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Research Article

Light Scattering From a Cluster Consists of Layered Axisymmetric Objects

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Abstract

Random-orientation scattering properties of a plane wave scattered by an arbitrary, symmetric cluster consists of coated spherical and coated spheroidal particles are presented. The calculation is based on a method that calculate the cluster T-matrix, and from which the orientation-averaged scattering matrix and total cross sections can be analytically obtained. Numerical results for the random-orientation scattering matrix are presented. The cluster consists of particles ensemble the form of a densely packed cluster and linear chains.

Keywords: *Light Scattering, Cluster, Layered Axisymmetric Objects*

1. Introduction

Many types of particles in nature can be modeled as coated or layered particles. For example the shape of plant and animal cells, algal and bacteria, microcavities, aerosols in the atmosphere, and encapsulated material are coated or layered objects. Also the particles that are formed in natural or in technological processes will possess complicated morphologies. Frequently, however, the morphological complexity of small particles arises from aggregation of individual particles that, by themselves, possess a simple shape. Fractal clusters for example, are formed by different particles which aggregate and combined into sparse random fractal clusters (V. M. Shalaev, 1997). Understanding the radiative properties from a cluster consists of small coated particles is relevant to a wide variety of applications in many fields as, pharmaceuticals industries processes, nanotechnology, chemistry, astrophysics, biology and health sciences (N. A. Gumerov *et al*, 2005). A model of light scattering by fractal clusters in the atmosphere for pollution identification and characterization for example presents an important application for such type of particles. A cluster of small particles, as a whole, is nondeterministic in shape, therefore the usual light scattering calculations using the known established theories based on regular particles (such as spheres, spheroids, cylinders) cannot, be directly applied to aggregated particles. However, if the individual particles making up the aggregated cluster possess shapes that admit analytical solutions to the wave equations, then by appropriate superposition techniques, the radioactive properties of the cluster can be calculated (D. W. Mackowski *et al*, 1996). This approach has been established for clusters of spheres (D. W. Mackowski *et al*, 2011), cluster of two prolate spheroids (Y. Jin *et al*, 1996), clusters of fibres (T. Wriedt *et al*, 2008) and microscopic grains dusting of surfaces of larger host particles (M. I. Mishchenko *et al*, 2011).

In this paper, we improve the method introduced by Mackowski and Mishchenko ((D. W. Mackowski *et al*,1996), (D. W. Mackowski *et al*, 2011)) and combine them with the previously published techniques ((E. E. M. Khaled *et al*, 1994), (A. Quirantes, 2005)) to get a modified method that can be applied for a cluster consists of adjacent layered particles. The modified codes can calculate the efficiency factors and scattering matrix elements of clusters for either fixed or random orientation with respect to the incident wave or light. The incident wave can be either a plane wave or a focused shaped beam. In addition, the codes can calculate the intensities of the interior and exterior electric fields distributions.

Our aim is to exploit the scattering description of the *T*-matrix for a cluster consists of layered axisymmetric particles, from which the random-orientation cross sections and scattering matrix can be obtained analytically. The *T*-matrix method is faster and requires less computer memory than the alternative approach based on matrix inversion and numerical quadrature. The method also allows calculations of the random-orientation scattering properties of the cluster.

2. Theoretical Analysis

The scattered field from a cluster consists of N_S layered axisymmetric particles is resolved into partial fields scattered from each particle in the cluster ((D. W.

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Mackowski *et al*,1996), (D. W. Mackowski *et al*, 2011)), i.e.

$$\mathbf{E}_s = \sum_{i=1}^{N_S} \mathbf{E}_i \tag{1}$$

where each partial field $\mathbf{E}_i$ is represented by an expansion of vector spherical harmonics (VSH) that are manipulated with respect to the origin of the i^{th} object:

$$\mathbf{E}_i = H \sum_m \sum_n D_{mn} \left[f^i_{emn} \mathbf{M}^3_{emn}(k\mathbf{r}) + f^i_{omn} \mathbf{M}^3_{omn}(k\mathbf{r}) + g^i_{emn} \mathbf{N}^3_{emn}(k\mathbf{r}) + g^i_{omn} \mathbf{N}^3_{omn}(k\mathbf{r}) \right] \tag{2}$$

where H, and D_{mn} are normalization factors, $\mathbf{M}^3(k\mathbf{r})$ *and* $\mathbf{N}^3(k\mathbf{r})$ are the VSH of the third kind (outgoing wave functions) obtained from the VSH of the first kind. The coefficients $f^i_{emn}, f^i_{omn}, g^i_{emn}$ *and* g^i_{omn} are the scattered field expansion coefficients for the i^{th} object, and *e, o* stand for even and odd respectively. All the parameters and details of the analysis are given in (M. I. Mishchenko *et al*, 2005- P. W. Barber *et al*, 1990).

