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Research Article

Performance of Sphere Decoder for MIMO System using Radius Choice Algorithm

Suneeta V. Budihal[a*] and R. M. Banakar[a]

[a]B.V.Bhoomaraddi College of Engineering and Technology, Hubli

Abstract

Maximum likelihood (ML) decoding is an optimal detector with high-performance for multiple-input-multiple-output (MIMO) communication systems. While it is attractive due to its superior performance in terms of BER, its complexity using an exhaustive search which grows exponentially with the number of antennas and order of the modulation. It becomes infeasible to apply to practical systems as it searches through all lattice points in the constellation. Sphere decoding (SD) is a promising method to reduce the average decoding complexity without compromising performance. It provides optimal performance with reduced complexity as it searches the points within the specified radius of sphere. The complexity of the sphere decoder depends on the initial radius selection of the sphere, to begin search process. Attention is drawn to initial radius selection strategy, since an inappropriate initial radius can result in either a large number of lattice points to be searched, or a large number of restart actions. The simulations are performed for constellation size of 4-QAM, 8-QAM and 16-QAM for antenna size of 2X2 MIMO. It is observed that the performance of Probabilistic Tree Pruning (PTP)-SD converges with ML by taking less time and maintaining the same performance. It is proposed that a Look Up Table (LUT) for initial radius Using Radius Choice Algorithm is generated. The complexity reduces by 13% as the number of FLOPS required reduces.

Keywords*: Multiple Input Multiple Output, Maximum-likelihood decoding, Sphere Decoding, Sphere Decoding, Radius Choice algorithm.*

1. Introduction

Due to the large available bandwidth on a scattering-rich wire-less channel (G. J. Foschini, 1996), multiple-input multiple-output (MIMO) system has been extensively used in the communication system. MIMO uses multiple antennas at both the transmitter and receiver to improve the performance of the communication system. It has attracted attention in wireless communications; because it offers significant increase in data throughput without any increase in transmit power. It achieves this goal by spreading the same total transmit power over the antennas that improves the spectral efficiency.

The complexity of any decoding system increases exponentially with the number of transmits antennas and the constellation size. It is known that Maximum Likelihood Decoding (MLD) is the optimum decoding method as it searches each points in the constellation and chooses the best of them. But the requirement of exhaustive full search makes it unrealizable in a practical system. Sphere decoding (SD) is one of the methods to reduce the complexity of MLD with sub-optimal solution as it searches the points which are present within the specific radius of the sphere. The conventional SDA is very complex for hardware implementation and the throughput of current SDA implementation is below the requirement of next generation high-speed wireless communications. Choosing the initial radius of the sphere and updating its radius whenever the required lattice point is not found within the sphere, contribute to the complexity of the system.

Many algorithms were proposed to further reduce the complexity of SD such as Maximum-Likelihood detection and the search for the closest lattice point method (M. O. Damen, H. E. Gamel, and G. Caire, 2003). Closest point search in lattices method (B. Hassibi and H. Vikalo, 2005), (B. Shim and I. Kang, 2008), minimum mean square error (MMSE) detection method, Radius Choice Algorithm (Shiliang WANG, Xiaolong GUO, Mala Umar Mustpha Bakura, Songlin SUN, Xiaojun JING, Hai HUANG, 2012) to set the initial radius and Increasing Radii Algorithm (IRA), Probabilistic Tree Pruning Sphere Decoding (PTP-SD) algorithms for updating the radius. The number of visited nodes determines the complexity of SD. This can be reduced by removing the unlikely branches in early stage of sphere search. The sphere constraint of the SD algorithm offers a loose necessary condition in the early layers of search.(M. O. Damen, H. E. Gamel, and G. Caire, 2003) and (B. Hassibi and H. Vikalo, 2005) choose ∞ as the initial radius. In this situation, the first point

*Corresponding author: **Suneeta V. Budihal**

obtained by SD is known as Babai point or Zero-Forcing Decision Feedback Equalization (ZF-DFE) point. The radius can be updated as the distance between Babai point and the received point. (Qianlei Liu and Luxi Yang, 2007) uses Minimum Mean Square Error (MMSE) detection to obtain the initial point. The two methods mentioned above ensure that there is at least one point in the sphere, but the radii are often too large due to the poor performance of ZF-DFE and MMSE. Hence, it does not reduce the complexity of SD considerably. Most of the calculations in standard sphere decoders are redundant, in the sense that they either calculate quantities that are never used or calculate some quantities more than once.

