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Research Article

Improving Consistency of Comparison Matrices in Analytical Hierarchy Process

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Abstract

In the field of decision-making, the concept of priority is archetypal and how priorities are derived influence the choices one makes. Priorities should not only be unique but should also reflect the dominance of the order expressed in the judgments of pair wise comparison matrix. In addition, judgments are much more sensitive and responsive to small perturbations. They are highly related to the notion of consistency of a pair wise comparison matrix simply because when dealing with intangibles, if one is able to improve inconsistency to near consistency then that could improve the validity of the priorities of a decision. This paper endeavors to accomplish nearly consistent matrices in pair wise comparisons by subsiding the effects of hypothetical decisions made by the decision makers. The proposed methodology efficiently improves group decisions by incorporating corrective measures for inconsistent judgments.

Key words: *Multi Criteria Decision Making (MCDM), Eigen Value, Eigen Vector, Reciprocal Matrices, Analytical Hierarchy Process (AHP), Consistency.*

1. Introduction

Multi Criteria Decision Making (MCDM) is a sub-discipline of decision sciences that explicitly considers multiple criteria in decision-making environments. This concept is designed to make better choices when faced with complex decisions involving several dimensions. MCDM tactics are especially helpful when there is a need to combine hard data with subjective preferences, to make trade-offs between desired outcomes and to involve multiple decision makers. In the decision making process, there are typically multiple conflicting criteria that need to be assessed simultaneously with about same degree of precedence. This craves the search for an approach which deals the predicament with necessary sagacity to obtain a clear and unambiguous conclusion. It has been established in previous researches that the pair wise comparison methods can always be used to draw the final conclusions in a comparatively accessible and intelligible way.

The concept of pair wise comparisons is more than two hundred years old. Borda(1781) and Condorcet(1785) introduced it for voting problems in eighteenth century by using only 0 and 1 in the pair wise comparison matrices. The method was efficiently regulated by Thorndike(1920) to tackle the classical techniques of experimental psychology in early twentieth century. Thurstone(1927) also used pair wise comparisons for social values in twentieth century. Over the last three decades, a number of methods have been developed which use pair wise comparisons of the alternatives and criteria for solving MCDM problems. AHP proposed by Saaty(1980) has been a very popular approach to MCDM that involves pair wise comparisons for an objective inquisition. It has been applied during the last thirty years in many decision making situations and a wide range of applications in various fields. Advances in the decision sciences have led to the development of a number of approaches intended to minimize inconsistencies in pair wise comparisons to get closer to pragmatic scenario. Koczkodaj(1993) proposed a new definition of consistency in computational modeling. Ishizaka and Lusti(2004) designed a module to improve the consistency of AHP matrices. Antonio(2006) and Bozoki & Rapcsak(2007) introduced a new approach to gauge consistency of pair wise comparisons. Pair wise preference information is needed as input in many interactive multiple criteria decision making scenarios. The decision makers are required to make holistic binary comparisons among feasible alternatives. Information obtained from these comparisons is used to decrease the number of comparisons that have to be made when searching for the best (most preferred) alternative. Different approaches, with different assumptions for this problem have been presented. The simplest case of combinatorial consistency was analyzed by Davis(1963), other works are by Korhonen et al.(1984), and Koksalan & Sagala(1992) and many more, but only widely accepted measure of inconsistency is due to Saaty(1980).

In AHP, the calculated priorities are presumable only if the comparison matrices are consistent or near consistent. This condition is reached if (and only if) within the pair

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wise comparison process the transitivity and reciprocity rules are respected. However it is often assumed that the decision produced by a group will always be better than that supplied by an individual. This seems plausible because multiple participants can bring differing expertise and perspectives to carry out any complex decision. Ideally, we should endeavor to curtail group imperfections and yet capitalize on inherent group advantages. In this study, it is unveiled that the consistency of a positive reciprocal matrix can be considerably improved by analyzing the stimulating factors of inconsistencies in positive reciprocal matrices. A convenient correction method based on the relative error is brought up which not only aspires to minimize the inconsistencies of such matrices but also to substantially reduce number of pair wise comparisons in decision making. This method can fully retain the effective information of original positive reciprocal matrix. It helps to solve practical problems effectively and enriches the theories and methods of decision analysis.

The paper is organized in six sections, Section 1 is introductory. Section 2 gives a brief introduction of methodology which laid the foundation of the present work. Antecedent of inconsistencies are analyzed and a corrective application based approach is introduced in section 3 which helps the decision-maker to build a consistent matrix with a controlled error by appreciably less number of pair wise comparisons. Section 4 illustrates the application part using proposed methodology to provide near consistent matrices. Section 5 elucidates the proposed methodology in group decision making. Finally, we present some findings and draw some conclusions, followed by giving recommendations for further research in section 6.

