Modelling and Control of Single Link Manipulators for Flexible Operation by using Linearization Techniques

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Abstract

The Euler-Bernoulli’s method is used to model the dynamics of single link flexible manipulator. Model is highly nonlinear in nature, consists of partial differential equations, and so with the help of MATLAB & SIMULINK is taken. The state space model is designed by using S function block in SIMULINK and it’s M-file in MATLAB. The response of the model is obtained in SIMULINK, which consists of angular position and tip vibration of flexible manipulator. It is found to be unbounded. To control the angular position and tip vibration of the manipulator, different type of controllers PID, Pole Placement and LQR has been designed. PID controller is designed by considering the flexible link as a rigid one and its response is observed in SIMULINK, it is found to be bounded. LQR and Pole Placement techniques are used and their responses are observed in SIMULINK, which are found to be bounded and satisfactory. Comparison of three type of controllers is made and found that LQR control is better than others.

Key Words: Single Link Flexible Manipulator, PID Controller, LQR Technique, Pole Placement Technique, Euler-Bernoulli’s method

1. Introduction

The robot technology is advancing rapidly. The industry is moving from the current state of automation to robotization, to increase productivity and to deliver uniform quality. C.Xiaotao et al. (2006) Proposed that Robots like manipulators are now commonly employed in hostile environment, such as at various places in an atomic plant for handling radioactive materials. Yinchuan et.al (2006) done research on Robots are being employed to construct and repair space stations and satellites. There are now increasing number of applications of robots such as in nursing and aiding a patient. Robots like systems are now employed in heavy earth-moving equipment. It is not possible to put up an exhaustive list of robot applications. One type of robot commonly used in the industry is a robotic manipulator or simply a manipulator or a robotic arm. Arantza Sanz et al.(2005) Proposed that Manipulator: a machine that has functions similar to human upper limbs, and moves the objects spatially. Large weight manipulator introduces limitations in terms of speed, energy consumption, and mobility. A suitable approach to overcoming these disadvantages is to design flexible manipulators. Flexible manipulators possess a lot of advantages over their traditional rigid link counter parts, like less weight, less power input, higher speed response and less energy consumption etc. Santanu Kumar Pradhan, 


Since a flexible link has an infinite number of modes of vibration, it is impossible to control all these modes simultaneously. Therefore a finite number of modes must be chosen for feedback control to get good angular position and rectified or damped vibration of the manipulator.

2. Dynamic modelling of Flexible manipulator

The co-ordinates of the point p1 on the beam at a distance of x from the origin

\[ p_x = x \cos \theta - w \cos \theta. \]  

(1)
\[ p^1_\gamma = x \sin \theta + w \cos \theta. \] Position vector of a point \( p^1 \) is given by

\[
\begin{bmatrix}
  p^1_x \\
  p^1_y \\
  p^1_z 
\end{bmatrix} =
\begin{bmatrix}
  x \cos \theta - w \cos \theta \\
  x \sin \theta + w \cos \theta 
\end{bmatrix}
\]

Velocity vector of point \( p^1 \) is represented by

\[
\begin{bmatrix}
  \dot{p}^1_x \\
  \dot{p}^1_y \\
  \dot{p}^1_z 
\end{bmatrix} =
\begin{bmatrix}
  x \sin \theta \dot{\theta} - w \cos \theta \dot{\theta} - \sin \theta \dot{\theta} \\
  x \cos \theta \dot{\theta} - w \sin \theta \dot{\theta} + \cos \theta \dot{\theta} 
\end{bmatrix}
\]

\[ \dot{p}^1 \]

2.1 Kinetic energy of the flexible link

Dynamic modelling of the manipulator is done by taking the link parameters as shown. Length \( (L) = 0.5m \), Width \( (B) = 25mm \), Thickness \( (T) = 2mm \). Tip load \( (m^0) = 100 gm/200 gm \).