Different methods have been proposed to limit the complexity of the sphere decoding algorithm. Most of them still have a variable complexity depending on the channel conditions. They can be classified in different ways. The first is to modify in the existing algorithm to marginally reduce the complexity associated with additional operations. The second is to simplify the algorithm for specific constellation types. The basic concept is to search a N-dimensional hyper sphere of some predefined radius R within the code space. The choice of the radius R to search over is a tradeoff between exponentially increasing search complexities as R increases. The probability of error in decoding will be more as the R reduces i.e. the correct code point may not be inside the search radius. The recent analysis in (L. G. Barbero and J. S. Thompson, 2007) has shown that the complexity of sphere decoding algorithm at high SNR for 16-QAM (Quadrature Amplitude Modulation) and 64-QAM modulations can be reasonably implemented with current processors.

2. Sphere Decoding

Considering an uncoded MIMO system with M transmit and N receive antennas (M N), the received complex signal at each instant time is given by

$$y_c = \frac{1}{\sqrt{ME}} H_c x_c + n_c \qquad (1)$$

Where x_c is the transmitted symbol vector whose components are elements of a Quadrature Amplitude Modulation (QAM) signal set $\overset{o}{A}$ with size A. We assume all vectors are transmitted with the same probability. H_c is a complex channel matrix known perfectly to the receiver. n_c is a circular symmetric complex Gaussian noise vector. E is the average power of the transmitted symbol. If the signal-to-noise ratio is ρ, the variance of the component of n_c is $1/\rho$. In order to use SD, the complex number signal model in (1) needs to be reformulated to a real number signal model as follows.

$$y = \begin{bmatrix} \Re(y_c) \\ \Im(y_c) \end{bmatrix}$$

$$x = \begin{bmatrix} \Re(X_c) \\ \Im(X_c) \end{bmatrix}$$

$$v = \begin{bmatrix} \Re(V_c) \\ \Im(V_c) \end{bmatrix}$$

$$H = (1/\sqrt{ME}) \begin{bmatrix} \Re(H_c) & -\Im(H_c) \\ \Im(H_c) & \Re(H_c) \end{bmatrix} \qquad (2)$$

Where $\Re(\cdot)$ and $\Im(\cdot)$ are the real and imaginary parts of its argument. Then the real number signal model is given by

$$y = Hx + v \qquad (3)$$

Let m = 2 M, n = 2 N, hence, H is a (n × m) real matrix. The real MLD is given by,

$$\hat{x} = argmin|y-Hx|^2 \qquad (4)$$

SD reduces the complexity by limiting the search space in a hyper sphere $S(y, \sqrt{C})$ centered at y, where C is the squared radius of the sphere. SD can be expressed as

$$|y-Hx|^2 \le C \qquad (5)$$

Performing QR-decomposition of H as $H = [Q \quad Q'][R^T \quad 0^T]^T$, where R is an m × n upper triangular matrix with positive diagonal elements, 0 is a zero matrix, Q and Q' are an n × m and n × (n–m)) unitary matrices respectively. The inequality (5) is equivalent to,

$$|[Q \quad Q']^T y - \begin{bmatrix} R \\ 0 \end{bmatrix} x|^2 \le C \qquad (6)$$

$$|Q^T y - Rx|^2 \le C - |(Q')^T y|^2 \qquad (7)$$

$$|y' - Rx|^2 \le c_0 \qquad (8)$$

Where $y' = Q^T y$ and $c_0 = C - |(Q')^T y|^2$.