2. Overview of Pair wise Comparisons Approaches

MCDM is concerned with structuring and solving decision making problems involving multiple criteria (Figure1).

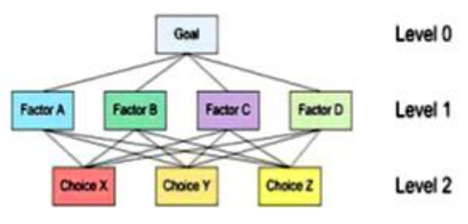

Figure1. MCDM Problem

Typically, there does not exist a unique optimal solution for such problems and it is necessary to use decision maker's preferences to get prioritized solutions. AHP is one of the most widely used methods to handle these types of problems.

2.1 Analytical Hierarchy (AHP)

The analytical hierarchy process (AHP) is a decision making approach designed to aid in the solution of complex multiple criteria problems in number of application domains. The outcome of AHP is a prioritized weighting of each decision alternative. The first step in the analytical hierarchy process is to model the problem as a hierarchy. The hierarchy is a structured mean of describing the problem at hand. It consists of an overall goal at the top level, a group of options or alternatives for reaching the goal and a group of factors or criteria that relate the alternatives to the goal. In most cases the criteria are further broken down into sub criteria, sub-sub criteria and so on in many levels as per the requirement of the problem. Once the hierarchy has been constructed, the participants use the AHP to establish priorities for all its nodes. In this, the elements of a problem are compared in pairs with respect to their relative impact on a property they share in common. The pair wise comparison is quantified in a matrix form by using the scale of Relative Importance given in Saaty (1980) as shown in Table 1.
This scale has been validated for effectiveness, not only in many applications by a number of people, but also through theoretical comparison with a large number of other scales. During the elicitation process, a positive reciprocal matrix is formed in which $(i,j)^{th}$ element a_{ij} is filled by the corresponding number from the Table 1.

Table 1. Analytic Hierarchy Measurement Scale

Reciprocal Measure of Intensity of Importance	Definition	Explanation
1	Equal Importance	Two activities contribute equally to the objective
3	Weak importance of one over another	Experience and judgment slightly favor one activity over another
5	Moderate importance	Experience and judgments moderately favor one activity over another.
7	Strong Importance	An activity is strongly favored and its dominance is demonstrated in practice.
9	Absolute Importance	The evidence favoring one activity over another is of the highest possible order of affirmation
2,4,6,8	Intermediate values between two adjacent judgments.	When compromise is needed

The number is chosen according to the following criterion.

$$\begin{cases} a_{ij}, & \text{if } x_i \text{ dominates } x_j \\ 1/a_{ij}, & \text{if } x_j \text{ dominates } x_i \\ 1, & \text{if } x_i \text{ and } x_j \text{ do not dominate over one another} \end{cases}$$

The matrix so formed is called the reciprocal matrix. This reciprocal matrix is used to calculate the local priority weight of each criterion. The local priority weight (w) is the normalized eigen vector of the priority matrix corresponding to the maximum eigen value of the matrix. For detailed reasoning of this account we refer to Forman (1990), Lunging (1992), Ball & Srinivasan (1994) and Bryson & Mobolurin (1994). An interesting property of the priority matrix is that if in addition its elements are such that

$$a_{ij}\, a_{jk} = a_{ik}, \qquad i \le j \le k \tag{1}$$

then the derived priority vector **w** satisfies

$$w_i / w_j = a_{ij}, \qquad i < j \tag{2}$$

Any reciprocal matrix satisfying (1) is called consistent. However in practice, the priority matrix seldom satisfies (1), thereby making it more important to define some relax measuring of consistency check, Saaty [8] introduced the concept of consistency index CI of a reciprocal matrix as the ratio $\frac{\lambda_{max} - n}{n-1}$ where λ_{max} and n, respectively stand for the maximum eigen value and order of the reciprocal matrix. The obtained CI value is compared with the random index RI given in Table 2.

Table 2. Random consistency Index (RI)

N	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

The Table 2 had been calculated as an average of CI's of many thousands matrices of the same order whose entries were generated randomly from the scale 1 to 9 with reciprocal force. The simulation results of RI for matrices of size 1 to 10 had been developed by Saaty (1980) and are given in Table 2. The ratio of CI and RI for the same order matrix is called the consistency ratio CR. In general, a consistency ratio of 10% or less is considered very good. If consistency is poor, inconsistency of judgments within the matrix has occurred and the evaluation process should therefore be reviewed and improved.

