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Research Article

# Mechanical Properties of Hybrid Elliptical Fiber Reinforced Lamina with Equal Fiber Volume Fractions

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## *Abstract*

*A three-dimensional finite element model is developed and solved for the micromechanical prediction of Young's modulii and Poisson's ratios of an hybrid FRP lamina consisting of two different fiber materials (T-300 & S-Glass) embedded in a thermosetting polymer matrix. The finite element model of representative volume element of hexagonal pattern is generated in ANSYS software. The cross-section of each fiber is taken as ellipse. The effect of fiber volume fraction and ellipse aspect ratio on the predicted mechanical properties is discussed.*

***Keywords:*** *Hybrid, FRP, FEM, Micromechanics*

## 1. Introduction

A hybrid composite consists of two or more types of reinforcing fibers in one or more types of matrices. By hybridizing two or more types of fiber in a matrix allows a closer tailoring of composite properties to satisfy specific requirements compared with composites with only a single type of fiber. Modeling of composites made up of inclusions embedded in a matrix has been a subject of interest of many researchers in the past half-century. Noteworthy among the earlier models are the works of (Eshelby, 1957), (Hashin,1962), (Hill,1963&1965), (Hashin and Shtrikman, 1963), (Hashin and Rosen, 1964). (Hashin and Shtrikman, 1963) used Variational principles to obtain upper and lower bounds for the effective elastic moduli as well as the effective electrical and thermal conductivities of multiphase composites with quasi-isotropic global characteristics. Later on, (Milton, 1981&1982) obtained higher-order bounds for the elastic, electromagnetic, and transport properties of two-component macroscopically homogenous and isotropic composites given the properties of the individual constituents. More recently, (Drugan and Willis, 1996) and (Drugan, 2003), employed the Hashin–Shtrikman variational principles to analyze two-phase composites with random microstructure. A numerical implementation of this work was carried out by (Segurado and Llorca, 2002). Other significant early results can be found in the work of (Budiansky, 1965), (Russel, 1973). (Mori and Tanaka, 1973) in their micromechanical approach obtained closed-form expressions for the elastic properties of two-phase composites. (Ying Shan and Kin Liao, 2002) proposed a simple life prediction model for the hybrid composite.

The present research work deals with the analysis of micromechanical behaviour of unidirectional continuous hybrid elliptical fiber-reinforced composite by three-dimensional elasticity theory based finite element method. The analysis includes prediction of modulii and Poisson's ratios of the hybrid lamina subjected to longitudinal, in-plane transverse and out-of-plane transverse loads.

## 2. Methodology

The unidirectional continuous fiber reinforced composite lamina has been idealized as a large array of representative volume elements. Depending upon the arrangement of the fibers across the cross section of the lamina, different types of representative volume elements can be obtained such as square, hexagonal, staggered square patterns etc. In any pattern repetition of a particular volume of the lamina can be observed, which is called the representative volume element (RVE) or unit cell.

### *2.1 Hexagonal Array of Unit Cells*

A schematic diagram of the unidirectional fiber composite is shown in Fig.1 where the fibers are arranged in the hexagonal array. It is assumed that the fiber and matrix materials are linearly elastic. A unit cell is adopted for the analysis. The cross sectional area of the fiber relative to the total cross sectional area of the unit cell is a measure of the volume of fiber relative to the total volume of the composite. This fraction is an important parameter in

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composite materials and is called fiber volume fraction ($V_f$).

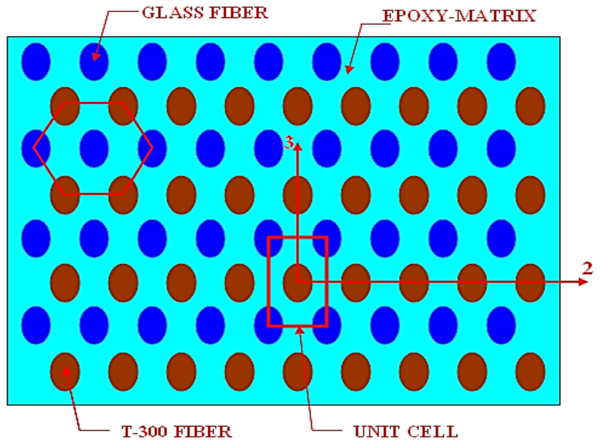

Fig. 1 Hybrid composite lamina

The isolated unit cell behaves as a part of large array of unit cells by satisfying the conditions that the boundaries of the isolated unit cell remain plane. It is assumed that the geometry, material and loading of unit cell are symmetric with respect to 1-2-3 coordinate system (1- is longitudinal direction of the fiber). Therefore, a one-eighth portion of the unit cell is modeled for the present work.

