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Research Article

Study of Newbie Fractal Controlled by TAN Function

Sunil Shukla[a*] and Ashish Negi[b]

[a]Department of Computer Science Omkarananda Institute of Management & Technology, RishikeshTehri Garhwal, 249192.
[b]Department of Computer Science & Engineering,G.B Pant Engineering College, Pauri Garhwal, 246001.

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Abstract

In this paper we have presented the complex dynamics of Newbie fractal controlled by tan function using Superior iterates. We have introduced the new fractal based on new function named as newbie fractal. This function is the variant of Mandelbrot function. These fractals are governed by initial controlled functions such as sin, cos, log, tan, conj, abs etc.

Key words: *Complex dynamics, Newbie Fractal*

1. Introduction

Fractal geometry is based on the idea of self-similar forms. In this paper we have presented the study of variants of escape time fractals. For this purpose we have taken one special function described as follows:
Modified pixel coordinate of ***C***-plain by augmenting it with a set of given function i.e.

$$c = fn_1\left(\frac{1}{pixel^{p1}}\right)$$

For the analysis purpose we have presented the study of complex dynamics of the new fractal defined above, with ***tan*** function.

2. Generation process

The fractals have been generated by iterative formula $z_{n+1} \leftarrow f(z_n)$ where z_0 is initial value of z, and z_i is the value of the complex quantity z. The Mandelbrot's self-squared function for generating fractals is $f(z) = z^{p_2} + c,\ p_2 \geq 2$, ***z*** and ***c*** are both complex quantities. We use of the transformation function $f(z) = z^{p_2} + c,\ p_2 \geq 2$ for generating fractal images with respect to superior iterates, where ***z*** and ***c*** are the complex quantities and the power $\boldsymbol{p_2}$ is real number. These fractal images are constructed as a two-dimensional array of pixels. Each pixel is represented by a pair of (x, y) co-ordinates. The complex quantities ***z*** and ***c*** can be represented as: $z = z_x + iz_y$ and $c = c_x + ic_y$, where $i = \sqrt{(-1)}$ and z_x, c_x are the real parts and z_y & c_y are the imaginary parts of ***z*** and ***c*** respectively. The pixel coordinates (***x***, ***y***) may be associated with (c_x, c_y) *or* (z_x, z_y) which is based on this concept, the fractal images can be classified as follows:

a) ***Z* – plane** fractals, wherein (x, y) is a function of $(z_x,\ z_y)$.
b) C - **plane** fractals, wherein (x, y) is a function of (c_x, c_y).

In the literature, the fractals for $p_2 = 2$ in z plane are termed as the Mandelbrot set while the fractals for $p_2 = 2$ in c plane are known as Julia sets.

3. Julia set

French mathematician Gaston Julia investigated the iteration process of a complex function intensively, and attained the Julia set, a very important and useful concept. At present Julia set has been applied widely in computer graphics, biology, engineering and other branches of mathematical sciences. Consider the complex-valued quadratic function $z_{n+1} = z_n^2 + c; c \in C$, where C be the set of complex numbers and n is the iteration number. The Julia set for parameter c is defined as the boundary between those of z_0 that remain bounded after repeated iterations and those escape to infinity. The Julia set on the real axis are reflection symmetric, while those with

*Corresponding author: Sunil Shukla

complex parameter show rotation symmetry with an exception to $c = (0,0)$ see Rani and Kumar and.

4. Mandelbrot set

The Mandelbrot set ***M*** for the quadratic $Q_c(z) = z^2 + c$ is define as the collection of all $c \in C$ for which the orbit of the point 0 is bounded, i.e

$$M = \{c \in C : \{Q_c^{\ n}(0)\}_{n=0,1,2,...} is\ \ bounded\}.$$

5. Superior iterates

Let A be a subset of real or complex numbers and $f : A \to A$. For $x_0 \in A$, construct a sequence $\{x_n\}$ in A in the following manner

$$x_1 = s_1 f\left(x_0\right) + \left(1 - s_1\right) x_0$$
$$x_2 = s_2 f\left(x_1\right) + \left(1 - s_2\right) x_1$$
$$\vdots$$
$$x_n = s_n f\left(x_{n-1}\right) + \left(1 - s_n\right) x_{n-1}$$

where $0 < s_n \le 1$ and $\{s_n\}$ is convergent to a non-zero number. The sequence $\{x_n\}$ constructed above is called Mann sequence of iterates or superior sequence of iterates. Let z_0 be an arbitrarily element of ***C***, construct a sequences $\{z_n\}$ of points of ***C*** in the following manner:

$$z_n = sf\left(z_{n-1}\right) + \left(1 - s\right) z_{n-1,} n = 1, 2, 3....,$$

where f is a function on a subset of ***C*** and the parameter ***s*** lie in the closed interval [0, 1].The sequence $\{z_n\}$ constructed above, denoted by $SO\left(f, z_0, s\right)$ is superior orbit for the complex-valued function f with an initial choice z_0 and parameter s. We may denote it by $SO\left(f, x_0, s_n\right)$. Notice that $SO\left(f, x_0, s_n\right)$ with $s_n = 1$ is $O\left(f, x_0\right)$. We remark that the superior orbit reduces to the usual Picard orbit when $s_n = 1$.

