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Review Article

Probabilistic Analyses of Slopes: A State of the Art Review

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Abstract

Slope stability analyses are evolving from deterministic analyses, which rely only on the mean value of each parameter, to more advanced probabilistic methods. Moreover, many researchers have shown the benefits of considering spatial variability of soil properties in probabilistic analyses of slopes that may lead to a more economical and/or safer design. Spatial variability is usually modeled by random fields theory in both limit equilibrium and finite element based probabilistic analyses. This paper presents a state of the art review of the literatures on probabilistic analyses of slopes that may help researchers to choose a suitable probabilistic method for their problems.

Key words: *Slope stability, probabilistic analysis, limit equilibrium method, finite element method, spatial variability.*

1. Introduction

Soil as the main material in geotechnical engineering is nature-made with a complex structure, in contradiction with steel or concrete, which are manufactured in the factory or site with exact proportions. Considering complex structure of soils, it is sensible to have so much uncertainty and scatter in the soil data that are measured in site or laboratory. Uncertainties that affect the results of geotechnical designs such as earth dams, roads, foundations, and slopes are reviewed in the works of A.L Malkawi et al, (2000), J.T, Christian, (2004) and R.Chowdhury et al, (2010). These uncertainties can be grouped into two types. First, human mistakes such as shortcomings in measurements and calculations, which can be reduced by gathering more information via increasing soil specimens and improving test devices. The uncertainties of soil strength parameters that are needed in slope stability analyses can be reduced based on available information. J. Ching et al. (2011) proposed a systematic way that uses the results of field and laboratory tests as input. The output of the works of J. Ching et al. (2011) for characterizing uncertainties is updated mean and standard deviations of the shear strength parameters like friction angles by simplified equations.

Second source of uncertainty is inherent variability that exists in soil deposits and cannot be reduced or ignored, like spatial variability. Spatial variability means having different value for a parameter in different spatial locations.

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For evaluating stability of the slopes against failure, there are two types of analyses: deterministic and probabilistic analyses. Deterministic analyses are traditional approaches like limit equilibrium methods (LEM) or finite element strength reduction methods (FE SRM). To check the stability of the slopes in the LEM, a slip surface is assumed. Soil mass above this slip surface is divided into slices and equilibrium equations are written for all slices. Factor of safety is calculated by one of the limit equilibrium methods like Spencer (E. Spencer, 1967), Bishop's simplified method of slices (A.W. Bishop, 1955), Janbu's method (N. Janbu, 1973) etc. A detailed description of the LEM can be found in J.M. Duncan and S.G. Wright (2005).

Plane strain, elastic-plastic analyses of finite element methods (FEM) that are used in slope stability by shear strength reduction (SSR) or shear reduction method (SRM) was first proposed by (O.C. Zienkiewics, 1975). Finite Element methods in the analyses of slopes are popular for not requiring any assumptions about inter slice forces, location or shape of the failure surfaces. FEM is able to model layered slopes with complex geometries. In each step of FE SRM, original shear strength parameters are divided into a trial factor of safety. Each set of these reduced strengths are used in the finite element analysis (D.Y. Xu et al, 2009), then shear stresses in each element are compared to a failure criteria like Mohr-Columb criteria. FE SRM is terminated when the stresses become unacceptably large or trial and error procedure do not converge to a solution (D.V. Griffiths & G.A. Fenton, 2001). A location is assumed yielded if the stresses in elements are greater than stresses of Mohr-Coulomb criterion. When a sufficient number of elements yield, a mechanism of failure develops that leads to overall failure of the slope (D.V. Griffiths and P.A. Lane, 1999).