3. Research Methodology

3.1 Inconsistency of reciprocity

Decision Makers are more likely to be cardinally inconsistent because they cannot estimate precisely measurement values even from a known scale and worse when they deal with intangibles (a is preferred to b twice and b to c three times, but a is preferred to c only five times). Consider the following judgment matrix for three alternatives.

$$\begin{bmatrix} 1 & 2 & 3 \\ 1/2 & 1 & a_{23} \\ 1/3 & a_{32} & 1 \end{bmatrix}.$$

To fulfill the criteria of consistency i.e. $a_{ij}\, a_{jk} = a_{ik}$, $a_{23} = a_{21} \cdot a_{13} = 3/2$ and consequently $a_{32} = 2/3$. So the judgment is consistent if (and only if) $a_{23} = 3/2$ and $a_{32} = 2/3$. Here the first reason of inconsistency appears. The comparison scale of AHP (Table 1) has no such value. To overcome this difficulty, it is more appropriate to use the numbers of the form $\{a/b : a, b \in \{I^+\}\}$, where I^+ represents the set of positive integers excluding 0. This modified scale allows the decision maker to present a consistent judgment. This is perhaps the simplest way for composing priorities.

3.2 Inconsistency of Transitivity

Whenever object a is related to b and object b is related to c, then the relation at hand is transitive provided object a is also related to c. In mathematical syntax:

$$(a\,R\,b \text{ and } b\,R\,c) \Rightarrow a\,R\,c,\ \forall a, b, c \in A.$$

The same property is respected in pair wise comparisons i.e. if A > B and B > C then A > C. Judgments are ordinally intransitive if A is preferred to B and B to C but C is preferred to A. One of the apparent reason is, large number of pairwise comparisons ($n(n-1)/2$) required to workout the attribute weights at a given level of hierarchy. This results in baffling responses relating to pair wise comparisons, as size of comparison matrix goes on increasing. Majority of available software entertain not more than nine attributes at a time. This is because of the fact that handling of a nine attribute matrix would need 36 pair wise comparisons at a time. It is an established fact that inconsistency of transitivity goes on increasing with the size of the matrix.

3.3 How to build a consistent matrix

Proposed methodology intends to materialize an augmentation to overcome the above mentioned lacuna to some extent. Procedure outlined to implement the proposed methodology is as follows:

Step 1: A decision-maker should first rank all the n attributes to be weighed, according to their importance in the preferred domain. Reorder them in an ascending order of priorities.

Step 2: Exercise $(n-1)$ comparisons among the consecutive criteria using the scale $\{a/b : a, b \in \{I^+\}\}$.

If any two or more criteria are equally significant, obvious priority of one over the other is 1 using the given scale.

Step 3: Priorities for remaining pairs (non-consecutive) can easily be computed logically as follows :

If B be prioritized *r* times to A and C is prioritize *s* times to B, then C is prioritized *r*×*s* times to A. Objective ratings to all potential pair wise comparisons can be provided in this manner and represented in a matrix form to provide weights to given set or criteria. It is conspicuous to mention here that priorities within a given pair of attributes are self-reciprocal, i.e. if B be prioritized *q* times to A then preference of A over B is *1/q* times.

Step 4: The procedure results in perfectly consistent comparison matrix supported by the fact $\lambda_{max} = n$ and hence

CI = 0. Eigenvector corresponding to this maximum eigenvalue provides the requisite criteria weights. Geometric mean or weighted geometric mean of individual judgments may be taken to accomplish aggregated matrices for the set of criteria at various levels of hierarchy.

Step 5: The nodes at each level are compared pair wise with respect to their contribution to the nodes above them to find their respective global weights. We rank each of the criteria in the final set by evaluating it with respect to upper level attributes separately. The evaluation process finally generates the global weights for each requisite criterion of interest. In a realistic scenario, the technique is very adaptable and can handle any number of attributes in a system. This simplification can reduce the calculation effort for the weights significantly, especially when judgment criteria are large in number and pair wise comparisons are difficult to be accomplished.