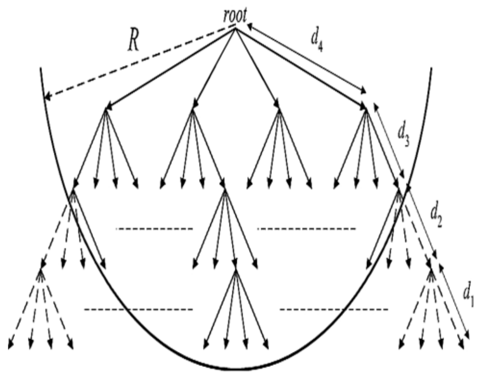

Figure 1. Illustration of sphere decoding in a tree.

In the above figure the numbers labeled for each node are the path metrics. Note that the dotted nodes are skipped since they are outside of sphere constraint.

3. Radius Choice Algorithm

In Radius Choice algorithm , the initial radius can be obtained corresponding to the expected number of points for particular values of SNR.

3.1 Expected number of points in sphere

The received symbol vector is denoted as $\tilde{x}$ and the actual transmitted symbol vector as x. Then,

$$y - H\tilde{x} = Hx + n - H\tilde{x} = \mathrm{H}\,(\mathrm{x} - \tilde{x}) + n = He + n \qquad (9)$$

Where e = x–$\tilde{x}$ is the error symbol vector. Therefore, the components of $(y - H\tilde{x})$ are i.i.d. $\bar{N}\,(0, \sigma^2 + \sigma_h^2|e|)$ random variables and $\lfloor y - H\tilde{x} \rfloor^2$ is a scaled chi- squared distribution with m degrees of freedom, where σ_h^2 is the variance of the component of *H*. When the decoding is perfect, $\tilde{x}$ equals to x so *y–Hx = n* is Gaussian random vector whose component is $\bar{N}\,(0, \sigma^2)$ random variable. Then when a definite radius C is given, we can obtain the probability that the lattice point is $\tilde{x}$ in the sphere,

$$F_{\tilde{x}}(C) = \int_0^{(C/\sigma^2 + \sigma_h^2|e|^2)} (1/2^{1/2}\Gamma(m/2))(u^{m/2-1})e^{-u/2}du \qquad (10)$$

$$F_{\tilde{x}}(C) = \Phi(C/(\sigma^2 + \sigma_h^2|e|^2)) \qquad (11)$$

Where σ^2 is the co-variance and σ_h^2 is the covariance of component of channel matrix H.

$$\sigma^2 = M(L^2 - 1)/6\rho \qquad (12)$$

Where L^2 is the QAM constellation, Γ(.) is the Gamma function and Φ(.) is the Cumulative Distributive Function (CDF) of chi-square distribution. Here, form a table of initial radius value for any expected number of lattice points for a given value of SNR. A sequence of number of points such as D1, D2, D3 and so on are considered with a constant incremental steps. Then radius values C1, C2, C3 and so on respectively using the following equations (5), (9) for a given value of SNR are calculated. For 16-QAM, the equation for the expected number of points is given by,

$$D(C) = (1/2^m)\Sigma_q \Sigma_{l=0}^{m} \binom{m}{l} g_{kl}(q) F_{\tilde{x}}(C) \qquad (13)$$

Where $g_{kl}(q)$ is the coefficient of x^q in the polynomial $(1+x+x^4+x^9)^l\,(1+2x+x^4)^{k-l}$. Similar results can be obtained for 64-QAM and other constellations. The initial radius C1 is chosen such that it should eliminate the too-large and the too-small conditions. The too-large condition implies that there are many points within the sphere. Hence, the complexity cannot be reduced effectively. The too small condition implies that there is no lattice point within the sphere which leads to repetitive search and hence, increases the complexity. If the search fails with C1, then we start the new search with C2 as the initial radius. If there is only one lattice point then the solution will be the ML solution.

In this paper, it has been proposed that the complexity of the PTP-SD can be reduced further by combining the SD algorithm with the radius choice algorithm. In PTP-SD algorithm, instead of starting the search radius from infinity, the points can be searched from the initial radius which is obtained from the Radius Choice algorithm table. Hence, in this paper, it has been shown that the combination of these two algorithms will lead to the significant reduction in the complexity maintaining the same performance.