For calculating factor of safety (FOS) - whether by LEM or FEM- mean value of soil properties that is gathered from lab or site samples is used. These data are few in comparison to slope dimensions and based on the previous studies, using mean value without further attention to dispersion of data influence the failure mechanism of the slopes, and hence the reported FOS is uncertain (S.E. Cho, 2007). If the reported FOS is estimated smaller than correct answer, design is not economic, and may make million dollars waste of money. For the cases that calculated factor of safety without modeling uncertainties is bigger than reality, safety is not guaranteed and this inattention may cause loss of life. The lives of the residents in the area near slopes or those who pass through the roads across the slopes are worthwhile to do additional analysis of probabilistic analyses. Moreover, repairing damaged earth structures are quite costly, so probabilistic analyses found their popularity in slope stability and entered in engineering standards. For example, Eurocode, which are 10 European standards (Eurocode7 for geotechnics), embedded Reliability Based Designs (RBDs) to be used by engineers since 1997 (K.K. Phoon, 2008). U.S Army Corps of Engineers also uses probabilistic methods as a required input to the annual prioritization of budget funds in all their projects including slope stability.

Probabilistic analyses have been applied in slope stability more than any other geotechnical problem. The rule of thumb is to take shear strength parameters, mainly cohesion and friction angle as random variables. Probability of failure (P_f) and reliability index (β) which are outputs of probabilistic analyses are calculated by one of the two procedures: simulation-based approach (e.g., Monte Carlo simulation; subset simulation) or analytical-based approaches (e.g., first-order second-moment; first-order reliability method). Both analytical and simulation based approaches require deterministic analysis that evaluate factor of safety. Therefore, alternative grouping for probabilistic analyses is based on their deterministic part that can be LEM-based probabilistic analyses or FE SRM-based probabilistic analyses.

Since the early 1970s, different techniques have been proposed to evaluate reliability of slopes (e.g., Wu & Kraft, 1970; E.E. Alonso, 1976; E.H. Vanmarcke, 1977). They have continued over the years and are still quite active (e.g., D.V. Griffiths and G.A Fenton, 2004; J.F. Xue and K. Gavin, 2007; J. Ching et al., 2009; J. Zhang et al., 2011). Results of the probabilistic analysis are used in combination with deterministic analysis, to give the designer a chance to see the uncertainties in his or her analysis for a safe and economic design. In this study, an attempt is made to review the most popular and well-known methods of probabilistic analyses of slopes. The results will help the designers to choose a probabilistic method that is more consistent to their problem for modeling the uncertainties that ignoring them may lead to unconservative results of slope stability analyses.

Spatial Variability

Physical properties of soil vary even in locations near to each other throughout the soil deposits and this natural property is called spatial variability that is one of the main source of uncertainty in geotechnical analysis like slope stability. G.B Baecher and J.T Christian (2003) presented the plot of two sets of data that share the same mean and standard deviation but one set has horizontal trend while the other set of data is erratic (Figure 1). Therefore, mean and standard deviation are not enough to describe and characterize spatial variability in probabilistic analyses.

Spatial variability is verified experimentally by M.B Jaksa (1995) over the period of 7½ years, from 1988 to 1995. He performed 223 electrical cone penetration test (CPT) in Adelaide city, Australia. CPT was done in 5×5 meters area to the depth of 5 meters at intervals of 5 mm to quantify small-scale spatial variability of the Keswick and Hindmarsh Clays. For gathering geotechnical data like shear strength of the soil in large scale, M.B Jaksa (1995) used 10140 data of 380 separate boreholes from one government agency and CPT data of 7 private geotechnical consultancies in Adelaide city area. Figure 2 shows how cone tip resistance that is used for calculating shear strength in horizontal and vertical directions is spatially variable (M.B. Jaksa, 1995).

