

## Research Article

# Principles of Transforms in Signal Processing

Vijaya C<sup>a\*</sup><sup>a</sup>Department of Electronics & Communication Engineering, SDM College of Engineering & Technology, Dharwad**Abstract**

Signal processing is a mathematical way of manipulating the signal for useful purpose. It may be for the purpose of transferring the signal from transmitter to receiver side, in which case the processing system is referred to as communication system. Signal processing in a communication system for example, involves limiting the base band signal with respect to frequency and amplitude within certain range, modulating the base band signal using a carrier signal and demodulating the same at the receiving end, improving the quality of weak signal received at the receiving end, selection of one channel out of multiple channels and so on. Complexity of signal processing varies and many algorithms have evolved to reduce the complexity. While repetitiveness in the operations is exploited to reduce the complexity of signal processing, change in domain of processing also has been known to reduce the complexity of processing the signal. Original domain, for example, for speech signal is time domain in which it is represented in time-amplitude form, while original domain for image is spatial domain in which intensity level in image is represented as a function of two spatial coordinates. Representation of the signal in its original domain is not always the best representation of the signal for most of the signal processing applications. Changing the representation of the signal from one form to another form by applying mathematical transformations is referred to as transform. In some applications original domain best suits for signal manipulation and in some other applications, the transform domain best suits the processing. For non stationary signal processing, time-frequency domain representation sometimes best suits in signal processing applications. In this invited paper, an attempt is made to discuss various transforms, time-frequency representations and their advantages in signal processing applications.

**Keywords:** Transform domain, Time-Frequency Representation**1. Introduction**

There are varieties of reasons for signal processing. The purpose may be to improve the quality of the noisy signal, which is a degraded version of original signal. Such a signal may be received at the receiver through either wired or wireless channel or recorded in a noisy environment. Even the limitations of the recording system i.e., dynamic range of the capturing equipment, such as light intensity to electrical signal conversion in camera in the case of image or sound intensity to electrical signal conversion in mike in the case of sound signal, also degrade the quality of the signal. While filters may be used to improve the quality of the signal (Sadouki Furui, 1989), knowing the mathematical model of the degradation system an inverse system can be employed to undo the degradation introduced (Anil K. Jain, 2002) and thus improve the quality of the signal. Purpose of signal processing may be to compress and reconstruct the signal. While transmitting the signal to receiver or when signal is to be stored for future purpose, the criteria is to save time for transmission or save space for storage. In both the cases signal may

have to be compressed. The expectation from a compression technique is that it should reduce the size of original signal considerably and at the same time, it should not degrade the quality of the signal which is verified after reconstructing it to its original size. There are lossy compression techniques and lossless compression techniques (Sayood K., 2000). Some applications may withstand loss due to compression and some may not. For example, in medical image processing, loss due to compression may result in faulty diagnosis of disease. In the case of entertainment, TV or music environment, loss is tolerable and sometimes it is not even observable. While compressing video signal, repetitiveness of information in successive frames is exploited. The advantage of compression also can be seen in the fact that there will be less number of numbers to process in compressed domain and hence least computation time for signal processing in compressed domain. There are other signal processing tasks such as qualitative / quantitative enhancement of signal (Anil K. Jain, 2002), pattern recognition (E. Goose, et.al., 1996), weather forecasting, and so on, from simple tasks to more sophisticated systems and hence involved processes.

---

\*Corresponding author **Vijaya C** is working as Professor

There are three levels of signal processing. Several primitive type of signal processing may be treated as low level signal processing. Making the machine to understand the human language or human behavior or even read mind and act accordingly are also field of exploration in signal processing and they are treated as medium to high level signal processing. Hand writing recognition, any biometric (face / finger / iris / ear shape) recognition, separating voice signal of a singer from music, finding out mineral / water resources from a satellite image, weather forecasting, are to name a few applications wherein medium or high level signal processing is involved. Usually low level signal processing is a small integral part of any medium or high level signal processing.