The field arriving at the surface of the i^{th} layered axisymmetric object consists of the incident field plus the scattered fields that originate from all other objects in the cluster. By use of the addition theorem for VSH the interacting scattered fields can be transformed into expansions of the fields about the origin of a certain i^{th} object (D. W. Mackowski *et al*, 1996), which makes possible to formulate an analytical formulation of the boundary conditions at the surface. After truncation of the expansions to $n = N_{O,i}$ orders, which is the maximum order retained for the scattered field expansion of the individual object, a system of equations for the scattering coefficients of i^{th} particle can be constructed. This system of equation can be obtained using the centered T-matrix of the cluster as follow ((D. W. Mackowski *et al*,1996), (D. W. Mackowski *et al*, 2011)),

$$\begin{bmatrix} f^i_{emn} \\ f^i_{omn} \\ g^i_{emn} \\ g^i_{omn} \end{bmatrix} = \sum_{j=1}^{N_s} \sum_{n'=1}^{N_{o,i}} \sum_{m'=-n'}^{n'} T^{ij}_{mn\ m'n'} \begin{bmatrix} a^j_{em'n'} \\ a^j_{om'n'} \\ b^j_{em'n'} \\ b^j_{om'n'} \end{bmatrix} \tag{3}$$

Typically, $N_{O,i}$ will be proportional to the size parameter x_i of the individual particles, and a_{mn}, b_{mn} denote the expansion coefficients for the incident field as described in ((E. E. M. Khaled *et al*, 1992), (P. W. Barber *et al*, 1990)), *e* and *o* refer to even and odd respectively.

The *T*-matrix elements depend on the particle's size, shape, composition and orientation, but not on the nature of the incident field. They can, therefore, be calculated in the so-called natural reference frame (*z*-axis along the revolution axis) and then be averaged for all incidence and scattering directions, which equals averaging on particle orientation.

In this paper we apply the method on a cluster consists of coated axisymmetric particles (a coated particle is a special case of layered particle and the method can be applied to the layered particle as well but with longer calculation). The procedure for calculating the *T*-matrix of a cluster is started with calculating the coated axisymmetric particle-centered matrix T^{ij} by using the code described in ((A. Quirantes, 2005), (E. E. M. Khaled, 1992), (H. L. Ibrahim, 2009)) for each coated particle with a core of radius R_c and refractive index m_c and a shell of radius R_s and refractive index m_s,

$$T^{ij} = -B * A^{-1} = -\left[B_s + B_{cs} * \left(-B_c * A_c^{-1}\right)\right] * \left[A_s + A_{cs} * \left(-B_c * A_c^{-1}\right)\right]^{-1} \tag{4}$$

where:

- $-B_c * A_c^{-1} = T_c$, is the *T*-matrix calculated for a particle with refractive index $m = m_c/m_s$ for the inner layer (core), and the size parameter as $R * m_s * x$. In other words, it is the *T*-matrix for the core alone without the shell, where $R = r_c/r_s$, x is the size parameter of the outer surface.
- B_s and A_s, are calculated using the refractive index equal to m_s for the outer layer and size parameter equal to x. So the product $-B_s * A_s^{-1}$ would be the *T*-matrix for a particle with no core,
- Matrices A_{cs} and B_{cs} are calculated in the same way as A_s and B_s except that the Bessel functions of the first kind with argument kr are replaced by Hankel functions with the same argument

Now the T^{ij} of the individual particle is merged into the cluster *T*-matrix through:

$$T_{nl} = \sum_{i=1}^{N_S} \sum_{j=1}^{N_S} \sum_{n'=1}^{N_{o,i}} \sum_{l'=1}^{N_{o,i}} j^{oi}_{nn'} \, T^{ij}_{n'l'} \, j^{jo}_{l'l} \tag{5}$$

where $N_{O,i}$ is the maximum order retained in the individual object scattered field expansions, J^{0i} and J^{j0} matrices are formed from the addition coefficients based on the spherical Bessel function, the other parameters and details of the analysis are given in ((D. W. Mackowski *et al*,1996), (D. W. Mackowski *et al*, 2011)). Note that, the orientation averaged scattering matrix elements can be analytically obtained from the *T*-matrix, following the procedures developed in ((E. E. M. Khaled *et al*, 1994), (A. Quirantes, 2005)). Since the T^{ij} matrix can be calculated then total cross sections in both fixed and random orientation can be obtained by performing the operations directly on T^{ij} matrix.

The Stokes parameters ***I***, ***Q***, ***U*** and ***V*** which define the relation between the incident and scattered light are specified with respect to the plane of the scattering direction ((M. I. Mishchenko *et al*, 2005), (P. W. Barber *et al*, 1990)). The transformation of the Stokes parameters upon scattering is described by the real valued 4×4 Stokes scattering matrix ***S***. Although each element of the scattering matrix depends on the scattering angle Θ^{sca}, there is no dependence on the azimuthal scattering angle Φ^{sca} for the collections of identical randomly oriented particles considered here. For a collection of randomly oriented particles, the scattering matrix reduces to:

$$\begin{bmatrix} I^{sca} \\ Q^{sca} \\ U^{sca} \\ V^{sca} \end{bmatrix} \propto \begin{bmatrix} S_{11}(\Theta) & S_{21}(\Theta) & 0 & 0 \\ S_{21}(\Theta) & S_{22}(\Theta) & 0 & 0 \\ 0 & 0 & S_{33}(\Theta) & S_{34}(\Theta) \\ 0 & 0 & -S_{34}(\Theta) & S_{44}(\Theta) \end{bmatrix} \begin{bmatrix} I^{inc} \\ Q^{inc} \\ U^{inc} \\ V^{inc} \end{bmatrix} \tag{6}$$