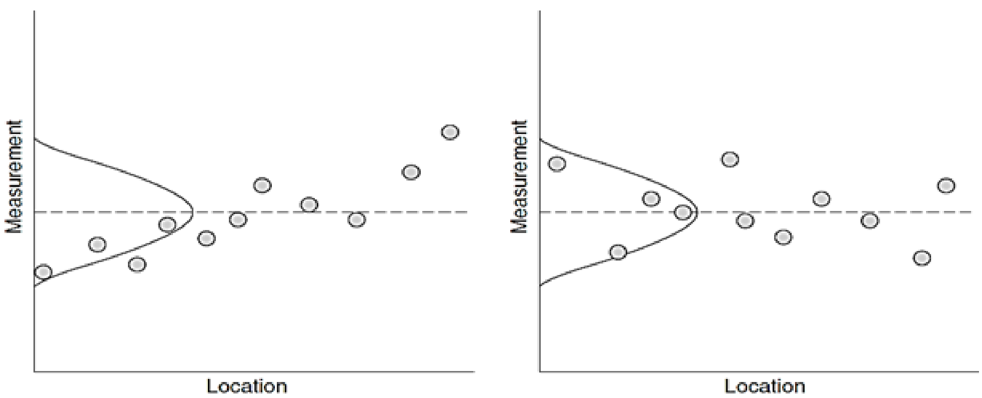

Figure 1: Different patterns of spatial variability in two sets of data with the same mean and standard deviation (After G.B Baecher & J.T Christian, 2003).

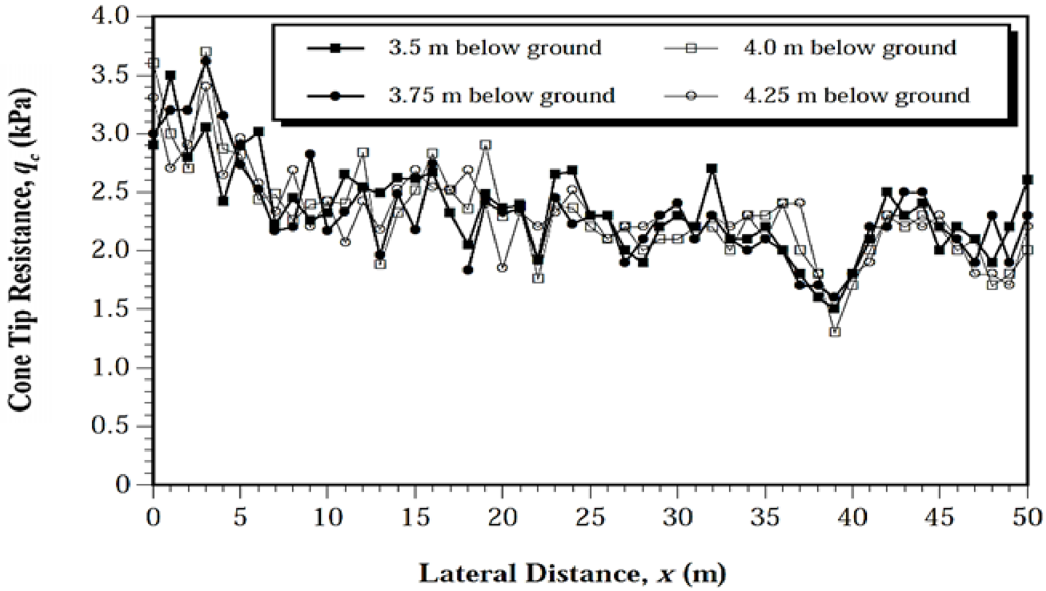

Figure 2: Experimental data of horizontal and vertical spatial variability in Keswick area (After Jaksa 1995).

If spatial variability is ignored, estimating the probability of failure is unconservative especially when factor of safety is low or coefficient of variation of soil strength parameters is relatively high (G.A Fenton & D.V Griffiths, 2008). In addition, Y. Wang et al, (2011) found that ignoring spatial variability, cause overestimation of the variance of FOS, and consequently overestimation of P_f. This is for cases that FOS=1 occurs at the lower tail of the probability distribution of FOS. If FOS=1 occurs at center or upper tail of probability distribution of FOS, there will be overestimation of the variance of FOS which leads to underestimation of P_f and this is unconservative.