There are numerous domains of signal processing in these three levels of signal processing. Neural network, artificial intelligence, nano signal processing, signal processing in signal domain itself, signal processing in transform domain are quite a few to name. Neural network approach for signal processing is close to natural way of signal processing by human being (Liberios Vokorokos, et.al., 2011). Conventional computers follow a set of instructions in order to solve a problem. The way of finding solution i.e., executing an algorithm should be known to the programmer so that he / she can write an optimized program for signal processing. Optimization may be in terms of speed or space utilization on hardware for implementation or both. Neural network is composed of a large number of highly interconnected processing elements (called neurons) working in parallel to solve a specific problem. High degree of parallelism can be modeled in neural network domain of signal processing. Artificial intelligence is about making computational models of human behavior / human intelligence. It is about automating tasks that have required human intelligence. A general purpose processor with some application software is a machine exhibiting artificial intelligence since it can perform all complicated arithmetic / logic operations. Robot is a sophisticated version of a general purpose processor. It can take decision based on vision, sound and other sensing devices just like a human being. Speech recognition / speaker identification, computer gaming, expert systems are some of the applications of artificial intelligence. Weather forecasting is yet another application. Mathematical model is developed for the previously available data such as temperature, humidity, altitude etc. and then prediction is done.

Nanoscience and nanotechnology are impacting physics, chemistry and biology, and creating new applications in different arenas. Medical application is one such field. It is possible to combine diagnosis and drug delivery into a single nanoscale system as multifunctional nanoparticles (Cordelia Sealy, 2009). Another entirely different field of application of nano science is finding out means of storing Hydrogen as a fuel for automotive applications is under exploration, including carbon nanotubes (CNTs) (G.K. Dimitrakakis, et al., 2008). Yet another application of nano science is in nano signal processing. It is observed that by selectively aligning a Ni-

capped ZnO nanowire in a magnetic field on a pre-fabricated electrode patterns formed by lithography, an air-gap field-effect transistor (FET) can be prepared (Sang-Won Lee et.al., 2010). This in turn has made it possible for realizing matrix-type devices based on one dimensional nano structures such as logic circuits, biochemical sensors, nanowires, nano cables, nanotubes, etc. and are regarded as promising candidates for the building blocks of various micro/nano-electronic, photonic, and spintronic devices due to reduced dimensionality. Novel design and simulation tool for quantum-dot cellular automata (QCA), namely QCADesigner are available. The paradigm of cellular-nonlinear-network universal machines (CNN-UM) provide a natural architectural concept for nano electronic signal processing (Árpád I. Csurgay, et.al., 2000).

There is significant development of signal processing techniques in the domain of the signal itself. Convolution as a response of FIR filtering, analog and digital IIR filters, and coding techniques: Huffman coding, arithmetic coding, linear prediction coding and statistical models, probability models for representing signals and systems, have been well established techniques working on signals in their respective domains. Various analog and digital modulation / demodulation techniques have been developed operating on signals in signal domain. Image enhancement can be performed in spatial domain of the image. It may be based on intensity level clipping, intensity level slicing, logarithmic based enhancement etc (Anil K. Jain, 2002). Diagnosis of disease can be based on image negative. There are algorithms for coloring a gray image. Some part of the image can be filled with color for medical diagnosis purpose.

The domains for signal processing are not independent of each other and also they are not that they cannot be independent of each other. Employing more than one signal processing technique in more than one domain sometimes results in better performance. For example, pattern classification can be achieved by developing neural network using fractional Fourier transform and ensuring least mean square error. It is found to be better than relying upon conventional Fourier transform and least mean square error (H. M. Ozaktas, et.al., 1994). Image compression can be achieved by performing DCT through neural network. Signal compression can be achieved by employing fractional Fourier transform followed by SPIHT encoding technique (Vijaya C., et.al., 2006).

The necessity of signal processing has given rise to many dimensions of technological developments contributing to signal processing. It may be development of algorithms in which case new techniques of signal processing have evolved. It may be development of special purpose processor for signal processing, known as digital signal processor evolving with its architecture suitably designed for signal processing purpose, in which case optimization of algorithm and its hardware implementation have witnessed new dimension. It may be optimizing the space for hardware implementation, in which case smaller and smaller nano meter scale VLSI design is emerging. Not only that, even programming techniques for software

development for the purpose of signal processing also a new dimension added by signal processing in technological development.

In the following sections, technique of transform domain signal processing and its importance is highlighted. Various transforms in signal processing are covered. Time-frequency representation and such transforms are discussed. Conclusion is drawn at the end, highlighting the importance of transforms in signal processing.