### *2.2 Finite Element Model*

The element used for the present analysis is SOLID 95 of (ANSYS,2008), which is developed, based on three-dimensional elasticity theory and is defined by 20 nodes having three degrees of freedom at each node: translation in the node x, y and z directions. Finite element meshing of one eighth portion of the unit cell (i.e. one quarter in the cross section and one-half in the longitudinal direction of the fiber) is shown in Fig. 2. The dimensions of FE model are taken as 50, 86.6 and 10 units in x- y- and z- directions respectively. The dimensions of ellipse are obtained according to the fiber volume fraction and the ellipse aspect ratio (‘a’= axis length in 3-direction by axis length in 2-direction).

### *2.3 Boundary Conditions and Loading*

The loading, boundary conditions and other multipoint constraints are applied in such a way that the faces of the FE model remain plane during and after deformation. i) Uni-axial state of stress of 1MPa is applied in longitudinal direction of the fiber for the prediction of $E_1$, $\nu_{12}$ and $\nu_{13}$ respectively, ii) $E_2$, $\nu_{21}$ and $\nu_{23}$ are obtained from in-plane transverse load and iii) $E_3$, $\nu_{31}$ and $\nu_{32}$ from out-of-plane transverse load.

### *2.4 Materials*

The arrangement of fibers in hybrid composite is as shown in Fig. 2. The mechanical properties of the constituent materials used in the present analysis are given in Table 1.

Table 1. Mechanical properties of the constituent materials

| Property | T-300 fiber | S-Glass fiber | HM Polymer Matrix |
|---|---|---|---|
| $E_1$ (GPa) | 220.6 | 85.5 | 5.17 |
| $E_2$ (GPa) | 13.79 | 85.5 | 5.17 |
| $E_3$ (GPa) | 13.79 | 85.5 | 5.17 |
| $\nu_{12}$ | 0.2 | 0.2 | 0.35 |
| $\nu_{23}$ | 0.25 | 0.2 | 0.35 |
| $\nu_{13}$ | 0.2 | 0.2 | 0.35 |
| $G_{12}$ (GPa) | 8.96 | 35.62 | 1.91 |
| $G_{23}$ (GPa) | 4.83 | 35.62 | 1.91 |
| $G_{13}$ (GPa) | 8.96 | 35.62 | 1.91 |

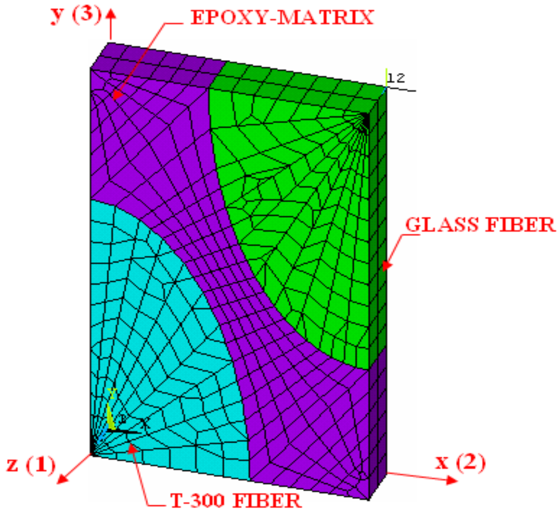

Fig. 2 FE mesh on one-eighth portion of the unit cell (one quarter in cross section and half in length direction)

## 3.0 Results

The finite element software ANSYS is successfully executed for the analysis. The elastic properties are evaluated using the normal strains in 1, 2 and 3 directions calculated from the normal deformations of the unit cell obtained from finite element analysis. The displacements in x, y and z directions, $U_x$, $U_y$ and $U_z$ respectively of the finite element model are obtained from the finite element solutions. The corresponding normal strains are determined from the displacements. The longitudinal Young’s moduli and Poisson’s ratios due to the longitudinal load are determined from the following expressions

$$E_1=\sigma_1/\varepsilon_1 \qquad \nu_{12}= - \varepsilon_2/ \varepsilon_1 \qquad \nu_{13}= - \varepsilon_3/ \varepsilon_1$$

where 1(z), 2(x) and 3(y) are longitudinal, in-plane transverse and out-of-plane transverse directions respectively of the composite lamina. Remaining properties are obtained in similar fashion for in-plane and out-of-plane transverse loads. The results are obtained for hybrid lamina consisting of T-300 fiber, S-Glass fiber and Epoxy matrix. In this case the volume of both the fibers is taken equally.