6. Analysis of superior function controlled julia set NF Mandelbrot set and Julia set: for tan controlled function

For the ***tan*** controlled functions $\boldsymbol{fn}_1$ we find Mandelbrot fractal symmetric through *x* and *y* axis for $\boldsymbol{p}_1 = 2$ and $\boldsymbol{p}_2 = 2$, see Fig. 1. On increasing the value of $\boldsymbol{p}_1$ and keeping the value of $\boldsymbol{p}_2$ to be a constant i.e. $\boldsymbol{p}_2 = 2$, the number of corners follows the pattern p_1+1 and the ovoid in the centre follows the $2\boldsymbol{p}_1$ rule, *see* Fig. 2-4. Further, on increasing the $\boldsymbol{p}_2$ and keeping the value of p_1 to be constant i.e. 2, we observe the corners to be of order $2\boldsymbol{p}_1$ **-** 2 and the ovoid to be $2\mathbf{p}_1$, see Fig. 5-7. Keeping $\boldsymbol{p}_1 = \boldsymbol{p}_2 = 5$, we observe that an ovoids of order p*2, see Fig. 8-9. The corresponding superior Julia sets for $\boldsymbol{p}_1 = 5$, $\boldsymbol{p}_2 = 5$ and $\boldsymbol{s} = 1$ are presented in Fig. 10-11, where we found the Julia sets which resembles with Arrow or more likely to Fishbone. We observe that the superior Julia sets show the symmetric architecture along both the axis, see Fig. 12. On increasing $\boldsymbol{p}_1$ and keeping $\boldsymbol{p}_2 = 2$, we found spiral dendrite of order 4*$\boldsymbol{p}_1$ on the boundary, see Fig. 13.

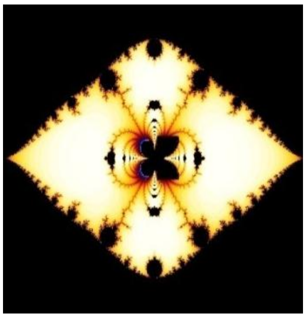
Fig. 1

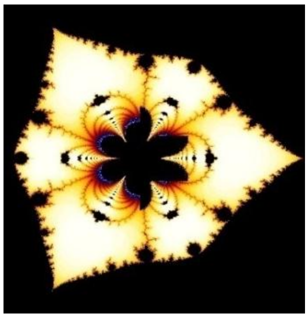
Fig. 2

Fig. 1. For *D* (*z*), fn_1 = *tan*, $p_1 = 2$, $p_2 = 2$ and $s = 1$
Fig. 2. For *D* (*z*), fn_1 = *tan*, $p_1=3$, $p_2= 2$ and $s=1$

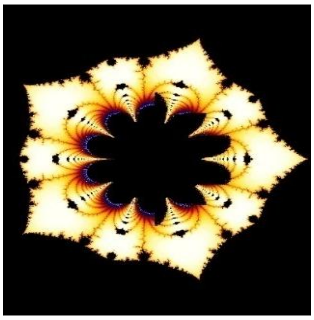
Fig.3

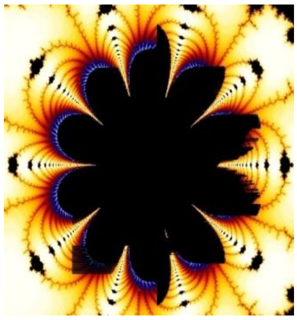
Fig.4

Fig. 3. For *D* (*z*), fn_1 = *tan*, $p_1=5$, $p_2= 2$ and $s=1$
Fig. 4. For *D* (*z*), fn_1 = *tan*, $p_1=5$, $p_2= 2$ and $s=1$
(Zoom of Fig 3. Note the number of ovoid = 2 p_1)