**Transform Domain Signal Processing:** Changing the representation of the signal from one form to another form by applying mathematical transformations is referred to as transform. Signal is decomposed in terms of orthogonal basis functions. Transform can also be viewed as projection of the signal on a set of basis functions. Each coefficient is the inner product of basis function with the given signal. Weightage given to each of the basis function in expressing a signal in transform domain is called transform coefficient. Transform domain brings out hidden information in the signal. There is no change in the information content. There is change only in the way the signal is viewed. Referring to the fig. 1, (x,y) is the list of coordinate points in x-y plane representing certain information. When the reference axis is changed to ('x new' and 'y new'), one of the coordinates, i.e., 'y new' becomes insignificant. Even if any value of 'y new' is set to zero, it is not effecting the position of the points in the plane. However, setting either x or y value to zero changes the position of the points in the plane implying that transform coefficients are highly decorrelated (Vijaya C., 2008). High decorrelation among the coefficients makes it possible to encode different coefficients with differing importance.

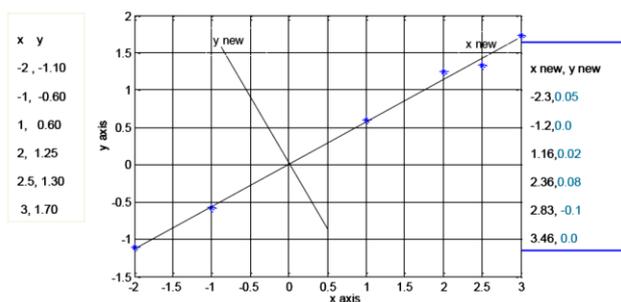


Fig 1. An Example of Decorrelation

Transforms have tendency to pack a large fraction of energy of the signal into a relatively few components of the transform coefficients. This feature of transform can also be observed in the example quoted in fig. 1. While when points are represented in terms of (x,y), all 12 numbers for various (x,y) are required whereas when points are represented in terms of (x new, 'y new'), only 6 numbers of 'x new' are significant. Thus there will be less number of numbers to manipulate in transform domain. Also it is possible to encode or allocate more bits for significant coefficients and least bits or zero bits for insignificant coefficients. Encoding the coefficients in this

fashion reduces total number of bits and high degree of compression can be achieved. For most of the transforms, the information carried by the signal is preserved even after transform which is evident by the fact that signal can be obtained by taking inverse transform. The transform domain provides an excellent compromise between the computational complexity and performance, because transform domain manipulations are simple and sometimes more advantageous than time domain operations. For example, in signal denoising applications, noise component can easily be separated from the noisy signal as the signal and noise occupy different regions in transform domain as shown in fig. 2.

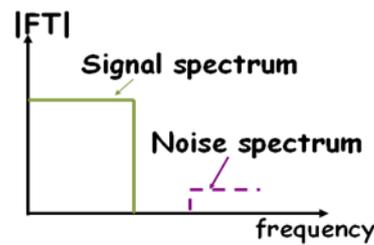


Fig.2 Spectrum of signal and noise

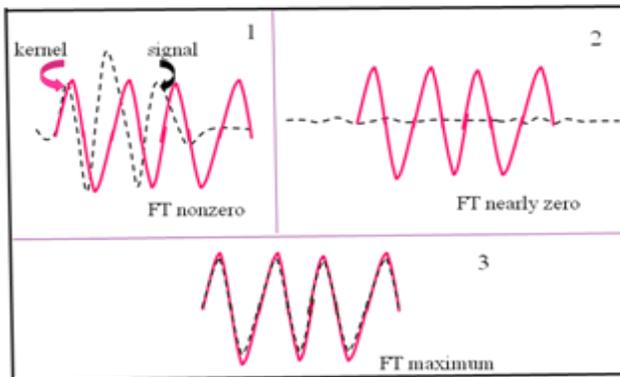
Noise suppression is achieved by applying a mask, a low pass filter, which is equivalent to multiplication of the transform of noisy signal with the mask. Filtering in time domain is process of convolution of input signal with the transfer function of the system. It involves fold, shift and cumulative multiply-add operations. In FT domain, by the property of FT, it is simplified to the multiplication of FT of the two signals to be convolved. After multiplication, inverse FT is applied to get only the signal component. Transform may also be viewed as searching for the pattern of basis function of the transform in the given signal. It is depicted in fig. 3 taking FT as the transform. Whenever the signal pattern matches with the basis function of FT, the corresponding coefficient value will be a maximum. This is so for any other transform also. FT searches for a frequency, in the given signal and since the summation in the transform expression is for all values of time, FT consolidates the presence of that frequency in the entire signal.