### *3.1 Validation*

The finite Element model is validated for the longitudinal Young's modulus using Rule of Mixtures (ROM). The results are presented in the Table 2. A very close agreement is observed between the FE and analytical results.

Table 2 Validation of $E_1$

| $V_f$ | $E_1$ (ROM) | $E_1$ (FEM) | % Variation |
|---|---|---|---|
| 0.10 | 19.958 | 19.974 | 0.080168 |
| 0.20 | 34.746 | 34.771 | 0.071951 |
| 0.30 | 49.534 | 49.569 | 0.070659 |
| 0.40 | 64.322 | 64.361 | 0.060632 |
| 0.50 | 79.11 | 79.148 | 0.048034 |
| 0.60 | 93.898 | 93.939 | 0.043664 |
| 0.70 | 108.686 | 108.726 | 0.036803 |

Fig. 3 shows the variation of $E_1$ with respective to the fiber volume fraction ($V_f$) for the values of 'a' ranging from 0.5 to 1.5 as represented in legend. As the volume fraction increases, the range of 'a' decreases and 'a'=1 (circle) beyond $V_f$=0.75. From Fig. 3 it is observed that $E_1$ of the lamina is increasing with respective $V_f$ in a linear manner but no variation due to aspect ratio 'a'. This is true since rule of mixtures uses cross-sectional area of the fiber but not the shape of the fiber.

Figs. 4 and 5 show the variation of $\nu_{12}$ and $\nu_{13}$ with respective to $V_f$ for the stated values of 'a'. It is found that both the Poisson's ratios decreases with increase in $V_f$ . A decreasing trend of $\nu_{12}$ is observed with respect to 'a' and $\nu_{13}$ is increasing with 'a'. Increase in $V_f$ increases resistant of the material in all the directions but in the transverse directions the action of the matrix is more and therefore the rate of increase in resistance of the material in transverse directions is less when compared to the longitudinal direction, as a result the Poisson's ratio's $\nu_{12}$ and $\nu_{13}$ are decreasing with respective $V_f$ . Same reason can be attributed for the variation of these Poisson's ratios with respect to 'a'.

Figs. 6 and 7 show the variation of $E_2$ and $E_3$with respect to $V_f$ and 'a'. It is observed that $E_2$ increases with $V_f$ and 'a' where as $E_3$ increases with $V_f$ but decreases with 'a'. At lower $V_f$, there is no significant variation of $E_2$ and $E_3$ with 'a' and they vary at larger rate at higher $V_f$. increase in $V_f$ reduces the gap between fiber to fiber resulting in increase of stiffness of composite. This effect increases in 2-direction with increase in 'a' but decreases in 3-direction causing for reduction of $E_3$. Rate of variation of $E_2$ is more when compared to $E_3$ with respect to 'a'. This is due to the reason that in 3-direction, though the gap between fibers increases, the stiffness is not affected that much due to overlapping of fibers and elimination of clearance between fibers in an RVE in 2-direction.

Figs. 8 and 9 show the variation of minor Poisson's ratios $\nu_{21}$ and $\nu_{31}$ with respect to $V_f$ and 'a'. $\nu_{21}$ decreases up to certain value of $V_f$ depending up on 'a' and later increases. $\nu_{31}$ decreases continuously. $\nu_{21}$ increases with 'a' at higher $V_f$ but there is no significant variation of $\nu_{31}$ with 'a'. The reasons discussed for the variation $E_2$ and $E_3$ are applicable for these cases also.

Figs. 10 and 11 show the variation of transverse Poisson's ratios $\nu_{23}$ and $\nu_{32}$ with respect to $V_f$ and 'a'. $\nu_{23}$ increases up to certain value of $V_f$ depending up on 'a' and later decreases. $\nu_{32}$ decreases continuously except at lower values of 'a' where its variation is similar to that of $\nu_{23}$. There is no uniform trend of $\nu_{23}$ with respect to 'a' but there is a decreasing trend in $\nu_{32}$ with respect to 'a' which is drastic at higher $V_f$ . These variations are resulting from the change in stiffness of composite in one transverse direction due to load in other transverse direction.