4. Simulations

In this section, we present the results of simulations for different system configurations.

4.1 Performance analysis of ML and Sphere decoder

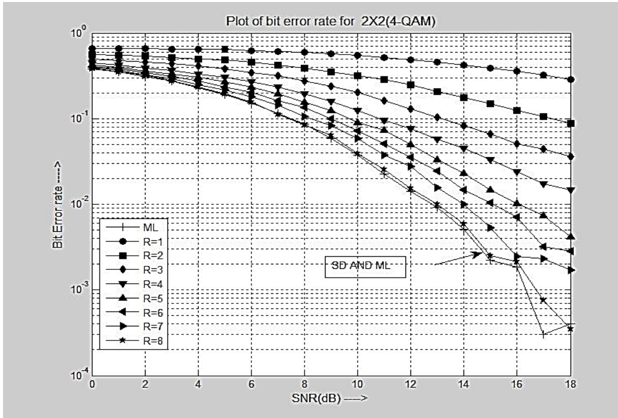

Figure 2. BER Vs SNR for M=N=2 and 4-QAM.

Figure 2 is a plot of bit error rate for 2X2 MIMO and 4-QAM constellation size. It shows the plot for ML decoder and SD for various values of initial search radius values. It shows a plot BER Vs SNR for the initial radius values from R=1 to 8. From the graph it is found that the performance of SD approaches that of ML at larger values of R i.e. at R=8. The arrow indicates this in the graph.

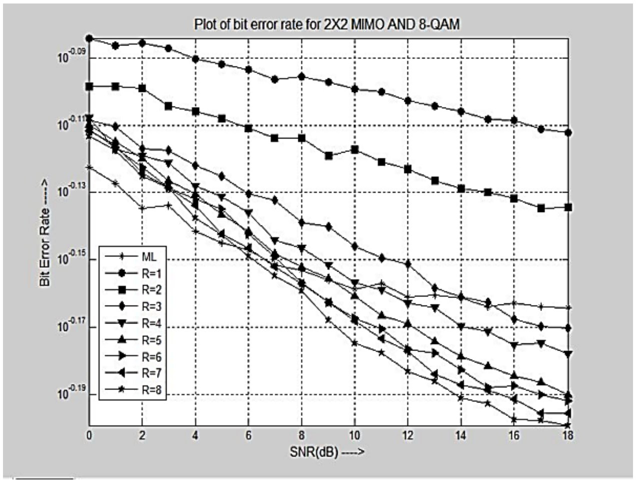

Figure 3. BER Vs SNR for M=N=2 and 8-QAM.

The Figure 3, Figure 4 and Figure 5 show the performance for 8-QAM, 16-QAM and 32-QAM constellation size respectively, keeping the other parameters constant. From the figures, it is seen that the performance of ML can be achieved with SD with reduced complexity by eliminating the redundant calculations multiple number of times. The Table 1 gives the details of the time taken by the different modulation techniques with different constellation sizes for ML and SD. This shows that the time reduces for SD

Table 1. The table of elapsed time for ML decoding and for the SD for radius values from 1 to 8.

MIMO SYSTEM Antenna size 2 X2	Time taken in seconds			
	For 4-QAM	**For 8-QAM**	**For 16-QAM**	**For 32-QAM**
Maximum Likelihood decoder	**112. 401190**	**119.271497**	**123.752901**	**129.212373**
Sphere decoder with radius R=1	0.000879	0.001121	0.001026	0.00125
R=2	0.000834	0.000878	0.001105	0.001326
R=3	0.000877	0.000918	0.001145	0.001613
R=4	0.000898	0.001064	0.001181	0.00164
R=5	0.000903	0.00107	0.001235	0.001663
R=6	0.000981	0.001081	0.001306	0.001848
R=7	0.000993	0.001286	0.001353	0.002042
R=8	**0.001044**	**0.001474**	**0.001559**	**0.002453**

Table.2. Initial radius Look Up Table for 4 X 4 MIMO with 16QAM when D=1,2,....and 8.