M.B Jaksa (1995) while characterizing spatial variability of Keswick clay examined accuracy of his experimental data by mathematical theory of random fields. Using random fields theory is a common way for modeling spatial variability, following the works of E.H. Vanmarcke (1983) who was the first one that used random field theory in geotechnics. For modeling spatial variability, random variables over a space, are mapped onto a n ($n \in N$, N=1,2,…) dimensional domain which are referred to as random fields. Random field $\xi(h,v)$ on D (i.e. $(h,v) \in D^2 \subset \mathbb{R}^n$) is a function whose values are random variables for any $\xi(h,v) \in \mathbb{R}^2$. In slope stability, spatial variability of a shear strength parameter like cohesion is modeled by random fields.

Analytical-Based Approaches

To perform a probabilistic analysis, in addition to the mean value of soil strength parameters, variance or standard deviation, covariance and correlation function between uncertain parameters are needed.

First order second moment (FOSM): First order second moment method use first order of Taylor series expansion of FOS about second moments that are mean and variance. FOSM evaluates mean value ($\mu_{(FOS)}$) and standard deviation ($\sigma_{(FOS)}$) of factor of safety that are needed for calculating reliability index (β) as follows:

$$\beta = \frac{\mu_{(FOS)}-1}{\sigma(FOS)} \tag{1}$$

$\mu_{(FOS)}$ is obtained by using mean values of cohesion and friction angle in the factor of safety equation that can be calculated by any of limit equilibrium methods. For calculating the variance of factor of safety ($\sigma(_{FOS})$), one should differentiate the factor of safety with respect to cohesion and friction angle in the mean value and multiply it by the variance of them separately, then sum up them all as it is usual in Taylor's series expansion.

$$\sigma^2_{(FOS)} = (\frac{\sigma(FOS)}{\sigma(c)})^2 \times \text{variance}(c) + (\frac{\sigma(FOS)}{\sigma(\varphi)})^2 \times \text{variance}(\varphi) \tag{2}$$

After deriving reliability index, probability of failure P_f is given by Equation 3 where ψ is standard cumulative distribution function.

$$P_f = \psi(-\beta) \tag{3}$$

Beside FOSM method is popular in geotechnical applications, in most practical cases if the function of factor of safety is highly non-linear in random variables, computation of the derivatives are impossible or inconvenient and make the FOSM results inaccurate (R.D. Suchmol et al, 2010).

First Order Reliability Method (FORM): A.M. Hasofer and N.C. Lind (1974) proposed the method of First Order Reliability Method (FORM) that is an improvement on the FOSM. This method requires iteration and represents the reliability index as a measure of distance in dimensionless space between the peak of distribution of the uncertain parameters and a function defining the factor of safety. Beside there is no method that is comparable to FORM in simplicity, R. Rackwitz (2001) and K.K. Phoon (2008) found that FORM works only for the slopes with small P_f or high reliability index. D.V. Griffith et al. (2010) discussed the cases that FORM results are inaccurate and conservative after D.V. Griffith et al, (2007) illustrated the limitations of FORM in details. Results of FORM and two simulation methods, which are Monte Carlo Simulations (MCS) and Important Sampling (IS) for three numerical cases have been compared and it was concluded that FORM may underestimate P_f of slopes (J. Ching et al. 2009)

Point estimate method: Another probabilistic method applied to slope stability is point estimate method introduced by E. Rosenblueth (1975) that is analogous to the numerical integration methods. Point estimate method has a simple procedure that obtains the variance of the factor of safety by evaluating the factor of safety function at a set of specifically chosen discrete points. This method is widely employed in practice by researchers such as N.C. Lind (1983), T.F. Wolff (1996), and J.M Duncan (1999). Limitations of Point Estimate Method are discussed in details by G.B Baecher and J.T Christian (2003). For example, two points may not be adequate to obtain accurate estimates of the probability of failure in particular cases (G.B Baecher and J.T Christian, 2003).