The elements of the scattering matrix can be used to define specific optical observables corresponding to different types of polarization state of the incoming light. For example, if the incident radiation is unpolarized, then the (1,1) element characterizes the angular distribution of the scattered intensity in the far-field zone of the target, while the ratio $-S_{21}(\Theta)/S_{11}(\Theta)$ gives the corresponding angular distribution of the degree of linear polarization. If the incident radiation is linearly polarized in the scattering plane, then the angular distribution of the cross-polarized scattered intensity is given by $\frac{1}{2}[S_{11}(\Theta)-S_{22}(\Theta)]$ as described in ((M. I. Mishchenko *et al,* 2005), (P. W. Barber *et al,* 1990)). All of the S_{ij} can be written in terms of S_1, S_2, S_3, S_4. For example, $S_{34} = -\mathrm{Im}\left(S_1 S_2^* - S_3 S_4^*\right)$. The scattering matrix relating the Stoke's parameter has its basis in the amplitude scattering matrix,

$$\begin{bmatrix} E_{\parallel}^{s} \\ E_{\perp}^{s} \end{bmatrix} = \frac{e^{ikr}}{-ikr} \begin{bmatrix} S_2 & S_3 \\ S_4 & S_1 \end{bmatrix} \begin{bmatrix} E_{\parallel}^{i} \\ E_{\perp}^{i} \end{bmatrix} \tag{7}$$

where $k\mathbf{r}$ is the argument of the vector spherical wave function, and $\mathbf{r}$ is a position vector. The incident field has been evaluated at z=0. $E_{\parallel}$ is the electric field component polarized parallel to the *X-Z* scattering plane, $E_{\perp}$ is the electric field component polarized perpendicular to the *X-Z* scattering plane, and

S_1= the $\perp$ scattered field amplitude for $\perp$ incident
S_2= the $\parallel$ scattered field amplitude for $\parallel$ incident
S_3= the $\parallel$ scattered field amplitude for $\perp$ incident
S_4= the $\perp$ scattered field amplitude for $\parallel$ incident.

3. Numerical Results

The procedure for calculating the *T*-matrix of a cluster is started with calculating the matrix T^{ij} of the coated axisymmetric centered particle by using the code described in ((E. E. M. Khaled *et al,* 1994), (A. Quirantes, 2005)). Then merge the T^{ij} into the cluster *T*-matrix through Eq (5). Finally calculating the random orientation scattering matrix expansion coefficients, using Eq (6), can be performed. The random orientation scattering matrix described in (D. W. Mackowski *et al,* 1996) and its code is modified here to deal with not only cluster consisting of spheres but also consisting of different shapes of coated axisymmetric objects. The veracity of that modified technique is proven by several tests.

In the following we present a comprehensive examinations of the scattering properties of different zclusters using our modified technique with the previous techniques shown in ((E. E. M. Khaled *et al,* 1994), (A. Quirantes, 2005), (E. E. M. Khaled, 1992)) for a cluster of different special cases to show the capabilities and truthful results that can be obtained using the presented technique. For the present purposes we present a small sample of results and point out some salient features. Four examples of different clusters are presented. First, a linear chain of identical five coated spheres in a cluster is presented as illustrated in Fig.1. Second, a packed cluster consists of five identical coated spherical particles as illustrated in Fig.2. Third, a linear chain consists of five identical coated oblate spheroidal particles as in Fig. 3. Last example is a packed cluster consists of five identical coated oblate spheroidal particles as in Fig. 4.

To qualify the performance of the present technique some of cases, which are published in the literature ((D. W. Mackowski *et al,* 1996), (D. W. Mackowski *et al,* 2011), (Y. Jin *et al,* 996)) are recomputed here using the presented technique and codes for the purposed of comparison. The considered case for this purpose is the cluster shown in Fig.1. with the ratio R=a_c/a_s=1 and the refractive indices m_s=1.0, m_c=1.5+0.005i for the shell and the core, respectively, i.e, the cluster becomes a linear chain consists of five identical homogenous spherical particles as the case presented in (D. W. Mackowski *et al,* 1996) with size parameter x=2πa_s/λ=5 for each sphere. The results are typical with the results in Ref (D. W. Mackowski *et al,* 1996) as shown in Fig.5.

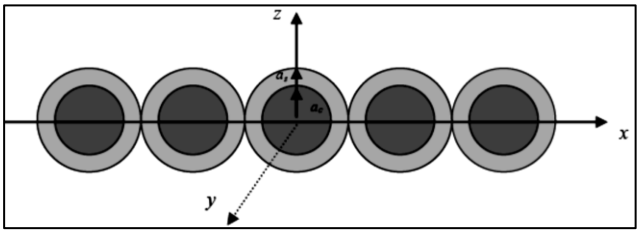


Fig.1 A linear chain cluster consists of five identical coated spherical particles

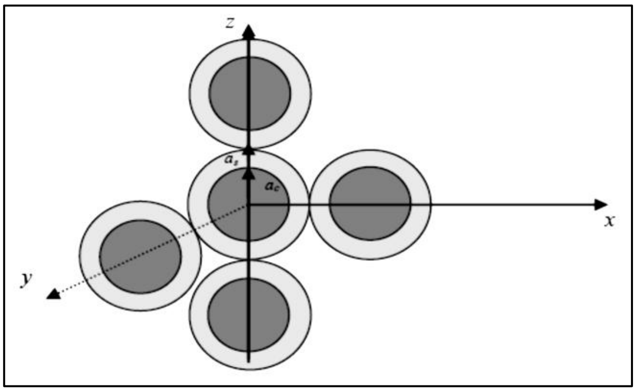


Fig.2 A packed cluster consists of five identical coated spherical particles.