4. Numerical Example

We now illustrate the methodology by an independent survey conducted on referees R_1, R_2 and R_3. The three are catechized to rank four attributes P, Q, R and S in ascending order of priorities. Suppose the ranking awarded by R_1 to four attributes is (Q, R, S, P) in ascending order of priorities where R is prioritized 2 times over Q, S is prioritized 4 times over R and P is prioritized 5 times over S. Subject to R_2's ranking (S, R, P, Q) where S & R are equally ranked, P is prioritized 3 times over R and Q is prioritized 7 times over P. Prioritized responses acceded by R_3 in ascending order is (R, Q, P, S). Here Q is prioritized 3 times over R, P is prioritized 2 times over Q and S is prioritized 5 times over P. Remaining priorities are calculated logically supervened by the methodology explained in section 3.3. Table 3 depicts the priorities procured by the four attributes accorded by the three referees.

Table 4 portrays the prioritized weights acquired by the four attributes tendered by the referees R_1, R_2 and R_3 subject to their rankings, guided by the methodology explained in section 3.3.

Table 3. Prioritized Weights Accorded by R_1, R_2 and R_3

		P	Q	R	S
A_1, A_2, A_3 =	P	1	40, 1/7, 2	20, 3, 6	5, 3, 1/5
	Q		1	1/2, 21, 3	1/8, 21, 1/10
	R			1	1/4, 1, 1/30
	S				1
$\lambda_{max} = 4$, C.I. = 0, C.R. = 0, $\forall$ Ai, i = 1, 2, 3					

Table 4. Prioritized Weights Accorded by R_1, R_2 and R_3

	R_1	R_2	R_3
P	0.784314	0.115385	0.15
Q	0.0196078	0.807692	0.075
R	0.0392157	0.0384615	0.025
S	0.025641	0.0384615	0.75

An aggregated comparison matrix is worked out by taking the geometric means of corresponding priorities in various components of each cell of Table 3. Table 5 shows the final rankings evolved by synthesizing the rankings provided by the referees R_1, R_2 and R_3.

Table 5. Aggregated Matrix Showing Final Weights of the Attributes using Proposed Methodology

	P	Q	R	S	Weights
P	1	2.2525	7.1138	1.4422	0.439024
Q	0.4439	1	3.152	0.6403	0.194812
R	0.1406	0.3172	1	0.2027	0.0617422
S	0.6934	1.5618	4.9334	1	0.304421
$\lambda_{max} = 4$, C.I. = 1.78862e − 07, C.R. = 1.99e − 07					

Final ranking is R, Q, S, P with CR approximately zero. Thus the efficiency of proposed methodology is substantially established in decision making scenarios.

5. Group Consistency

Group decision making is becoming increasingly important in decision scenarios associated with MCDM problems. AHP apparently arms the judgments which are consistent or near consistent (having CR < 0.1), whereas it discards inconsistent judgments affected by any of the above mentioned speculations. In realistic scenarios, only a handful of acceding judgments are taken into account defying the very objective of a legitimate conception. The proposed methodology provides an insight into the impediments to effective group processes and on techniques that can improve group decisions. A group decision-making methodology is being introduced as an effective approach for improving the targeted resolutions. It combines the following three components:

(1) The survey analysis
(2) The Analytic Hierarchy Process to produce judgments
(3) A logistic numerical assessment of irrational judgments.

We now illustrate the proposed methodology via an example in which eight referees say R_1, R_2, R_3, R_4, R_5, R_6, R_7 and R_8 are put to an inquisition to rank four attributes P, Q, R and S. To begin with, we first discuss the results working with the classical AHP methodology. Tables 6, 7, 8, 9, 10, 11, 12 and 13 exhibit the prioritized weights procured using AHP accorded by referees respectively. Note each decision maker R_i, i = 1, 2,…, 8, has to make $4(4 - 1)/2 = 6$ comparisons, viz. P with Q, R, S; then Q with R, S; and finally R with S, using scale from Table 1.

Table 6. Response to Inquisition by Referee R_1

	P	Q	R	S	Weights
P	1	1/9	1/2	1/6	0489075
Q	9	1	7	3	0.590915
R	2	1/7	1	1/5	0.0771508
S	6	1/3	5	1	0.283027
	λ_{max} = 4.09571, C.I. = 0.031905, C.R. = 0.03545				

Table 7. Response to Inquisition by Referee R_2

	P	Q	R	S	Weights
P	1	1/5	3	6	0.211829
Q	5	1	7	9	0.657806
R	1/3	1/7	1	2	0.0827088
S	1/6	1/9	1/2	1	0.0476561
	λ_{max} = 4.14228, C.I. = 0.047426, C.R. = 0.05270				

Table 8. Response to Inquisition by Referee R_3

	P	Q	R	S	Weights
P	1	1	1/2	1/7	0.0801702
Q	1	1	1/2	1/7	0.0801702
R	2	2	1	1/8	0.132531
S	7	7	8	1	0.707129
	λ_{max} = 4.08661, C.I. = 0.0288702, C.R. = 0.032078				

