Research Article

A Hierarchical Data Structure for A Non-Manifold B-Rep Modeler for Thin Components

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Abstract

This paper proposes a Hierarchical Data Structure (HDS) for modeling of non-manifold thin objects, consisting of seven nodes named as Thin Object, Sheet, Feature, Face, Part Edge, Edge and Vertex. This paper modifies and validates a general topological invariant $s + e_L + e_{NL} + e_{FF} + b + j - v - g_n - f_P - f_{NP} - f_{FF} = 0$ regarding the number of sheets, linear edges, non-linear edges, freeform edges, bends, joints, vertices, non-manifold genuses, planer faces, non-planer faces and free form faces respectively for non-manifold thin objects. Corresponding Euler operators are derived, providing a basis for a modeling system for thin-walled objects.

Keywords: Non-manifold Thin Objects, Topological Invariant, B-Rep, Euler Operators.

1. Introduction

Solid modeling has developed rapidly, especially for manifold geometries. Various data structures for solid modeling exist but most of them are based internally on manifold topology and manifold operators. However, many real solids do not comply with manifold topology as these do not validate modified Euler-Poincáre (Lipson et al., 1998) relation $f + v - e - w - b + 1 - m = 2s$, where $f$, $v$, $e$, $w$, $b$, $l$, $m$ and $s$ represents faces, vertices, edges, welds, bends, loops, volumes and bodies respectively. One such case is that of a thin-walled object based on the fundamental thin-walled primitive, as illustrated in Fig. 1.

Fig.1. A schematic thin-walled object

This paper discusses the topological properties of this type of objects and proposes a topological invariant which can be used as a basis for modeling thin-walled constructions in combination with the standard boundary representation and constructive solid geometry paradigm. Thin-walled objects are prevalent in many engineering disciplines, such as sheet metal, composite materials, injection molding in plastics as well as ornaments made of gold and other related metals. Dedicated CAD systems often use a thin-walled representation (SheenH, D.P et al., 2010) to describe the main geometry of thin objects, with a thickness parameter to represent the third dimension as an additional attribute varying along the main geometry. Although geometrical models of thin products can also be represented by full solid primitives, a better geometrical representation could be achieved by using a scheme as presented in this paper for standard topology of non-manifold (Mantyla M et al., 1987) thin objects.

This paper aims to propose the following:

- Thin-walled primitives (sheets) to represent thin-walled objects according to the boundary representation paradigm.
- A topological invariant for all thin-walled objects which may be used as a necessary condition for verifying topological validity and for reasoning about topological configurations.
- A Hierarchical Data Structure (HDS) for modeling of non-manifold thin objects, consisting of nodes named as Thin Objects, Sheet, Feature, Face, Part Edge, Edge and Vertex.
- A set of topological ‘Thin-walled Euler operators’, which may be used as basic building blocks for constructing and manipulating a thin-walled model representation.

2. The Fundamental Thin-Walled Primitive (Sheet)

A thin-walled (Lee, K et al., 1995; Lee, S.H et al., 2009) object can be considered as consisting of one or more thin elements that are joined together. Hence, the product is
composed of basic thin-wall sheets. A thin-walled sheet, as illustrated in Fig. 2, can be planer or non-planer (freeform). However, its topology always consists of a single face with a continuous single boundary. The boundary is composed of linear, non-linear or freeform edges joined at vertices, as shown in Fig. 2.

These thin-walled sheets may join along common edges, vertices and faces. The junction may occur not only along a complete or a part of edge and face but also an edge can join a face as shown in Fig. 3. The following sections establish the existence of relationships, and their validation, among the three elements of a thin-wall sheet (facet, edges and vertices) for a model composed of one or more sheets.

3. Topological Properties of the Sheet

We proceed to analyze a single thin-wall sheet consisting of vertices, edges (linear, non-linear and free form), and faces (planer, non-planer and freeform) (Goel, V.K et al, 2006). The modified Euler-Poincaré Law (Lipson, H et al, 1998) is further refined as

$$f_p + f_{NP} + f_{FF} + v - e_L - e_{NL} - e_{FF} - w - b + l - m = 2s$$

(3.1)

where $f_p$, $f_{NP}$, $f_{FF}$ represent the number of planer, non-planer and freeform faces and $e_L$, $e_{NL}$, $e_{FF}$ the number of linear, non-linear and freeform edges sheet (including the exterior face). Since, by definition, thin objects are combination of sheets contains a single facet (the exterior face does not represent a facet) surrounded by a single boundary of edges, which asserts that

$$f_p + f_{NP} + f_{FF} + v - e_L - e_{NL} - e_{FF} - w - b + l - m = s$$

(3.2)

where’s’ represents the number of sheets (or number of objects).