For more investigation of the qualification of the presented technique we consider the case illustrated in Fig.1. The ratio R=a_c/a_s for each sphere is changed to three different values which are R= 0.9, 0.8, 0.7 to represent three separate cases. The refractive indices for all cases are m_s=1.36 and m_c=1.5+0.005i for the shell and the core respectively for each sphere. The results of scattering matrix elements as a function of scattering angle Θ are shown in Fig.5. The results of each successive case (R=0.7, 0.8, 0.9, 1.0) are changed gradually which gives logical interpretation.

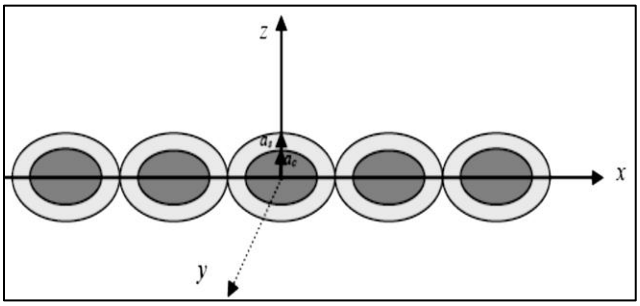


Fig.3 A linear chain cluster consists of five identical coated oblate spheroidal particles

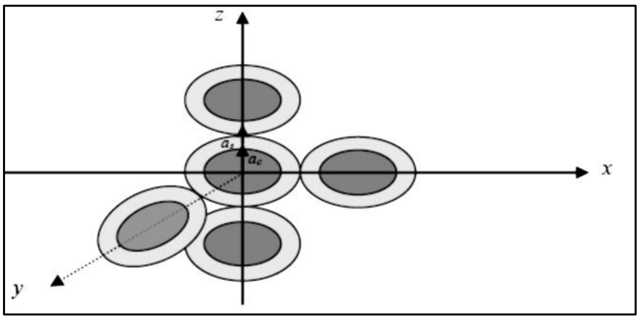
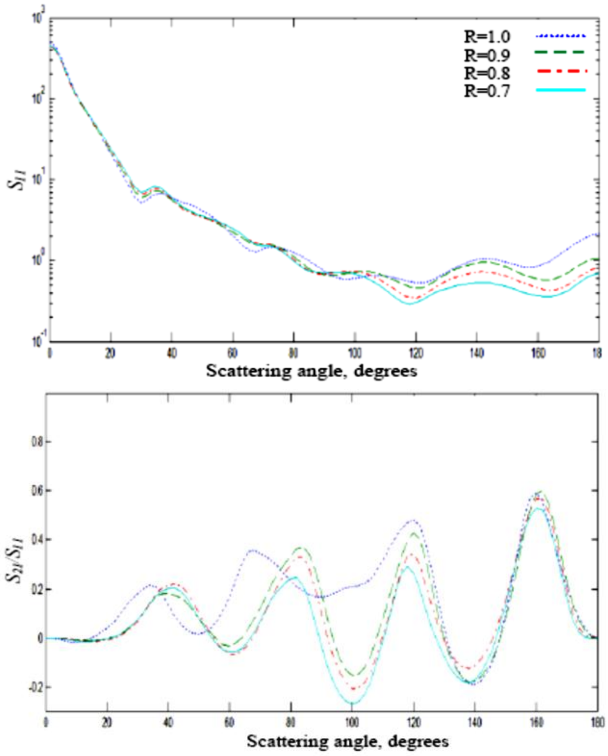


Fig.4 A packed cluster consists of five identical coated oblate spheroidal particles.