Simulation-Based Approaches

Monte Carlo Simulations (MCS): Monte Carlo method by a simple numerical algorithm is a simulation-based method for calculating probability of failure and reliability index. Regardless of the problem that MCS is used for, first step is generating random numbers from a probability density functions (pdf) like normal or lognormal pdf. Each of these random numbers is called a realization or a Monte Carlo seed.

For calculating P_f of the slopes by LEM-based probabilistic analysis, critical slip surface is searched and minimum factor of safety by using each realization of soil strength parameters as their mean value is determined. Probability of failure is ratio of the number of realizations that have a factor of safety smaller than one to the number of realizations (n).

$$P_f = \frac{\sum_{i=1}^{n} I(FOS_i < 1)}{n} \tag{4}$$

Where I [·] is an indicator function that when the FOS_i is smaller than one, have the value of 1, otherwise it is zero. A property that affects the accuracy of MCS is type of the probability density function that is used. In addition, number of realizations is a decisive factor. To obtain an optimum number of realizations (n), the probabilities of failure for several numbers of realizations are estimated. In the plot of P_f against realizations number, the point that P_f does not change any more with increasing number of realizations, indicates for optimum number of realizations. It means there is no need to increase number of realizations any more for an unbiased estimate of the probability of failure. Relation between number of realizations and reliability index by using 4 kind of limit equilibrium methods (Ordinary, Bishop, Janbu & Spencer) as deterministic part, shows no significant difference in the reliability index as the realization number exceed 700 in a sample slope by A.H. Malkawi (2000). In some cases, thousands of random numbers are needed for a Monte Carlo simulation. J. Ching et al, (2008) suggests that $10/P_f$ deterministic analyses are needed to make the coefficient of variation of factor of safety smaller than 30%.

Estimated probability of failure by MCS is logical but MCS is time-consuming for slopes with small failure probabilities. In such cases, importance sampling that is a modified Monte Carlo simulation is used instead of traditional Monte Carlo simulations (J. Ching et al. 2008). Important Sampling (IS): To run Monte Carlo simulations in the important sampling, a new probability density function is defined for generating realizations of Monte Carlo. Important sampling probability density function (IS pdf) is much closer to the failure region. Therefore, less realizations are needed in important sampling for the slopes with low P_f and this make the IS more efficient than Monte Carlo Simulations. Full description of defining IS pdf and other aspect of implementing IS are available in J. Ching et al. (2009).

LEM- based probabilistic analysis

Despite the popularity of LEM has among practical engineers, few works model spatial variability in LEM compared to FE SRM-based probabilistic analyses (S.E. Cho, 2007). A short review on the LEM-based probabilistic analyses that model spatial variability is done by D.V. Griffiths.et al. (2010). However, these LEM based works have three shortcomings: 1) the shape of failure surface is limited to the circular. 2) Non-rigorous methods, which only satisfy either force or moment equilibrium, or even a second-order polynomial function, are used to calculate FS. 3) Spatial variability is not modeled, or only modeled as one-dimensional which is much less accurate than two-dimensional. The consequences of ignoring these shortcomings will result in a biased and potentially unconservative estimation of the system reliability of slope (D.V. Griffiths et al. 2009; J. Ching et al. 2010; D.V Griffiths et al. 2010).

Another deficiency for modeling spatial variability in LEM based method is that they search for the critical slip surface by using realizations of random field for a fixed slip surface. Then, calculate FS and probability of failure for this predetermined slip surface. However, S.E. Cho (2010) modeled spatial variability by two-dimensional random fields that were generated based on a Karhunen-Loève expansion and studied the effects of spatial variability on the stability of slopes based on the limit equilibrium methods. Various failure modes caused by spatial variability are assessed that would be neglected if spatial variability were modeled only through fixed slip surface.

The drawback of S.E. Cho (2010) is that he used only circular slip surface that is sufficient for homogenous slopes but not for the cases that spatial variability is considered. If spatial variability is modeled in 2-D, many weak points that are distributed erratically appear in each realization of random fields and circular slip surface cannot capture them properly.