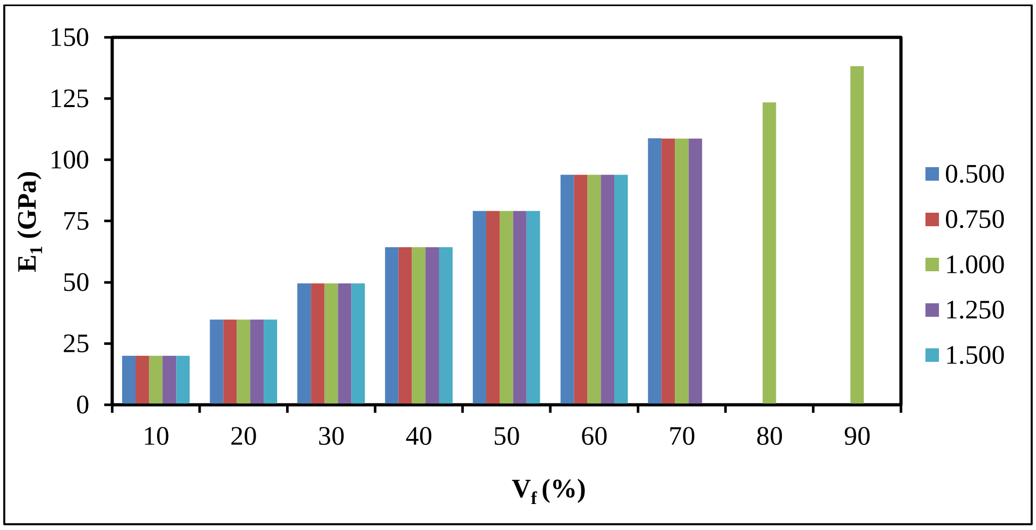

Fig. 3 Variation of $E_1$ with respect to $V_f$ for different 'a'

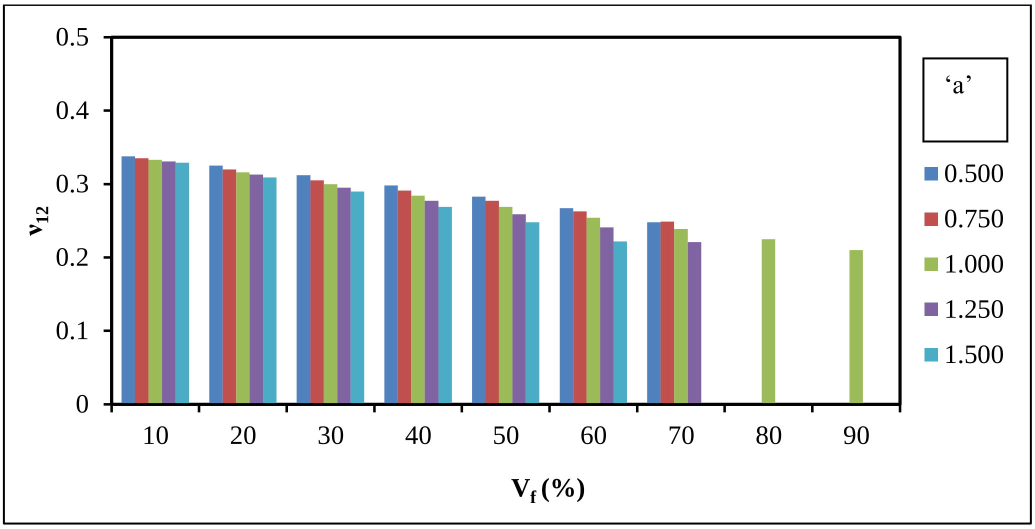

Fig. 4 Variation of $\nu_{12}$ with respect to $V_f$ for different 'a'