Table 9. Response to Inquisition by Referee R_4

	P	Q	R	S	Weights
P	1	1/2	6	4	0.368319
Q	2	1	1/2	5	0.327285
R	1/6	2	1	1/3	0.164289
S	1/4	1/5	3	1	0.140107
	λ_{max} = 5.64218, C.I. = 0.547395, C.R. = 0.60822				

Table 10. Response to Inquisition by Referee R_5

	P	Q	R	S	Weights
P	1	1/5	4	6	0.292937
Q	5	1	1/2	7	0.425984
R	1/4	2	1	3	0.238303
S	1/6	1/7	1/3	1	0.0427761
	λ_{max} = 5.42098, C.I. = 0.473658, C.R. = 0.52629				

Table 11. Response to Inquisition by Referee R_6

	P	Q	R	S	Weights
P	1	1/2	8	1/4	0.257225
Q	2	1	5	1/6	0.214766
R	1/8	1/5	1	7	0.245865
S	4	6	1/7	1	0.282144
	λ_{max} = 9.33837, C.I. = 1.77946, C.R. = 1.97718				

Table 12. Response to Inquisition by Referee R_7

	P	Q	R	S	Weights
P	1	2	3	1/4	0.266575
Q	1/2	1	1/5	1/6	0.0492226
R	1/3	5	1	7	0.416396
S	4	6	1/7	1	0.267806
	λ_{max} = 6.30652, C.I. = 0.768841, C.R. = 0.85427				

Table 13. Response to Inquisition by Referee R_8

	P	Q	R	S	Weights
P	1	1/2	1/3	1/4	0.0503552
Q	2	1	8	1/6	0.341091
R	3	1/8	1	5	0.272895
S	4	6	1/5	1	0.335659
	λ_{max} = 7.85978, C.I. = 1.28659, C.R. = 1.42954				

An aggregated reciprocal matrix is developed by taking geometric mean of corresponding values of all the eight matrices for further calculations. Table 14 depicts the final weights of the attributes P, Q, R and S using classical AHP, taking into account all consistent and inconsistent judgments.

Table 14. Aggregated Matrix Showing Final Weights Using AHP

	P	Q	R	S	Weights
P	1	0.42729	1.86121	0.69361	0.212604
Q	2.34033	1	1.62658	0.67798	0.30739
R	0.537285	0.614787	1	1.36778	0.205152
S	1.44173	1.47497	0.731112	1	0.274855
	λ_{max} = 4.31047, C.I. = 0.103489, C.R. = 0.11499				

Clearly resultant matrix shows incommensurate results as the attribute Q having awarded highest priority, yet not prioritized to S and similar other observations. Now we illustrate our proposed scheme on the same problem and simultaneously provide a comparison with the aforementioned result. We first seek the priorities for the four attributes P, Q, R, S, in ascending order from Table 14.

Following information is provided: attribute P is prioritized 1.86121 times over R, priority of S over P is 1.44173 times and that of Q over S is 0.67798 times.

Awarding the priorities to the remaining pair wise comparisons logically as explained in step 3 of the proposed methodology (section 3.3), we construct a pair wise a comparison matrix given by Table 15.

Table 15. Synthesis of Priorities Accorded by Proposed Methodology

	P	Q	R	S	Weights
P	1	1.02306	1.86121	0.693611	0.25275
Q	0.97746	1	1.81927	0.67798	0.247054
R	0.537285	0.549671	1	0.372667	0.135799
S	1.44173	1.47497	2.68336	1	0.364397
	$\lambda_{max} = 4$, C.Ic. = 1.43707e − 12, C.R. = 1.59678e − 12				

Table 15 provides highly commensurate results with all the attributes having provided with prioritized weights.

6. Concluding Remarks

We have described the implementation of a corrective model, assisting the decision-maker in the construction of a consistent comparison matrix. It is conspicuous to mention that not only pair wise comparisons are substantially reduced, from $n(n-1)/2$ to $n-1$, but also appreciably legitimate results are shown, as evident by CR of Table 15 which is significantly lower than CR of Table 14. Another momentous advantage is that any number of attributes may be entertained in one set without any baffling responses. We hope that the proposed methodology can refocus the attention of researchers from the race of finding better judgments for inconsistent and near consistent matrices. Future work will include incorporation of the proposed approach into practical software applications, i.e. case studies will be processed and evaluated. Detailed evaluation of the present approach with other similar approaches can be obtained through these case studies.

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