3.1 Genuses and Volumes

A sheet can contain two types of holes as illustrated in Fig. 4. The first type is a ring: a hole (blind or through) interior to a sheet disconnected from the external boundary of the sheet. Traditionally, such rings (holes) are considered as special topological elements and are counted explicitly in a modified Euler-Poincaré formula for manifold objects. Since a ring is local to single face and is disconnected from the main component topology, it can either be ignored or modeled as a feature (Farsi, M.A et al, 2009) on the sheet. The second type is genus or handle - a hole that crosses or touches one or more of the sheet's edges. It is generated when two or more sheets are joined together, leaving a ‘gap’ between them. Therefore, the genus of an object corresponds to the connectivity of two or more sheets, as connectivity is a topological quality so these are very much included in the discussion of modified topological invariant for thin objects and local loops l are replaced by non-manifold genus $g_{NM}$ in Eqn. 3.2. So, Eqn. 3.2 is modified as

$$f_p + f_{NP} + f_{FF} + v - e_L - e_{NL} - e_{FF} - w - b + g_{NM} - m = s$$

(3.3)

where $g_{NM}$ represents no. of genus or handles.

3.2 Bends and Joints

As thin objects are geometrically modeled as combination of thin sheets and various features like bends (slotting, hamming, flanging, etc.) and joints (welding, seams, etc.) along a linear, non-linear or freeform edge so Eqn. (3.4) takes the more generalized form as

$$f_p + f_{NP} + f_{FF} + v - e_L - e_{NL} - e_{FF} - j - b + g_{NM} = s$$

(3.5)

where j is no. of joints. Rearranging Eqn. (3.5)
\[ s + e_L + e_{NL} + e_{FF} + b + j - v - g_a - f_P - f_{NP} - f_{FF} = 0 \]  

(3.6)

which is desired and more general topological invariant for thin objects.

4. The Data Structure for Modeling of Nonmanifold Thin Objects

The data structure used to model non-manifold thin objects is an extension of the data structure proposed by Mantyla for representing manifold sheets. It is referred as half-edge data structure and is a variation of the full winged edge data structure.

In this work a hierarchical data structure, consisting of nodes of type Sheet, Feature, Face, Part Edge and Vertex is proposed. The hierarchical features of the various node types are described below:

4.1 Thin Object

It either contains a single sheet or a combination of more than one sheet. A node of a thin object forms the root node of an instance of the hierarchical data structure. It gives access to nodes of sheets, features, faces, edges, part edges and vertices of the model through doubly linked pointers.

4.2 Sheet

The sheet node represents a three dimensional face and gives access to nodes of features, faces, edges, part edges and vertices of the model through doubly-linked pointer. All sheets are also linked to the next and the previous sheet through pointers.

4.3 Feature

The feature node represents one or more features associated with list of features which includes bend, joint, cut, hole, handle, fillet, rib, bump, cavity and any freeform feature which can be geometrically defined. It gives access to nodes of faces, edges, part edges and vertices of the model in hierarchy through doubly-linked pointer.

4.4 Face

A face node represents planer, non-planer or freeform surface defined by a parametric equation or no. of control points. As faces with multiple boundaries are included into the data structure, therefore, each face is associated with a list of part edges, representing one part of a boundary curve of the face. To realize the doubly-inked list of all faces of a sheet each face includes pointers to the previous and the next face in the list. Finally, each face has a pointer to its parent sheet and children part edges.

4.5 Part Edge

A part edge node describes part of one curve segment of a face. It consists of a pointer to its parent face and a pointer to the vertex of the curve segment in the direction of the face. Pointers to the previous and the next part edge realize a doubly-linked list of part edges of a face. Each Part edge includes an additional pointer to its child edge.

4.6 Edge

An edge node associates two or more part edges with each other; it combines the two or more parts of a full edge together. It consists of pointers to the direction of part edge. The doubly-linked list of edges is realized by means of pointers to the next and the previous edge.