The problems of LEM-based methods increase for the slopes with more than one failure modes. J.M. Duncan and S.G. Write (2005) discussed such situations. J. Ching et al. (2010) found that difficulties of LEM-based probabilistic analysis in the presence of weak seem increase. If the slip surface in the LEM is limited to circular shape, many failure mode that pass through weak seems cannot be captured and probability of failure is under-estimated (J. Ching et al. 2010).

FE SRM-based probabilistic analyses

In the FE-SRM analysis, gravity loads are applied to element mesh and the soil is weakening systematically until sufficient number of failure mechanism forms. (D.V Griffiths et al. 2007). To perform FE SRM-based probabilistic analysis, Random Finite Element Method (RFEM) with open source software developed by G.A Fenton and D.V. Griffiths (2008) may be used. RFEM is a combination of finite element analysis, random fields, and Monte Carlo simulations. For considering spatial variability, RFEM based on defined statistical distribution generate desired number of realizations for random variables .

D.V. Griffith et al. (2010) compared the P_f in LEM-based probabilistic analysis with FE SRM-based probabilistic analysis and found that for different scale of fluctuations (Figure 3), probability of failure found by S.E. Cho (2007) is smaller than P_f by RFEM. The benefits of using RFEM for modeling spatial variability in probabilistic analysis over LEM are discussed by

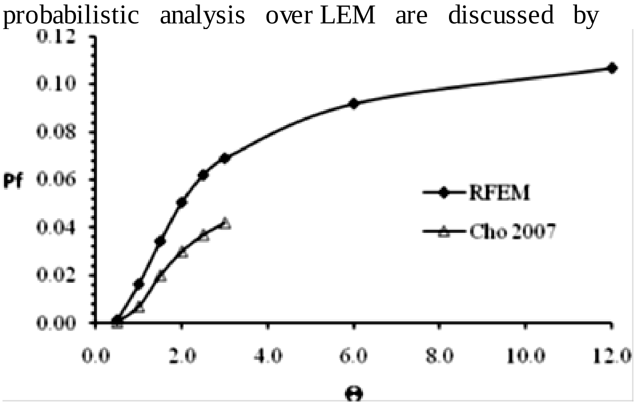

Figure 3: Comparison of the LEM-based probability of failure calculated by S.E. Cho (2007) and RFEM by Griffiths et al, 2010 (After D.V. Griffiths et al. 2010).

D.V. Griffiths et al. (2010). RFEM is free of assuming a fixed deterministic slip surface that is common in LEM based methods. In the RFEM failure mechanism can develop through weakest paths in every realization of random field and also Monte Carlo simulations.

Conclusion

This paper presented a state of the art review on the application of probabilistic analyses in the slope stability problems. Probabilistic analyses can provide a rational framework to deal with uncertainties in input parameters such as cohesion and friction angle. Spatial variability as a main source of uncertainty in slope stability analysis can be modeled by random field theory in both FEM and LEM based probabilistic methods. Ignoring the effects of spatial variability may overestimate or underestimate the probability of failure depending on the problem. As shown in this review, the complex structure of analytical based probabilistic methods, which require derivatives of the objective function, makes its application area for iterative function of FOS questionable. However, simulation based probabilistic analyses (e.g., Monte Carlo simulation) evaluate an unbiased estimate of the probability of the failure. It is also shown that the difficulties in LEM such as assumption on interslice force and shape of failure surface limit the use of LEM in probabilistic analyses of slopes that consider spatial variability. In contrast, FEM is free of the mentioned difficulties and seems to work properly in probabilistic analyses of slopes. Hence, random finite element method (RFEM) can be regarded as one of the few proposed technique up to now that can provide an unbiased estimate of the reliability of slopes in a reasonable time.

Reference

A.H. Malkawi, W.F Hassan and F.A. Abdulla, (2000), Uncertainty and reliability analysis applied to slope stability, *Structural Safety*, No.22, pp: 61-187.