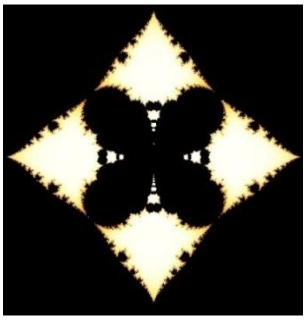
Fig.5

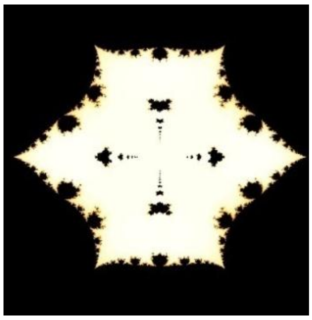
Fig.6

Fig. 5. For *D* (*z*), fn_1 = *tan*, $p_1=2$, $p_2= 3$ and $s = 1$
Fig. 6. For *D* (*z*), fn_1 = *tan*, $p_1 = 2$, $p_2= 4$ and $s = 1$

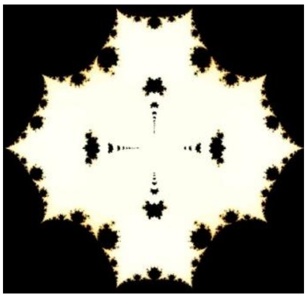

Fig.7

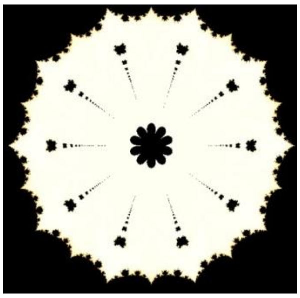

Fig.8

Fig. 7. For D (z), fn_1 = tan, p_1=2, p_2= 6 and s = 1
Fig. 8. For D (z), fn_1 = tan, p_1 = 5, p_2= 5 and s = 1

As and special case we observe that on initializing the value of z to the function fn1 rather than to start value i.e. (0, 0), a formation of an ovoid of order $2\boldsymbol{p}_1$ took place at the center of the image see Fig. [8].

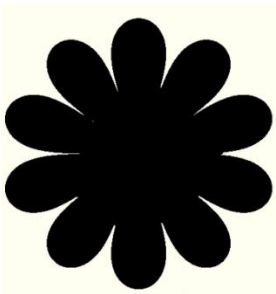

Fig.9

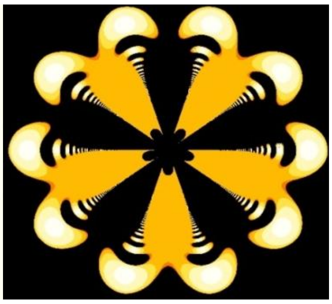

Fig.10

Fig. 9. For D (z), p_1 = 5, p_2= 5 and s = 1
(Zoom of Fig 8. Note the number of ovoid = 2 p_1)
Fig. 10. For D (z) = (-0.02, 0.03), fn_1 = tan, p_1=5, p_2= 5 and s = 1

Fig.11

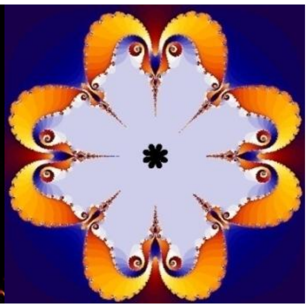

Fig.12

Fig. 11. For D (z) = (-0.02, 0.03), fn_1 = tan, p_1 = 5, p_2= 5 and s = 0.1
Fig. 12. For D(z) = (1.364, 0.002), fn_1 = tan, p_1 = 4, p_2= 2 and s = 0.1

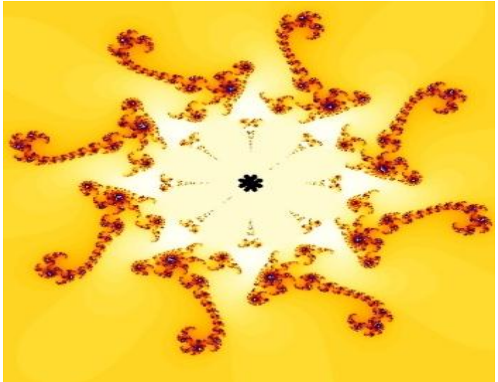

Fig. 13

Fig. 13. For D (z) = (0.298, 0.489), fn_1 = $sqrt$, p_1 = 4, p_2= 2 and s = 1

Conclusion

In this paper we have presented the analysis of superior function controlled julia set using ***tan*** function. Further we have analysis superior iterates at different power of $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ see Fig. [1] – [13]. Further we have presented the geometric properties of superior Julia sets along different axis. We also find an interesting image which resemble to Arrow or more likely to Fishbone see Fig. [11].

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