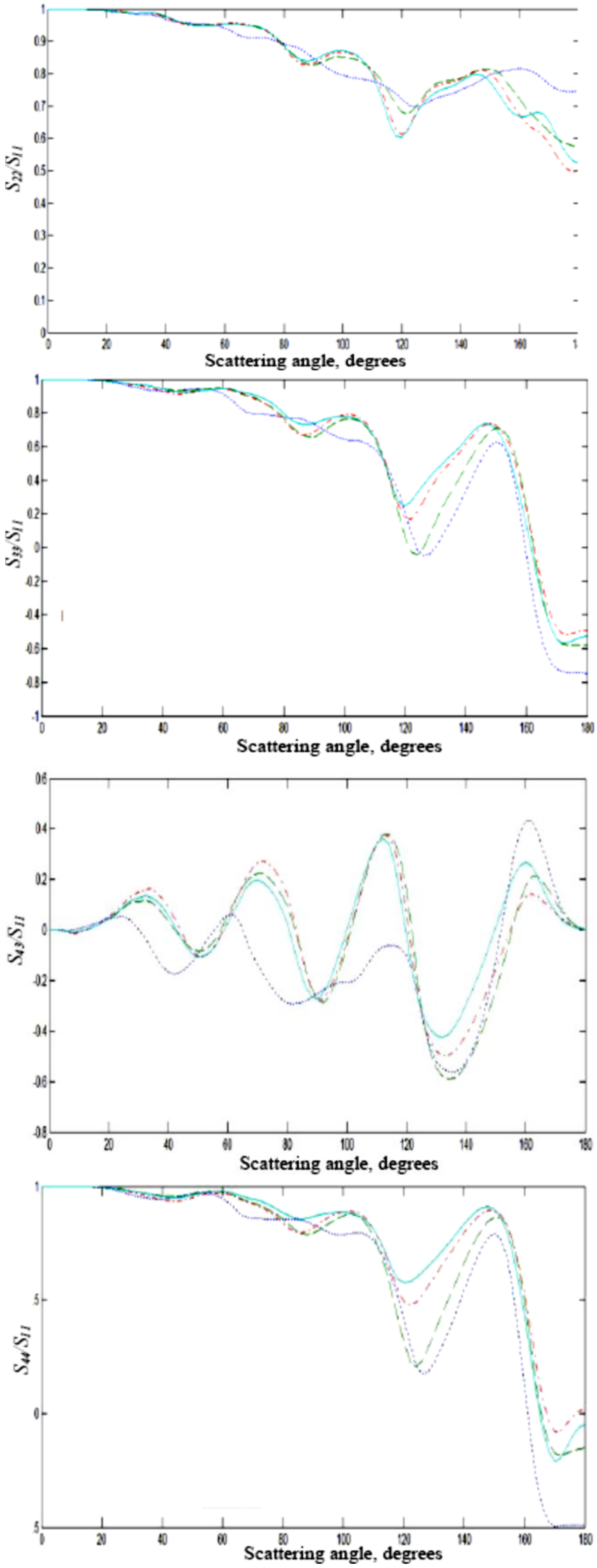
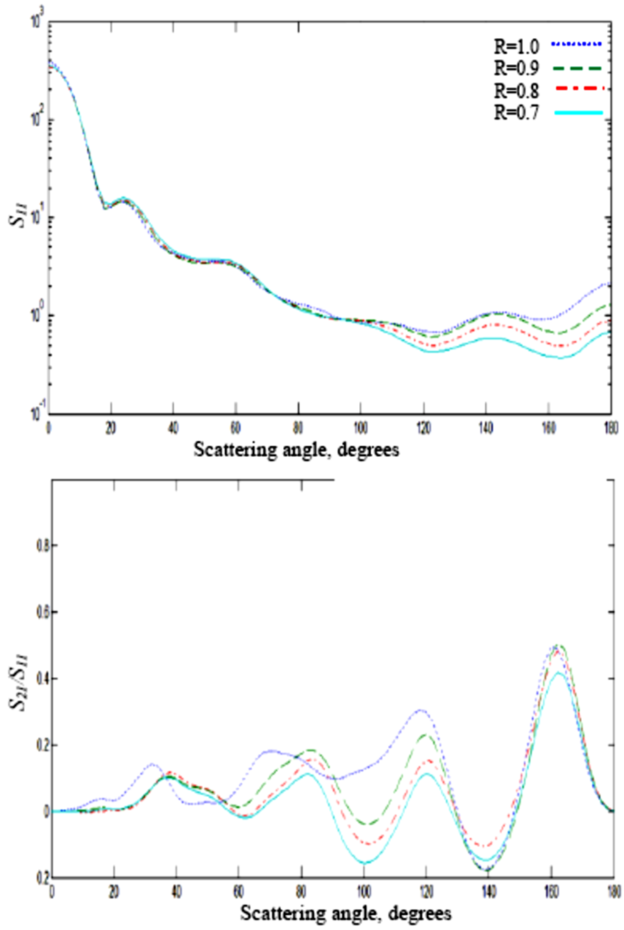


Fig.5. Orientation-averaged scattering matrix elements for a linear chain of five identical coated spherical particles, the ratio R=1, 0.9, 0.8 and 0.7 for each case, the size parameter is $x=2\pi a_s/\lambda=5$, and $m_s=1.36$, $m_c=1.5+0.005i$.

The cluster is illuminated with a plane wave, propagating in the *z*-direction. The case R=1 represents a cluster consists of homogeneous particle.

Figure 2 shows the first new case for a packed cluster (not chain clusters) consists of coated spheres which is not considered before in the literature. The coated spheres are constructed to shape a hexagonal lattice. The ratio R= a_c/a_s is changed to 0.9, 0.8 and 0.7 for all particles to represent three different cases of clusters each cluster in each case consists of identical particles. The size parameter of each individual coated sphere is 5, and the refractive indices m_s=1.36 and m_c=1.5+0.005i for the shell and the core respectively. The results of the scattering matrix elements corresponding to each case are illustrated in Fig. 6. The value of the element S_{11} change with the ratio R= a_c/a_s around the reverse direction but in the forward direction it is nearly independent of the ratio R. The locations of the oscillations of the scattering elements for the coated packed cluster are nearly the same for all the cases of homogenous coated spherical particles in the backward direction with the exception, of S_{22}/ S_{11}. On the other hand, the oscillations increase for the off-diagonal elements than for the diagonal elements of the scattering matrix. There are large changes in the values of on-diagonal elements around the forward scattering from those around the backward scattering. These results will be useful in characterization processes of clusters.