SNR	D=1	D=2	D=3	D=4	D=5	D=6	D=7	D=8
1	4.038472	0.069118	0.076905	0.082987	0.088056	0.092442	0.096332	0.099843
2	2.023659	0.063823	0.071013	0.07663	0.08131	0.08536	0.088952	0.092194
3	1.349106	0.062058	0.069049	0.074511	0.079062	0.082999	0.086492	0.089645
4	1.009618	0.061176	0.068067	0.073451	0.077937	0.081819	0.085262	0.08837
5	0.804156	0.060646	0.067478	0.072815	0.077263	0.081111	0.084524	0.087605
6	0.665707	0.060293	0.067086	0.072392	0.076813	0.080639	0.084032	0.087095
7	0.565551	0.060041	0.066805	0.072089	0.076492	0.080302	0.083681	0.086731
8	0.489329	0.059852	0.066595	0.071862	0.076251	0.080049	0.083417	0.086458
9	0.429062	0.059705	0.066431	0.071685	0.076063	0.079852	0.083212	0.086245
10	0.379964	0.059587	0.0663	0.071544	0.075914	0.079695	0.083048	0.086075
11	0.338988	0.059491	0.066193	0.071428	0.075791	0.079566	0.082914	0.085936
12	0.304105	0.059411	0.066104	0.071332	0.075689	0.079458	0.082802	0.08582
13	0.273908	0.059343	0.066028	0.07125	0.075602	0.079368	0.082708	0.085722
14	0.247393	0.059285	0.065963	0.071181	0.075528	0.07929	0.082626	0.085638
15	0.223823	0.059234	0.065907	0.07112	0.075464	0.079222	0.082556	0.085565

as compared to Ml giving the same performance w.r.t bit error rate. It is also found that as the size of constellation increases time taken also increases. Here the measure of complexity of SD is the time taken by the system to give the above mentioned performance.

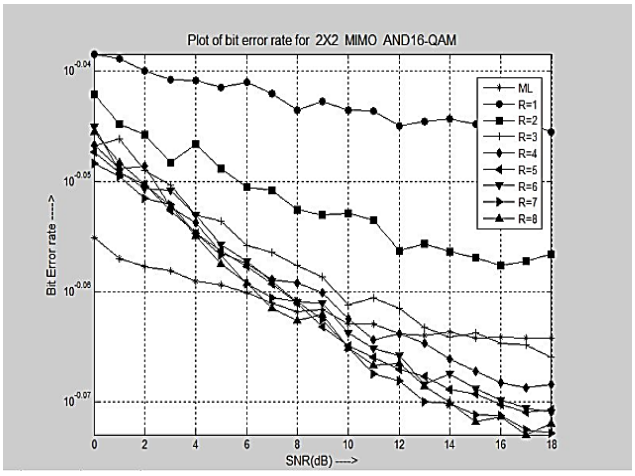

Figure 4. BER Vs SNR for M=N=2 and 16-QAM.

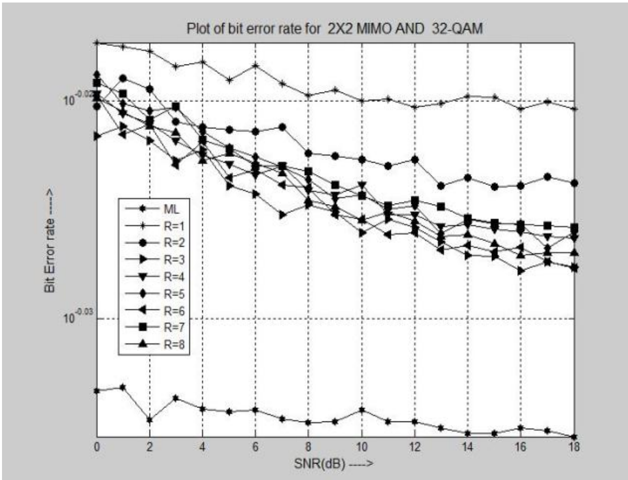

Figure 5. BER Vs SNR for M=N=2 and 32-QAM.