Mass density per unit volume \( (\rho_h) = 7800 kg/m^3 \), Youngs modulus \( (E) = 210*(10^9) N/m^2 \).

Finally the mass matrix \( (M) \) and stiffness matrices are found for first two modes as shown below.

\[
M = \begin{bmatrix}
0.0163 & 0.0551 & 0.0092 \\
0.0551 & 0.1950 & 0.0000 \\
0.0092 & 0.0000 & 0.1950 \\
\end{bmatrix}
\]

and

\[
K_1 = \begin{bmatrix}
13593 & 0 & 0 \\
0 & 346.0621 & -0.2070 \\
0 & -0.2070 & 15550 \\
\end{bmatrix}
\]

The state space variables and state space model

\[
x(1) = \theta \\
x(2) = q_1 \\
x(3) = q_2 \\
X(1) = \dot{x}(1) \\
X(2) = \dot{x}(2) \\
X(3) = \dot{x}(3) \\
X(4) = \dot{x}(4) \\
X(5) = \dot{x}(5) \\
X(6) = \dot{x}(6)
\]

3. S-function block

An S-function is a programmatic description of a dynamic system. Each S-function block with in a SIMULINK model has the following general characteristics. A vector of inputs, u, a vector of outputs, y, and a vector of states, x. M-file for S-function is programmed for S-function block for above state space model so that whole system can represent as S-function block.

3.1 Open loop response of state space model

\[
\begin{bmatrix}
P^1_x \\
P^1_y \\
P^1_z 
\end{bmatrix} =
\begin{bmatrix}
x \sin \theta + w \cos \theta \\
x \cos \theta - w \sin \theta + \cos \theta \\
0 
\end{bmatrix}
\]

Step input is given as input and the outputs angular displacements and tip deflections are found that unbounded.

4. PID controller design

To control the unbounded responses we designed a PID controller using iterative method the values of \( K_p, K_d \) and \( K_i \) are 20, 2 and 2 respectively for getting good rise time, less peak over shoot and less stead state error.

5. Linear Quadratic Controller (LQR) Design

The dynamic models of the robotics are nonlinear in nature that’s why we are converting this S-function model into linear mode. For that we are using the linmod command in command file as shown in the program. For the application of this linmod we have to made a SIMULINK subsystem by giving one input port and two output ports as shown below and also by giving the name m_forlinmod. After that we are calling this in M-file.
By using these above matrices and by initializing the weights of matrix Q as shown below and take R=1 because here we are using single input. After that by applying the command “[K,P,E]=lqr(A,B,Q,R)” in command window, we will get the outputs of matrices K, P and E.

Gain Matrix

\[
K = \begin{bmatrix}
0 & 0.0079 & 0.0408 & 0.0450 & -0.0060 \\
0 & 0.0316 & 0.0079 & 0.0408 & 0.0079
\end{bmatrix}
\]

An M-file is designed for S-function in LQR technique and the gain matrix elements are feedback with state variables as shown below and the responses.

Fig.4 LQR controller design in simulink

The angular displacement and tip deflections are coming better than PID.

6. Pole placement method

Here in this case we are adding new poles at our desired locations such that most of the poles should be in the left half of the S-plane to bring the more stability of the system. The controller "u=−KX+r" is same as that of LQR controller except the K values.

\[
p = [-7+.005*i -7-.005*i -10+15*i -10-15*i -18+50*i -18-50*i]
\]

KK = place (A,B,p) by running this command in command window the KK matrix values are

\[
KK = \begin{bmatrix}
9.3555e-5 & 97.8 & 1006.4 & 3.3680e-005 & 1.1765e-004 & -0.4
\end{bmatrix}
\]

By using this KK values in series with state variables as shown in given model we will get the responses better than PID but not better than LQR.

7. Comparison of Pole placement and LQR technique

Here in this we are comparing the responses of angular displacement, tip deflections and control torque are observed. We can see that rise time and settling times are less in case of LQR for angular displacement. And we can observe that the range of tip deflection and control signal values are less in case of LQR.

