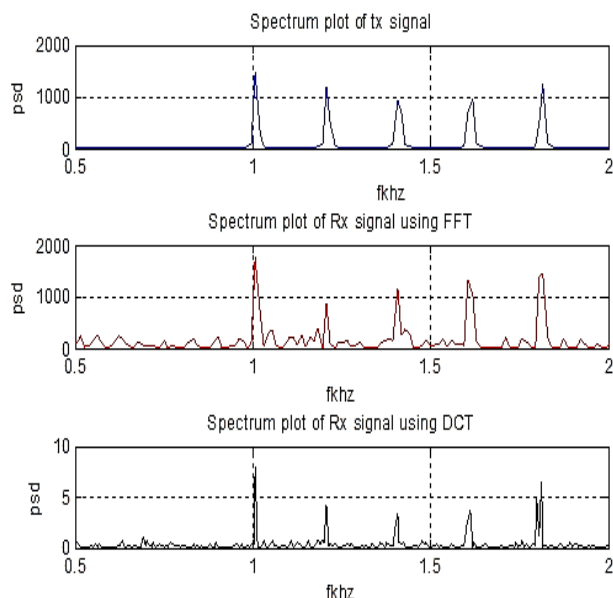


Fig. 2 PSD plot of received signal using FFT and DCT along with the transmitted signal.

Table IV. Variance of the estimated spectrum using FFT and DCT.

Transform	Variance
FFT	5.1315e+04
DCT	0.3794

C. Wavelet based Detection performance-ROC

The frequencies of the 3 primary users (PUs) at 1 kHz, 2 kHz and 4 kHz respectively are shown in fig. 3 indicates the presence of the PU which are transmitted and received at those frequencies is simulated. Simulation is performed using Daubechies wavelet for order 2, 4 and 10 and decomposition levels 2 and 3(I.Daubechies,1999).The detection performance is measured with respect to the ROC plots at different conditions

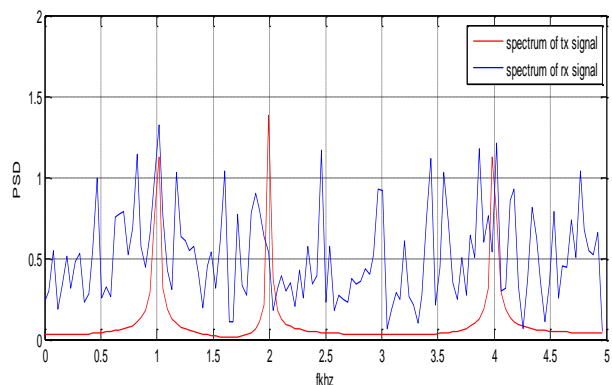


Fig.3.Spectrum plot of the transmitted and received signal.

Case I: In this scenario, two level decomposition is done for the signal and the performance of the detection is measured for different types of Daubechies (db) wavelets(Daubechies) as shown in fig. 4 which indicates that as the level of Daubechies coefficients are increased the performance of detection is increased (i.e.) compare to theoretical the probability of detection is more in db-2, which is still high in db-4 and further increase to db-10 leads to saturation where we can observe that the plots of db-4 and db-10 overlap each other even for the further increase in the Daubechies like db-20 etc. leads to the overlap with the plot of db-4 .

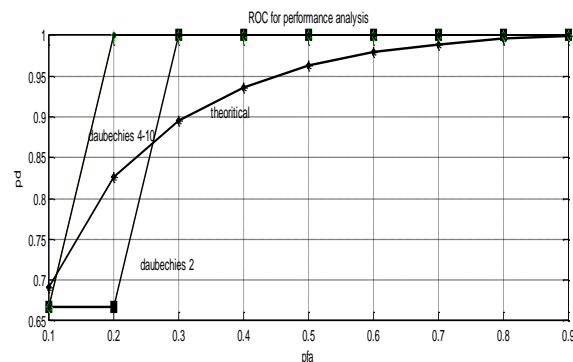


Fig. 4. ROC curves for different types of Daubechies wavelet

Case II: In this scenario, the performance of detection is measured for different levels of decomposition as shown in fig. 5 where we can observe that in 2-level decomposition the probability of false alarm is very high compare to the theoretical conditions whereas for 3-level the probability of false alarm is reduced very drastically. Therefore from the simulation results we conclude that the probability of detection can be increased if the number of decomposition levels are increased.