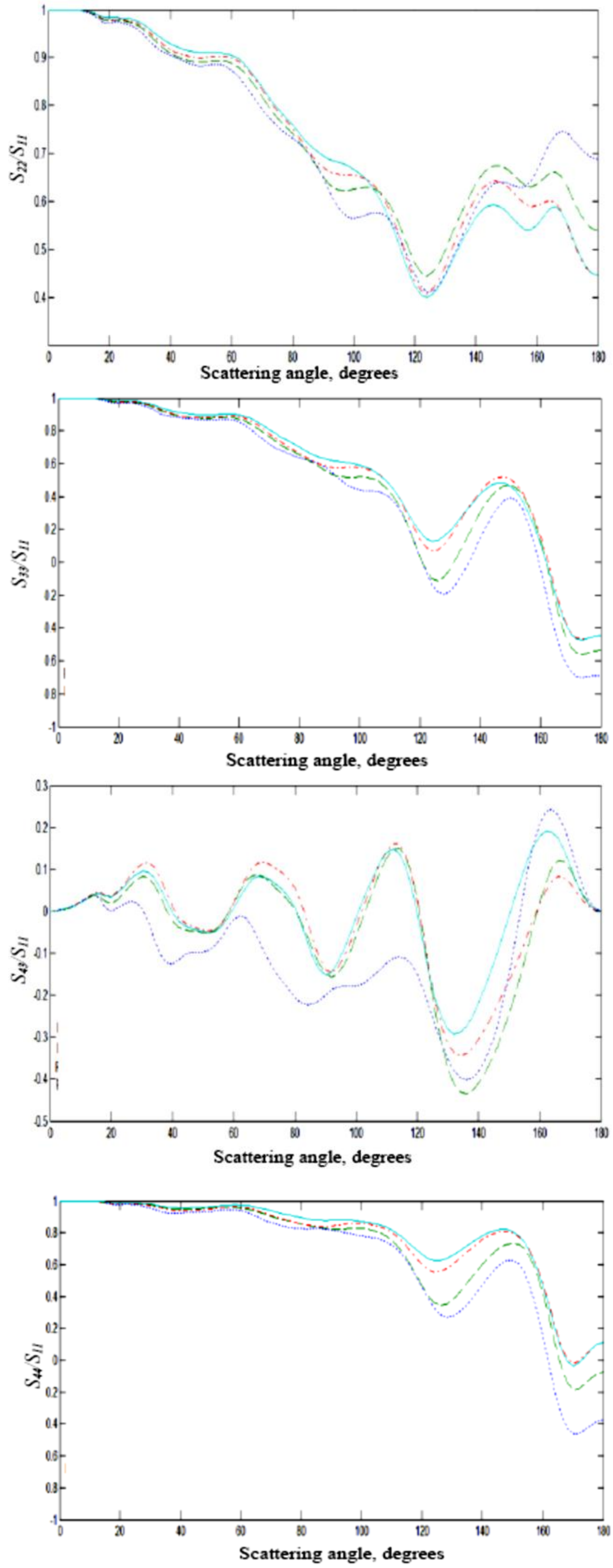
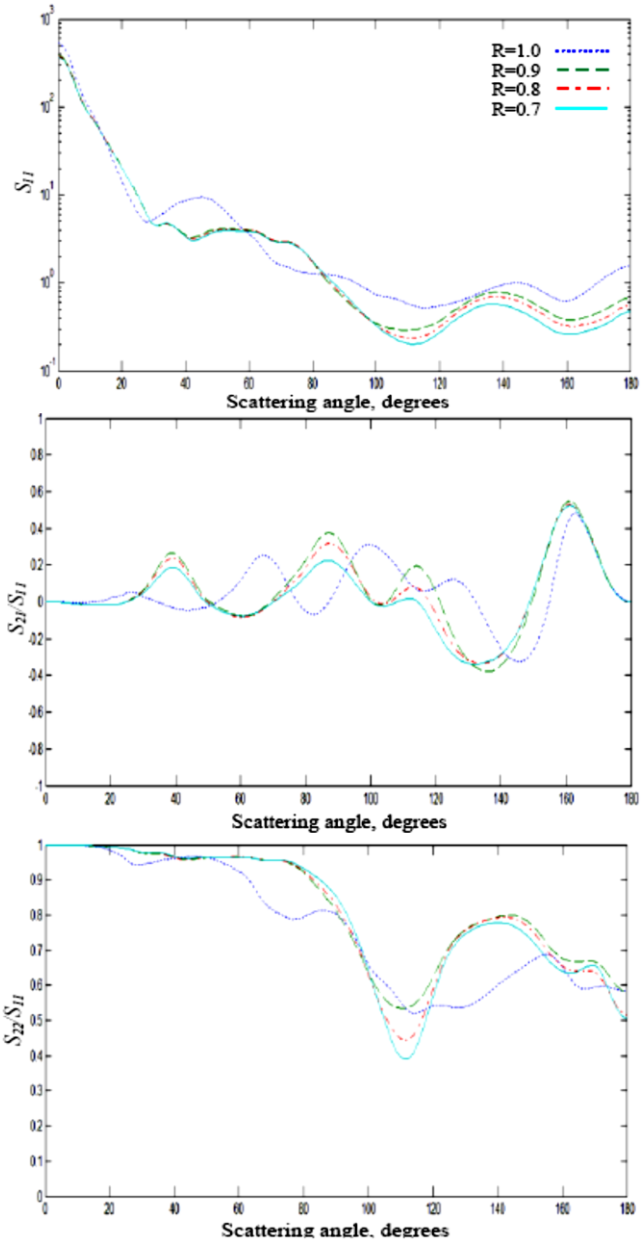


Fig.6. Orientation-averaged scattering matrix elements for a packed cluster of five identical coated spherical particles, the ratio R=a_c/a_s=1, 0.9, 0.8 and 0.7 each represents a separate case, each particle has a size parameter $x=2\pi a_s/\lambda=5$, and $m_s=1.36$, $m_c=1.5+0.005i$. The cluster is illuminated with a plane wave propagating in the *z*-direction.