4.2. Complexity analysis of ML and sphere decoder

From Table.1 it can be observed that the elapsed time for ML decoding is almost three times higher than that for SD. As elapsed time is directly related to the complexity, it can be deduced that the complexity of the ML is much higher than that of SD.

The Table.2 is generated using the above mentioned equations. These equations use the noise statistics, number of antennas at transmitter and receiver sides, available SNR and the number of points expected in the sphere and provide with a value if initial radius to begin the search process. Otherwise the initial radius was taken up as infinity. This reduces the complexity of SD further and hence provide with an optimal radius selection strategy.

The Figure 6 and Figure 7 shows the reduction in the number of floating point operations required to give the required performance for the MIMO systems using SD. It is observed that there is a reduction in the number of FLOPS by 13% at higher SNR i.e. above 5dB. Above this SNR the performance is unchanged after using the LUT for SD.

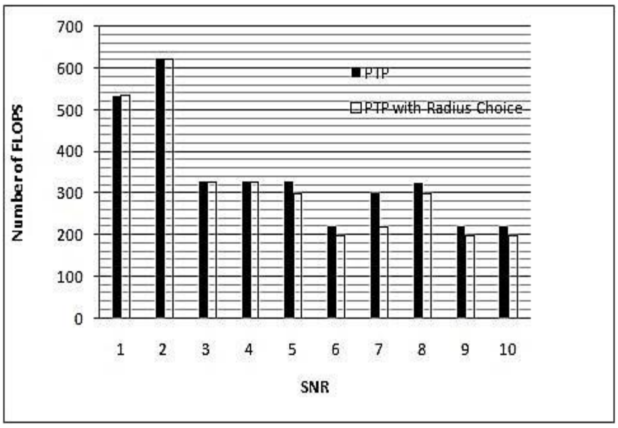

Figure 6. The plot of number of FLOPS Vs SNR for a 4 X 4 MIMO and 16-QAM.

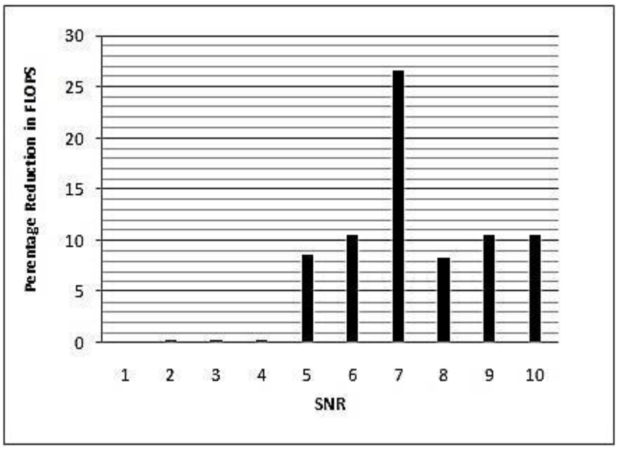

Figure 7. The plot of % reduction in FLOPS Vs SNR for a 4 X 4 MIMO and 16-QAM.

5. Conclusion

Sphere Decoding provides optimal performance with reduced complexity as it searches the points which are within the specified radius of the sphere. The complexity of the sphere decoding is dependent on the initial radius selection of the sphere, basically to begin search process. Attention is drawn to initial radius selection strategy, since an inappropriate initial radius can result in either a large number of lattice points to be searched, or a large number of restart actions. The simulations are performed for constellation size of 4-QAM, 8-QAM and 16-QAM for antenna size of 2X2 MIMO. It is observed that the performance of SD converges with ML by taking less time and maintaining the same performance. The radius choice algorithm for MIMO SD is based on expected number of lattice points in the sphere. The obtained radius can reduce the search space effectively and ensure that the sphere is not empty with high probability. The complexity of SD is further reduced by the combination of the radius choice algorithm and the PTP-SD algorithm as the search can be started from the radius obtained from the table instead of infinity. There is a reduction in the complexity. It is proposed that a LUT for Initial Radius Using Radius Choice Algorithm is generated. Thus the complexity reduces by 13% as the number of FLOPS required reduces.

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