8. Results and Discussions

Fig.6 Angular displacement of single link manipulator with open loop response

Fig.7 Tip vibration in open loop response

- Figure 6 shows that angular displacement of open loop single link manipulator. Mean while the graphical representation shows linearly increasing characteristics. On x-axis time and y-axis angular displacement was represented.
- Figure 7 shows that tip vibrations of open loop response for the single link manipulator. The graphical representation was shown totally blur and consisting sample mode of noise type vibration. On x-axis time and y-axis Tip vibration of single link manipulator was represented. Practically, the above two graphical representation are almost non-linear in nature. So, the single link manipulator was shows
dynamic characteristics. These are modified by using different types of techniques in state approach.

Fig.8 Angular displacement response of the Single link manipulator with PID Control strategy

- Figure 8 shows the angular displacement of the single link manipulator closed loop control with PID control strategy. This one helps to keep the angular displacement in linear position. From the figure 5 the angular displacement is increases from starting point to certain stage. The abrupt change of this graphical representation shows linearization characteristic.

- Figure 9 illustrates that tip of vibrations of manipulator in closed loop condition operated with PID Control strategy. From observation, the tip of vibrations from open loop to closed loop are quantized and they are promoted to liner one as shown in fig.6.

Fig.9 Tip vibration of the Single link manipulator with PID Control strategy

Fig.11 Angular displacement response of single link manipulator with LQR Control Strategy in closed loop condition

- Figure 10 illustrates that Tip deflection response of single link manipulator with LQR Control Strategy in closed loop condition. Basically by comparing the previous tip deflections of single link manipulator operating with LQR Technique shows better response than PID Control strategy. The graphical representations are shows better response characteristics other than that of utilization of PID controller.

- Figure 11 shows Angular displacement response of single link manipulator with LQR Control Strategy in closed loop condition. Here also the same linearity was applied and most stategetic one in case of an angular displacement of single link manipulator. Due to reliability in above two parameters like tip deflection and angular displacement, the produced to torque will be more and the manipulator flexibility will increases shown in fig.12.

Fig.12 Control torque response of single link manipulator with LQR Control Strategy in closed loop condition

Fig.13 Angular displacement response of single link manipulator with Pole Placement Control Strategy in closed loop condition

Fig.10 Tip deflection response of single link manipulator with LQR Control Strategy in closed loop condition
Figure 13 shows that angular displacement response of a single link manipulator with Pole Placement Control Strategy in closed loop condition. While operating with Pole placement technique the manipulator shows much better response than that of the PID controller. Figure 14 illustrates that tip deflection response of a single link manipulator with Pole Placement Control Strategy in closed loop condition. While operating with Pole placement technique the manipulator shows much better response with respect to vibrations.

Figure 15 shows the comparison graphical illustration for the operation of single link manipulator with LQR and PP Control strategies. By observing the fig. 12 with angular displacement, there is a slide change between LQR and PP rather than that of the PID controller. Figure 16 and 17 shows the comparison of tip deflections and control torque responses in different portions with respect to LQR Technique and Pole Placement. Finally, from the comparisons of angular displacement and control torque responses with respect to LQR and PP Techniques, the LQR will show the better performance and result oriented for the flexibility of a single link manipulator. Hence from three control design techniques PID, LQR and PP, LQC Technique shows somewhat better performance comparing with other control strategies.

Conclusion

In this paper, the model of the single link flexible manipulator is designed using Lagrange’s assumed mode method in a state space model form. With the help of this state space model, by giving the number of inputs, number of states and number of outputs an M-file for S-function block is made. Different controllers PID, pole placement, and LQR employed to S-function block for controlling the angular displacement and tip deflections. We observed that LQR controller giving better response out of all controllers.

For future scope of this project we can replace the conventional PID Controller with a Fuzzy PID controller because it is easily applicable to nonlinear models. And in the same way we can replace the Optimal Controller with a Time Optimal Controller still for better performance.

References


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