The third considered case is a cluster consists of five oblate coated particles as shown in Fig.3. Each particle in the cluster has refractive indices m_s=1.36 and m_c=1.5+0.005i for the shell and the core, respectively, with size parameter x=2$\pi a_s/\lambda$=4 and the ratio a_s/b_s=0.8. The case of R=1 and m=1.5+0.005i represents a linear cluster consists of five homogeneous oblates. The results of the orientation averaged scattering matrix elements as a function of the scattering angle Θ^{scat} are shown in Fig.7 for $R=a_c/a_s$ be 1.0, 0.9, 0.8, 0.7 as different cases of linear clusters. Fig.4 shows the last considered case in which the cluster is a backed cluster consists of five oblate coated particles. Each particle has the same parameters as those illustrated in the previous case. The results of the scattering matrix elements for this cluster are illustrated in Fig. 8.



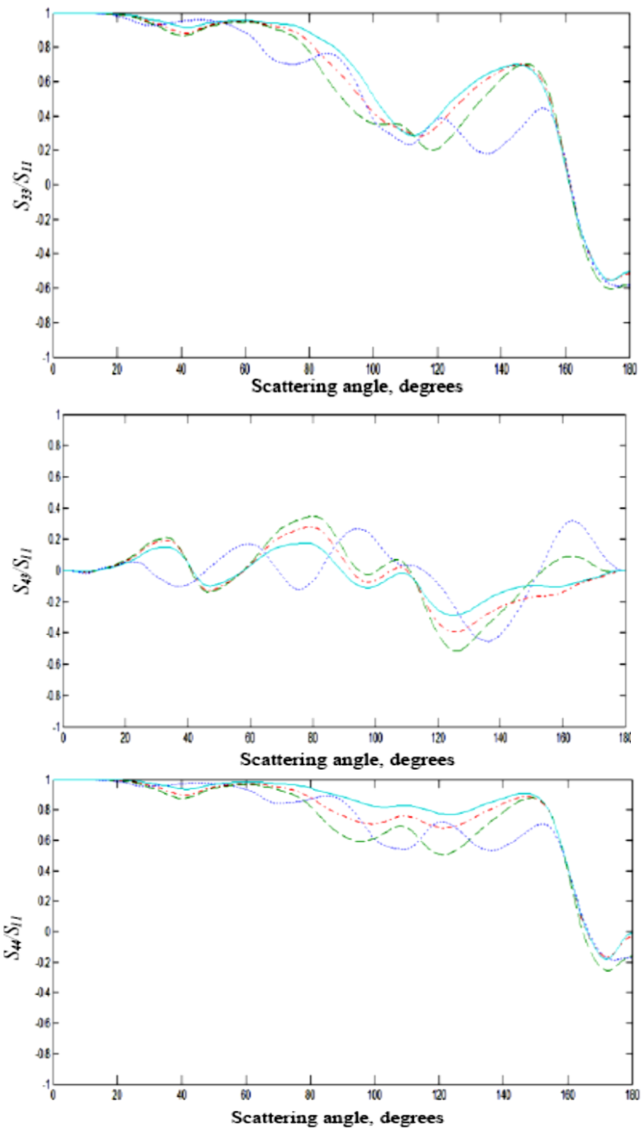
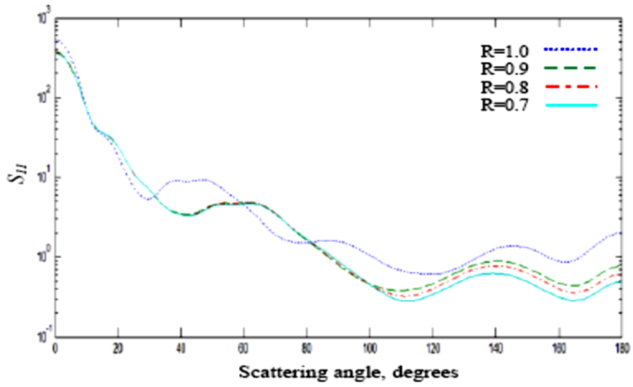
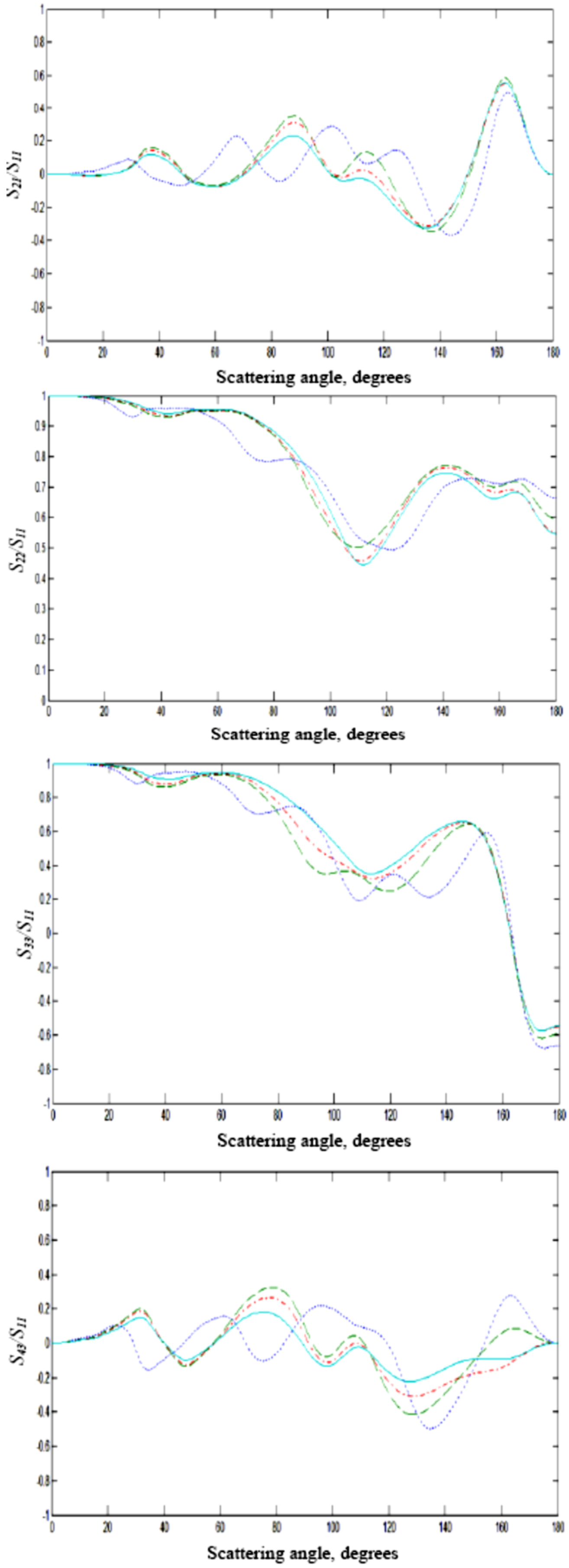