**Transforms in Signal Processing:** There are number of transforms frequently applied by engineers and mathematicians in signal processing. Because of the revolution in computational domain, most of the transforms are available in discrete form also. Table 1 lists some of the frequently used transforms in signal processing. They are (Anil K. Jain, 2002) discrete Fourier Transform (DFT), discrete cosine transform (DCT), discrete Sine Transform (DST), Hadamard transform etc. All the transforms listed here are linear transforms. Every transformation technique has unique area of applications, with advantages and disadvantages. As can be observed, they differ from each other with respect to the basis functions employed and the properties they satisfy. While the forward transform analyzes the signal in time domain, the inverse transform synthesizes the signal. By suitably

**Table 1:** Widely employed Transforms (Anil K. Jain, 2002)

S.No.	Transforms	S.No.	Transforms
1	<p><b>DFT:</b></p> $X(k) = \sum_{n=0}^{N-1} x(n)W_N^{kn}$ $k = 0 \dots N-1; W_N = e^{-j2\pi/N}$	2	<p><b>DCT:</b></p> $v(k) = \alpha(k) \sum_{n=0}^{N-1} u(n) \cos\left(\frac{\pi(2n+1)k}{2N}\right); 0 \leq k \leq N-1$ $\alpha(0) = \frac{1}{\sqrt{N}}; \alpha(k) = \frac{2}{\sqrt{N}} \quad 1 \leq k \leq N-1$
3	<p><b>DST:</b></p> $v(k) = \sqrt{\frac{2}{N+1}} \sum_{n=0}^{N-1} u(n) \sin\left(\frac{\pi(k+1)(n+1)}{N+1}\right)$ $0 \leq k \leq N-1$	4	<p><b>Hadamard transform:</b></p> $H_1 = \frac{1}{\sqrt{2}} [1, 1; 1, -1] \&$ $H_n = \frac{1}{\sqrt{2}} [H_{n-1}, H_{n-1}; H_{n-1}, -H_{n-1}]$

modifying / selecting the coefficients of a transform, a signal can be synthetically synthesized.



**Fig.3** Searching for pattern in applying FT

DFT is the most widely used transform (J.G. Proakis, et.al., 2000). Basis functions employed in DFT are complex sinusoids. By computing the DFT of a signal, it is decomposed into its individual, N frequency components,  $\omega_k = 2\pi k / N$ . DFT coefficients depict the relative significance or contribution of each frequency component represented as complex sinusoid. For this reason, DFT domain is sometimes referred to as frequency domain. Inverse transform has its kernel as complex conjugate of forward transform, thus easy to compute the same. The first coefficient represents zero frequency and the rest of the coefficients represent frequencies in the increasing order. Thus any type of filtering can be easily applied which amounts to selective selection of required coefficients. For most of the real time signals, audio / video DFT has coefficients in the decreasing order of values. This indicates the inherent property of real time signal that it is a band limited signal and has low frequency components. During signal compression, ignoring high frequency components does not affect the quality of the signal. Due to symmetry in the complex coefficients, number of numbers for representing the coefficients remains same as the number of samples in the signal. There are algorithms for fast computation of DFT