A.M. Hasofer, N.C. Lind, (1974), An exact and invariant first-order reliability format, *Journal of the engineering Mechanics Division*, pp:111–121.

A.W. Bishop, (1955), The use of the slip circle in the stability analysis of slopes, *Geotechnique (5) and Geoenvironmental Engineering*, No.135, pp:7-17.

D. V. Griffiths, and G.A. Fenton, (2001), Bearing capacity of spatially random soil: the undrained clay Prandtl problem revisited. *Geotechnique*, No.51, pp:351-359.

D.V. Griffiths, and G.A. Fenton, (2004), Probabilistic slope stability analysis by finite elements *Journal of Geotechnical and Geo environmental Engineering*, No.130, pp: 507-518.

D.V. Griffiths, D. V & P.A Lane, (1999). Slope stability analysis by finite elements. *Geotechnics*. No.49, pp:387-403.

D.V. Griffiths, D.V, Fenton, G.A and M.D. Denavit, M.D. (2007), Traditional and advanced probabilistic slope stability analysis, *Geotechnical Special Publication*, No. 170, pp: 19-39.

D.V. Griffiths, D.V, J, Huang, Gordon A. Fenton, G.A. (2010), Comparison of Slope Reliability Methods of Analysis. Geo Florida: *Advances in Analysis, Modeling & Design, ASCE.*

D.V. Griffiths, J. Huang, and G.A. Fenton, (2009), Influence of spatial variability on slope reliability using 2-D random fields. *Journal of Geotechnical and Geo environmental Engineering*, No.135, pp:1367-1378.

D.Y. Zhu, C.F. Lee, Q.H Qian, Z.S. Zou, F. Sun, F., (2001), A new procedure for computing the factor of safety using the Morgenstern-Price method. *Canadian Geotechnical Journal*, No.38. pp: 882-888.

E. Rosenblueth, (1975), Point estimates for probability moments. *Proceedings National Academy of Science*. No.72, pp:3812–3814.

E. Spencer, (1967), A method of analysis of the stability of embankments assuming parallel inter-slice forces, *Geotechnique*, No.17, pp:ll-26.

E.E. Alonso, (1976). Risk analysis of slopes and its application to slopes in *Canadian sensitive clays. Geotechnique*, No. 26, pp: 453-472.

E.H. Vanmarcke, (1977), Probabilistic modeling of soil profiles. *ASCE J Geotech Eng Div*, No.103, pp:1227-1246.

E.H. Vanmarcke, (1983), Random Fields: Analysis and Synthesis, *MIT Press*, Cambridge, Massachusetts.

G. B. Baecher, and J.T. Christian (2003), Reliability and statistics in geotechnical engineering. John *Wiley and Sons. England*.

G.A. Fenton and D.V. Griffiths, (2008), Risk Assessment in Geotechnical Engineering, *John Wiley & Sons, New York*.

J. Ching, J. Chen, K.K. Phoon, (2011), Updating uncertainties in friction angles of clean sands, *Geotechnical Special Publication* (224GSP), pp. 1133-1140.

J. Ching, K.K. Phoon and Y.G. Hu. (2009). Efficient Evaluation of Reliability for Slopes with Circular Slip Surfaces Using Importance Sampling. *Journal of Geotechnical and Geoenvironmental Engineering*. No.135, pp: 768-777.

J. Ching, K.K. Phoon and Yu-Gang Hu (2008), Efficient evaluation of slope stability reliability subject to soil parameter uncertainties using importance sampling, Landslides and Engineered Slopes, *Taylor & Francis Group, London*.

J. Ching, K.K. Phoon, and Y.G. Hu (2010), Observations on Limit Equilibrium–Based Slope Reliability Problems with Inclined Weak Seams, *Journal of Engineering Mechanics*, No.10, 136, pp:1220-1233.