Fig.7. Orientation-averaged scattering matrix elements for a linear chain of five identical coated oblate spheroidal particles, the ratio $R=a_c/a_s$=1, 0.9, 0.8 and 0.7 each represents a separate case, each particle has a size parameter $x=2\pi a_s/\lambda=4$, and $m_s=1.36$, $m_c=1.5+0.005i$ and the ratio a_s/b_s=0.8 for each coated spheroidal particle. The cluster is illuminated with a plane wave propagating in the z-direction.





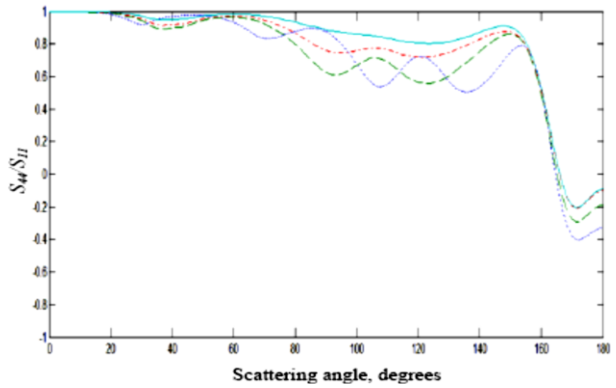


Fig.8. Orientation-averaged scattering matrix elements for a packed cluster consists of five identical coated oblate spheroidal particles, the ratio $R=a_c/a_s$=1, 0.9, 0.8 and 0.7 each represents a separate case, each particle has a size parameter $x=2\pi a_s/\lambda=4$, and $m_s=1.36$, $m_c=1.5+0.005i$ and the ratio a_s/b_s=0.8 for each coated spheroidal particle. The cluster is illuminated with a plane wave propagating in the z-direction.

The value of the scattering matrix element S_{11} for both the chain and packed cluster in the forward direction attain a form that is nearly independent of the value of $R=a_c/a_s$ from 0.9 to 0.7**.** Values of the oscillations in the matrix elements are large and bigger compared with those in the homogenous spherical particles. The locations of the oscillations for the coated chain and packed aren't identical to those for cluster of homogenous coated spheroidal particles.

4. Conclusions

A technique is modified and developed in this paper to determine the random-orientation scattering properties of a cluster consist of coated spherical and spheroidal particles. The considered cases here are linear chains cluster and packed clusters. Clustering particles affect the scattering by two mechanisms: far-field wave interference and near-field interactions, or, equivalently, multiple scattering. We present a sample of results and illustrate some salient features of scattering from different cases of clusters. Also computing the random-orientation scattering properties are made for different shell thickness of each particle in the cluster. The scattering matrix elements, as a function of scattering angle Θ^{sca}, for four different clusters are computed. The first case is a cluster formed by a linear chain consists of five identical coated spherical particles. The second case is performed by a packed cluster consists of five identical coated spherical particles. The third case is performed by a linear chain of five identical coated oblate spheroidal particles. Finally, packed cluster consists of five identical coated oblate spheroidal particles. Multiple scattering for packed-cluster configuration offers a much higher opportunity for multiple scattering among the coated spheres or spheroids. Therefore results of the packed-cluster have a greater difference in the random-orientation elements of the scattering matrix relative to on the chain clusters consists of coated spheres or spheroids**.**

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