known as fast Fourier Transform (FFT). While complexity in direct computation of DFT is of the order of  $N^2$ , FFT reduces it to the order of  $\log_2(N)$  where N is the number of samples in the signal. Fourier domain is useful in filtering as mentioned earlier. In real time signal processing, signal will have large number of samples to be processed. Instead of resorting to long procedure of convolution, multiplication in Fourier domain is advantageous. Long sequence is divided into overlapping small blocks and filtering is done on block basis. It also saves memory requirement during computation. Fourier domain finds one of its applications in separating the envelope and spectral fine structure in a speech signal. The spectral envelope represents slowly changing part in the speech spectrum as a function of frequency. It represents resonance and anti-resonance characteristics of articulatory organs, overall shape of the glottis and radiation characteristics at the lips and nostrils. The spectral fine structure represents fast changing part in the spectrum as a function of frequency. It represents voiced sound and periodicity of sound source. Speech signal, which is modeled as product of spectral envelope and spectral fine structure is given by eq (1). The product terms are separated in log domain, referred to as cepstrum analysis, by taking log of eq (1) as given in eq (2). By taking inverse transform and by applying low quefrequency (frequency equivalent in log domain) liftering (filtering in log domain), first term representing spectral envelope is extracted. Similarly, high quefrequency liftering is applied to find peaks in second term representing fundamental period of speech signal. To quote yet another application of Fourier domain, since the magnitude of the DFT of the shifted signal and of the signal itself does not differ, DFT domain is useful for pattern matching. Periodic noise which stands out as single frequency component in FT domain may be removed by suppressing it in FT domain.

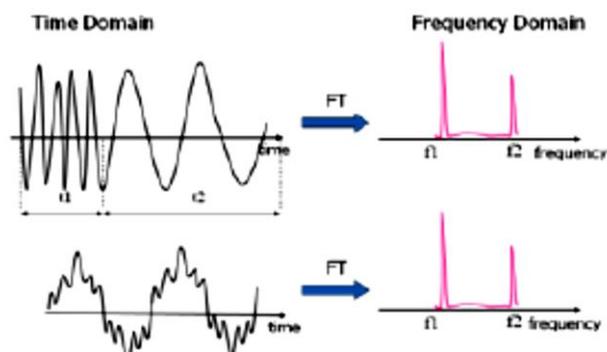
$$X(\omega) = H(\omega)G(\omega) \quad (1)$$

$$\log|X(\omega)| = \log|H(\omega)| + \log|G(\omega)| \quad (2)$$

$$IFT(\log|X(\omega)|) = IFT(\log|H(\omega)|) + IFT(\log|G(\omega)|) \quad (3)$$

DCT has cosine based real kernel, because of which its coefficients are real. It is observed that of all the transforms available, DCT represents the signal in compact form with its performance close to Karhunen and Loeve transform (KLT) which is an optimum transform. It exhibits highly decorrelated transform coefficients and hence excellent energy compaction in comparison with any other transform. The coefficients are in the increasing order of frequency of basis cosine term. It can be computed by computing DFT of symmetrically extended signal. It is the widely used transform in signal compression in multimedia applications such as in MPEG standards (V. Goyal, 2001). Inverse DCT has its kernel which is transpose of the basis matrix for forward transform. There are several DCT forms differing slightly in their basis function. There are fast techniques of computing DCT. Several architectures have been developed for the purpose. Because of its extensive use in multimedia signal processing, new algorithms are being developed for the purpose of signal processing in transform domain. Block edge detection in transform domain is one such example. KLT is termed as optimal transform because of high decorrelation among its coefficients (Anil K. Jain, 2002, V. Goyal, 2001). However, its kernel is signal dependent and it is computationally intensive. Basis vectors of KLT are given by the orthonormalized Eigen vectors of autocorrelation matrix of the signal. It has property of producing highly decorrelated coefficients.

**Time-Frequency Representation :** Although FT (DFT and DCT) is the widely used transform in signal processing, it does not suit all the requirements / situations of signal processing (Vijaya C., 2008, L. Cohen, 1989). As the computation of FT involves inner product of signal and the complex sinusoid over the entire time duration, there is no way to distinguish the local characteristics of the signal under analysis from its global characteristics during the computation of FT. Due to which FT fails to retain the information about the time duration of occurrence of different frequency components in the original signal. FT assumes the signal as stationary signal, i.e., a signal with no change in its frequency spectrum with respect to time.



**Fig. 4:** Time & Frequency Domain representation of signals

However, there exist natural and man-made nonstationary signals, which have time varying spectra. While sunlight is an example of signal with slowly varying spectral contents, speech signal is an example of signal with rapidly varying spectral contents. Inability of FT to bring out local characteristics is clear in fig. 4, that even if the two signals in time domain are entirely different, their FT are same. FT does not suit for the analysis of nonstationary signals, as the time resolution is zero in its domain.