4.7 Vertex

A vertex node contains a vector representing coordinates of a point of \( E^3 \). Two pointers to the next and the previous vertex realize a doubly-linked list of vertices of the sheet. Each vertex includes an additional pointer to one of the edges emanating from it.

The hierarchy is depicted in Figure 4 below including the names pointers.

![Fig 4. Schematic Diagram for Hierarchical Data Structure](image)

5. Thin Walled Building Operations

Modified Euler-Poincare law as per Eqn. 3.6 forms the basis to develop building operations (T Suzuki et al. 2007) to create boundary models of non manifold thin objects. These operators are classified as M for make, K for Kill, A for add, R for remove and MOD for modifications or change. These operations are described with the help of Table 2. Other operators are also available as shown in Table 2 to add convenience and flexibility to the construction process.
Table 1 Implementation of data structure with C++.

```cpp
class Hierarchalobject {
    int _objectid; // id of Hierarchal Object
    Attribute *attribute;
    public:
    int *_nextlink; // Pointer to Next entity of Hierarchal Object
    int *_prevlink; // Pointer to Previous entity of Hierarchal Object
    int *_parentlink; // Pointer to Parent of Hierarchal Object
    int *_childlink; // Pointer to child of Hierarchal Object
};
```

Table 2. Some thin-walled Euler Operators

<table>
<thead>
<tr>
<th>Description</th>
<th>Operations</th>
<th>Complement</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initialize database and begin creation</td>
<td>MSF_PV</td>
<td>KSF_PV</td>
<td>Make Sheet, Planer Face, Vertex</td>
</tr>
<tr>
<td>Create edges and vertices</td>
<td>ME_LV</td>
<td>KE_LV</td>
<td>Make Linear Edge, Vertex</td>
</tr>
<tr>
<td>Create edges and faces</td>
<td>ME_LF_P</td>
<td>KE_LF_P</td>
<td>Make Linear Edge, Planer Face</td>
</tr>
<tr>
<td>Addition of features</td>
<td>AH</td>
<td>RH</td>
<td>Add Hole</td>
</tr>
<tr>
<td></td>
<td>ARib</td>
<td>R Rib</td>
<td>Add Rib</td>
</tr>
<tr>
<td></td>
<td>ASKE_L</td>
<td>RSME_L</td>
<td>Add Sheet, Kill Linear Edge</td>
</tr>
<tr>
<td>Modification of Dimensions and type of Entities and Feature</td>
<td>MODE_L_E_F_P</td>
<td>MODF_P_F_P</td>
<td>Modify Linear Edge to Free Form Edge</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MODHDIM</td>
<td>Modify Planer Face to Freeform Face</td>
</tr>
<tr>
<td></td>
<td></td>
<td>MODHTYPE</td>
<td>Modify Hole Size</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Modify Type of Hole</td>
</tr>
<tr>
<td>Composite Operations</td>
<td>MME_L</td>
<td>KME_L</td>
<td>Make Multiple Linear Edge</td>
</tr>
<tr>
<td></td>
<td></td>
<td>ESPLIT</td>
<td>Edge-Split</td>
</tr>
<tr>
<td></td>
<td></td>
<td>KVE_L</td>
<td>Kill Vertex Linear Edge</td>
</tr>
</tbody>
</table>

Summary

In this paper, along with proposing a data structure and its implementation for modeling the geometry of inherently thin object, Euler-Poincaré’s equation is also modified for inclusion of non-linear and freeform entities and features, which provides a topological invariant supporting both manifold and non manifold objects. The validity of the proposed invariant constitutes a necessary condition for the validity of a geometrical representation of thin-walled objects from a topological point of view. Based on this invariant, a series of Euler operators are also defined which can serve as the fundamental tool set required in managing the topological representation of a thin part in a modeling system.

References

H SheenH, D.P.; Son, T.G; Myung, HD.K.H; Ryu, H.C.H; Lee, HS.H.H; Lee, HK.H; Yeo, HT.H (2010), Transformation of a Thin-walled Solid Model into a Surface Model via Solid Deflation. HComputer-Aided Design, 42H(8), 720-730 DOI:10.1016/j. cad. 2010.01.003 H.

Farsi, M.A.; Arezoo B ( May 2009), Feature Recognition and Design Advisory System for Sheet Metal Components, 5th International Advanced Technologies Symposium (IATS’09), Karabuk, Turkey, May 13-15, 1483-1487.