J. Ching, K.K. Phoon, Y.G. H, (2009), Efficient evaluation of reliability for slopes with circular slip surfaces using importance sampling, *Journal of Geotechnical and Geo environmental Engineering*, No.135, pp:768-777.

J. Ching, K.K. Phoon, Y.G. Hu, (2009), Efficient evaluation of reliability for slopes with circular slip surfaces using importance sampling *Journal of Geotechnical and Geo environmental Engineering*, No.135, pp:768-777.

J. M Duncan and S.G.Wright, (2005), Soil Strength and Slope Stability. *John Wiley & Sons. New Jersey*.

J. M. Duncan, (1999), The use of back analysis to reduce slope failure risk: The Seventh Annual Arthur Casagrande Memorial Lecture. Civil Engineering Practice, *Journal of the Boston Society of Civil Engineers Section, ASCE,* No.14, pp: 75–91.

J. Zhang, L.M. Zhang, W.H. Tang, (2011). New methods for system reliability analysis of soil slopes. *Canadian Geotechnical Journal*, No.48, pp: 1138-1148.

J. Zhang, L.M. Zhang, W.H. Tang, (2011). New methods for system reliability analysis of soil slopes. *Canadian Geotechnical Journal*, No.48, pp: 1138-1148.

J.F. Xue, K. Gavin,(2007), Simultaneous determination of critical slip surface and reliability index for slopes *Journal of Geotechnical and Geoenvironmental Engineering*, No.133, pp: 878-886.

J.T. Christian, (2004), Geotechnical Engineering Reliability "How well do we know what we are doing?" , The 39th Terzaghi Lecture, *Journal of Geotechnical and Geo environmental Engineering, ASCE*, Vol. 130, No.10, pp: 985–1003.

K.K. Phoon, (2008), Reliability-Based Design in Geotechnical Engineering. *Taylor & Francis, London and New York*.

K.K. Phoon, F.H. Kulhawy (1999), Evaluation of geotechnical property variability, *Canadian Geotechnical Journal*, No.36, pp. 625-639.

M.B Jaksa (1995), The influence of spatial variability on the geotechnical design properties of a stiff, over consolidated clay, Ph.D thesis, *University of Adelaide*.

N. Janbu, (1973), slope stability computations, *Journal of Embankment-Dam Eng*, pp. 47-86.

N.C. Lind, (1983), Modeling uncertainty in discrete dynamical systems, *Applied Mathematical Modelling*, No. 7, pp:146–152.

O.C. Zienkiewics, C. Humpheson, and R.W. Lewis, (1975), Associated and non-associated visco plasticity and plasticity in soil mechanics. *Geotechnique*. No.25, pp: 671-689.

R. Chowdhury, P. Flentje, G. Bhattacharya, (2010), Geotechnical Slope Analysis. *Taylor & Francis Group, London, UK*.

R. Rackwitz, (2001), Reliability analysis – a review and some perspectives. *Structural Safety*, No.23, pp:365–95.

R. Rackwitz, (2001), Reliability analysis—a review and some perspectives, *Structural Safety*, No.23, pp:365–395.

R.D. Suchomel, Mašín. (2010). Comparison of different probabilistic methods for predicting stability of a slope in spatially variable c–ϕ soil. *Computers and Geotechnics*. No.37, pp:132–140.

S.E. Cho (2007), Effects of spatial variability of soil properties on slope stability, *Engineering Geology*, No.92, pp: 97–109.

Spencer E. (1967), A method of analysis of the stability of embankments assuming parallel inter-slice forces, *Geotechnique*, No.17, pp: 11–26.

T.F. Wolff, (1996). Probabilistic slope stability in theory and practice, Uncertainty in the Geologic Environment, Madison, *WI, ASCE*, pp: 419–433.

Y. Wang, Z.J Cao, S.K. Au, (2011), Effect of spatial variability on reliability evaluation of an earth slope, *Conference on Applications of Statistics and Probability in Civil Engineering*, pp: 1666-1672.