For nonstationary signal analysis, there is a need for a transform, which is a joint function of time and frequency that describes the energy density or signal intensity simultaneously in the time and frequency plane. This is referred to as Time Frequency Representation (TFR) (L. Cohen, 1989).

Such a TFR finds fraction of energy in certain time and frequency range. Also, it would be a powerful tool for the construction or synthesis of signals with desirable time-frequency characteristics. In other words, TFR carry information of a signal in a convenient and precise way. Fractional Fourier transform, short time Fourier transform and wavelet transform are examples of linear TFRs. They are listed in table 2.

The principle of TFR can be understood with the help of short time Fourier transform (STFT) (L. Cohen, 1989, Haldun M. Ozaktas, et.al., 2000). In STFT, the entire time duration of the signal is divided into windows of short duration and FT is computed for each window. The window signal  $y(t)$  is of fixed duration. The window width is so chosen that the signal is stationary within the window so that FT is applicable. The window width conveys the duration of FT computation and this localizes the frequency analysis and hence the name for the technique as short time Fourier transform. With a longer time interval for the window signal, the band of frequencies covered is narrower leading to poor time resolution and better frequency resolution. With a shorter time interval for the window signal, the band of frequencies covered is wider. Hence it results in better time resolution and poor frequency resolution. There is no provision for changing the time interval during analysis. Such representation is generally called spectrogram. For the analysis of a slowly varying signal, wider window width is preferred and for a fast varying signal, narrow window width is preferred in order to capture the signal variation effectively. With the fixing of the analysis window, time and frequency resolutions are fixed throughout the analysis. Typical expectations from STFT analysis of a signal is shown in fig. 5. It depicts the presence of various frequency components during different time points. Because of fixed time and frequency resolution in STFT, it is not possible to resolve both the low and high frequencies of the signal simultaneously. A technique that has variable time and frequency resolution is required.

Varying time and frequency resolution is possible in wavelet transform (WT) (Raghuvver M. Rao, et.al., 2000). The analysis is carried out with scaled and shifted versions of basis function called wavelet as indicated in table 2. Scaling and shifting of wavelet during signal analysis results in variable time and frequency resolution. The

wavelet function can be chosen based on the characteristics of signal under analysis. Application of wavelet transform to a signal is shown in fig. 6. As the time width of the window reduces, frequency resolution becomes poor and vice versa as shown in fig. 7.

It implies that WT cannot discriminate signals with too close high frequencies. Uniqueness of WT is that it does not have a unique basis function. It can be selected depending on the signal. This feature helps in catching up with the rhythm of the signal. It localizes events in time and frequency domain. The basis function can be stretched or compressed dynamically by varying dilation parameter 'a' kernel can be shifted anywhere in the time domain dynamically by varying translation parameter 'b'. These two features result in variable time and frequency resolution. Thus, it is a technique that catches the rhythm of the signal. Thus, WT behaves as a mathematical microscope.

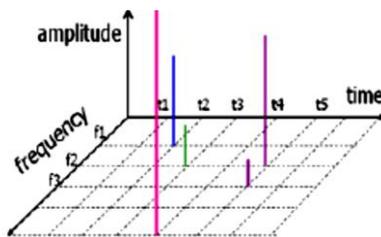


Fig.5 Typical view of STFT

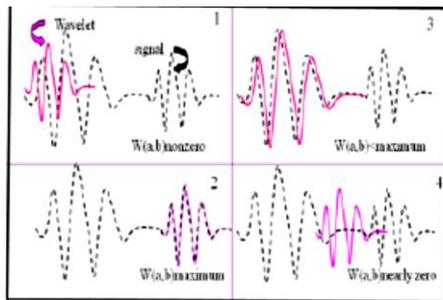


Fig.6 WT catches of rhythm of signal

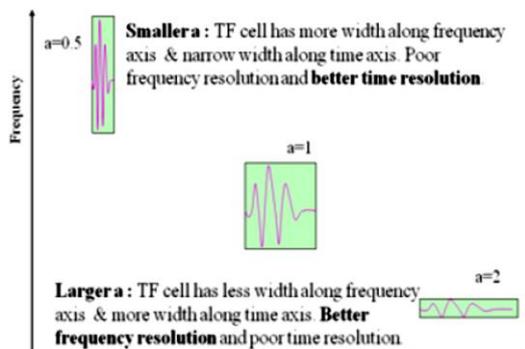


Fig.7 Time frequency resolution in WT

WT separates low and high frequency signals by decomposing the signal at various levels. It is also equivalent to passing the signal through low pass and high