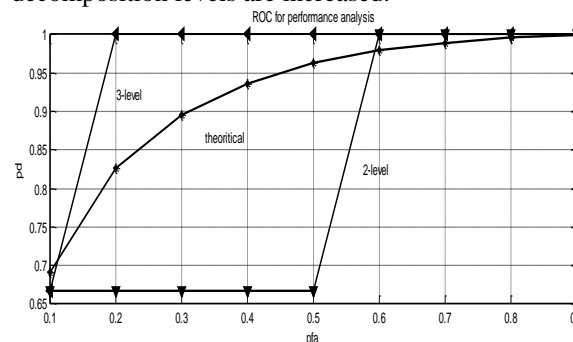


Fig. 5. ROC curves for different level of decomposition.

D. OFDM based Sensing

In the simulation 8 primary usersexist and each useris allocated on each subcarrier of OFDM symbol. Assuming that 2<sup>nd</sup>,4<sup>th</sup>, 7<sup>th</sup> user’s slots are free, the CR system senses user 2, user 4 and user 7 where the spectralholes at those corresponding frequencies exist. In order to detect the presence or absence of the user, the power spectral density (PSD) of each user are measured and compared with the Neymen-Pearson threshold. As shown in the fig. 6, the

PSD of user5, user 6 and user 7 are less than threshold which indicates the spectral holes.

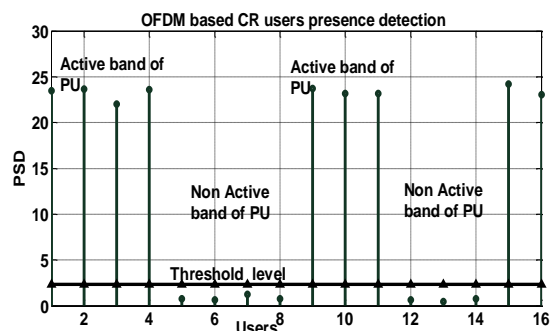


Fig. 6. PSD of CR using OFDM with spectrum hole.

### Conclusion & Futurescope

In this paper, we have studied the spectrum estimation based on parametric and Non parametric methods using DFT. Then DCT and FFT methods are compared. DCT yields better results compare to FFT as shown in the simulation results. This is mainly due to DCT being real transform and possess energy compaction property. The leakage effect is not there for DCT as compared to DFT. Spectrum sensing algorithm based on DWPT's performance at different wavelet decomposition is measured using ROCs curve through simulation. DWPT method is the robust method for spectrum sensing in Cognitive Radios when the noise is unknown. OFDM's underlying spectrum sensing capabilities together with its flexibility and adaptability make it probably the best transmission technology for CR systems. At present Non-cooperative method of spectrum sensing and estimation between the CRs is simulated. Future work is extended for the cooperative sensing and estimation of spectrum and also to extend this for OFDM and different family Wavelet packet based spectral sensing.

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### References

- FCC(2002),FCC. Spectrum Policy Task Force, ET Docket 02-135, November 2002.
- Mitola(1999),Mitola and G. Q. Maguire, "Cognitive Radio: Making Software Radios More Personal",*IEEE Personal Communications*, August 1999.
- Ariananda(2009),D. D. Ariananda, M.K. Lakshmanan, and H. Nikookar," A Survey on Spectrum Sensing Techniques for Cognitive Radio". *Cognitive Radio and Advanced Spectrum Management*, 2009 pages 74-79, May2009
- Narasimhan(2005),S.V Narasimhan and S Veena, *Signal Processing –Principles and Implementations*, vol1. Narosa Publications House 2005
- Young woo Youn(2009),Young woo Youn, HyongsukJeon, Hoiyoon Jung and Hyuckjae Lee, "Discrete Wavelet Packet Transform based Energy Detector for Cognitive Radios", in *IEEE vehicular Technology Conf. Spring, October 2009*
- Weon(1998),Weon-Ki Yoon and Michael J. Devaney,"Power Measurement Usingthe Wavelet Transform" *IEEE Transactions on Instrumentation andMeasurement*, vol. 47, no. 5, October 1998
- Daubechies(1992),I. Daubechies, *Ten Lectures on Wavelets*. Philadelphia, PA: SIAM, 1992
- Farhang(2008),Farhang-Boroujeny and R. Kempter, "Multicarrier communication techniques for spectrum sensing and communicationin cognitive radios," *IEEE Communications Magazine*, vol. 46,no. 4, pp. 80–85, 2008