pass filters. Low frequency signals are approximation of input and high frequency signals are minute details that are missing in making the approximation. Signal Compression can be easily carried out by retaining only approximated signal. Various end receivers / users in multimedia applications may prefer different resolutions of the signal. Receiver can reconstruct the signal using approximation coefficients along with different amount of detailed coefficients based on resolution that it desires. Similarly for de-noising purpose, only approximated coefficients are retained since noise is always high frequency signal which is in detailed coefficients of WT. It finds application in pattern recognition by considering coefficients at various levels of decomposition.

In nonstationary signal analysis, it is customary to use the time-frequency plane, with two orthogonal time and frequency axes. This enables representation of signal parameters such as signal energy, signal intensity etc. simultaneously with respect to time and frequency. FT can be interpreted as a rotation of time domain signal by an angle  $\pi/2$  in the time-frequency plane and represented as an orthogonal signal representation for sinusoidal signal. A rotation by integer multiple of  $\pi/2$  can thus be defined through repeated application of FT. A rotation angle  $\alpha = a\pi/2$ , with  $a$  being a real number, leads to a domain that represents the signal both with respect to time and frequency. Signal representation along this intermediate axis making an angle  $\alpha$  with time axis, in time-frequency plane has nonzero time and frequency resolutions. As the angle of rotation is fraction  $a$  of  $\pi/2$ , the transform is known as fractional Fourier transform (FRFT) with an order parameter  $a$ . Thus, FRFT is a generalization of the FT with an order parameter  $a$  (Vijaya C., 2008, Haldun M. Ozaktas, et.al.,2000). While first order FRFT, with  $a = 1$ , is the ordinary FT and the zero<sup>th</sup> order FRFT, with  $a = 0$ , is the signal itself. The FRFT is also called rotational Fourier transform or angular Fourier transform. It performs a rotation of signal in the continuous time-frequency plane by an arbitrary angle and serves as an orthonormal signal representation for the chirp signal, a signal whose frequency varies linearly with time. FRFT has been proved to relate to other time varying signal analysis tools, such as Wigner distribution, wavelet transform etc. Computational complexity of FRFT is same as that of FT. An added advantage of FRFT is the freedom in the choice of orientation of transform axis with respect to time axis. This makes FRFT a powerful tool in solving differential equations, optical signal processing, time variant filtering and multiplexing, swept frequency filters, pattern recognition, digital watermarking, time-frequency signal analysis, Fourier optics and optical information processing. In fig. 8, the spectrum of noisy signal is shown, which indicates that both in time domain and in frequency domain, noise and signal overlap. Whereas at different FRFT domains they are not overlapping and hence signal can be recovered free of noise. Similarly, attempts have been made to represent other transforms as fractional transforms. Fractional cosine transform,

Fractional Hilbert transform, Fractional Hartley transform and Fractional WT are some examples.

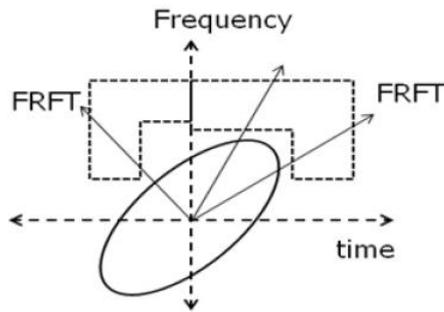


Fig.8 Filtering in FRFT domain

Table 2 Widely employed TFR Transforms

S.No.	Transforms
1	$STFT\{x(t)\} = STFT(t, f) = \int_{-\infty}^{\infty} x(\tau)y^*(\tau-t)e^{-j2\pi f\tau}d\tau$
2	<p>wavelet transform :</p> $WT\{x(t)\} = W(a, b) = \int_{-\infty}^{\infty} x(t)\Psi_{a,b}^*(t)dt$ $\Psi_{a,b}(t) = \frac{1}{\sqrt{ a }}\psi\left(\frac{t-b}{a}\right)$ $\Psi_{1,0}(t) = \psi(t) \quad \Psi_{1,b}(t) = \psi(t-b) \quad \Psi_{a,0}(t) = \frac{1}{\sqrt{ a }}\psi\left(\frac{t}{a}\right)$
3	<p>FRFT :</p> $f_{\alpha}(u) = \int_{-\infty}^{\infty} K_{\alpha}(u, u')f(u')du'$ $K_{\alpha}(u, u') = \frac{\sqrt{1-j\cot\alpha}}{2\pi} e^{j(\cot\alpha u^2 - 2\csc\alpha uu' + \cot\alpha u'^2)}$

**Conclusion**

Every application in the field of engineering involves signal of different types and their processing. There are several techniques of signal processing and domains of processing. They are not entirely independent of each other. Recent developments in signal processing have lead to special purpose processor for the purpose, development of software, optimizing performance of the software code, space required for its implementation and so on thus giving way to entirely new fields of research. An attempt is made to highlight significance of one of the domains of signal processing, i.e., transform domain. Principles of transform domain signal processing, its limitations and improvement over it by processing in time-frequency domain are highlighted in this paper, taking few examples of widely used transforms and TFRs.

**References**

Sadouki Furui, (1989), Digital Speech Processing, Synthesis and Recognition. *Marcel Dekker, Inc.*  
 Anil K. Jain, (2002), Fundamentals of Digital Image Processing, *Prentice-Hall of India*  
 Sayood, K., (2000), Introduction to Data Compression, Second Edition, San Mateo, CA: *Morgan Kauffman/Academic Press.*  
 E. Goose, R. Johnsonbaugh, S. Jost, (1996), Pattern recognition and image analysis, *Prentice Hall Inc.*  
 Liberios Vokorokos, Norbert Ádám (SAMI 2011), An Introduction to the Neural DF Architecture, 9<sup>th</sup> *IEEE International Symposium on Applied Machine Intelligence and Informatics*  
 Cordelia Sealy, (2009) Putting the squeeze on polymers *Nano Today*, vol. 4, pp. 3-7  
 G.K. Dimitrakakis, (2008) *Nano Lett.*, doi:10.1021/nl801417  
 Sang-Won Lee a, Moon-Ho Hama,1, Jyoti Prakash Kar, Woong Lee, Jae-Min Myoung (2010) Selective alignment of a ZnO nanowire in a magnetic field for the fabrication of an air-gap field-effect transistor *Microelectronic Engineering* vol. 87, pp.10-14  
 Árpád I. Csurgay, Wolfgang Porod, and Craig S. Lent, (2000), Signal Processing with Near-Neighbor-Coupled Time-Varying Quantum-Dot Arrays *IEEE Transactions on Circuits And Systems-I: Fundamental Theory And Applications*, Vol. 47, No. 8  
 H. M. Ozaktas, B. Barshan, D. Mendlovic, and L. Onural, (1994), 'Convolution, filtering, and multiplexing in fractional Fourier domains and their relation to chirp and wavelet transforms', *J. Opt. Soc. Am.A*11, pp.547-559  
 Vijaya C., J.S. Bhat, (2006), Signal compression using discrete fractional Fourier transform and set partitioning in hierarchical tree, *Signal Processing* vol. 86 pp.1976-1983  
 Vijaya C., (2008), Ph. D thesis 'Some studies on Digital Signal Processing'  
 J.G. Proakis, D.G. Manolakis, (2000), Digital Signal Processing Principles, Algorithms and Applications, *Prentice Hall of India*  
 V. Goyal, (2001), Theoretical foundations of transform coding, *IEEE Signal Process. Mag.* pp.9-21  
 L. Cohen, (1989), Time frequency distributions-a review, *Proc. IEEE* 77 (7) pp.941-981  
 Haldun M. Ozaktas, Zeev Zalevsky, M. Alper Kutay, (2000), Fractional Fourier Transform with Applications in Optics and Signal Processing., *John Wiley & Sons Ltd.*  
 Raghuvver M. Rao, Ajit S. Bopardikar, (2000), Wavelet Transform Introduction to Theory and Applications, *Addison Wesley*