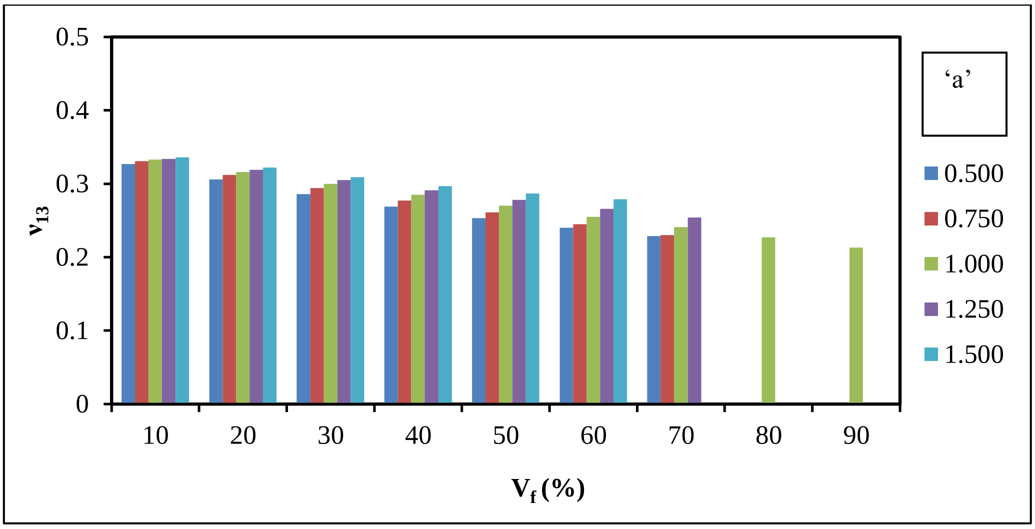

Fig. 5 Variation of $\nu_{13}$ with respect to $V_f$ for different 'a'

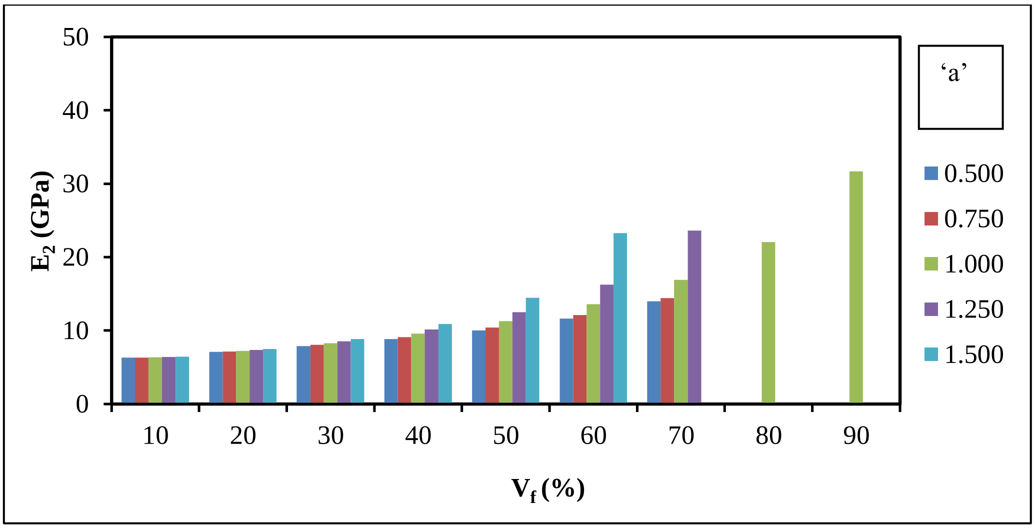

Fig. 6 Variation of $E_2$ with respect to $V_f$ for different 'a'

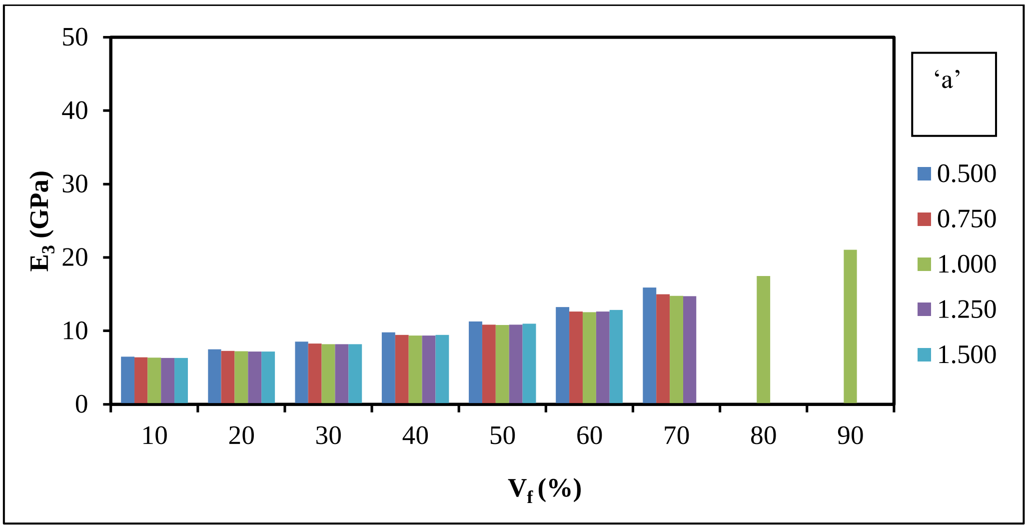

Fig. 7 Variation of $E_3$ with respect to $V_f$ for different 'a'

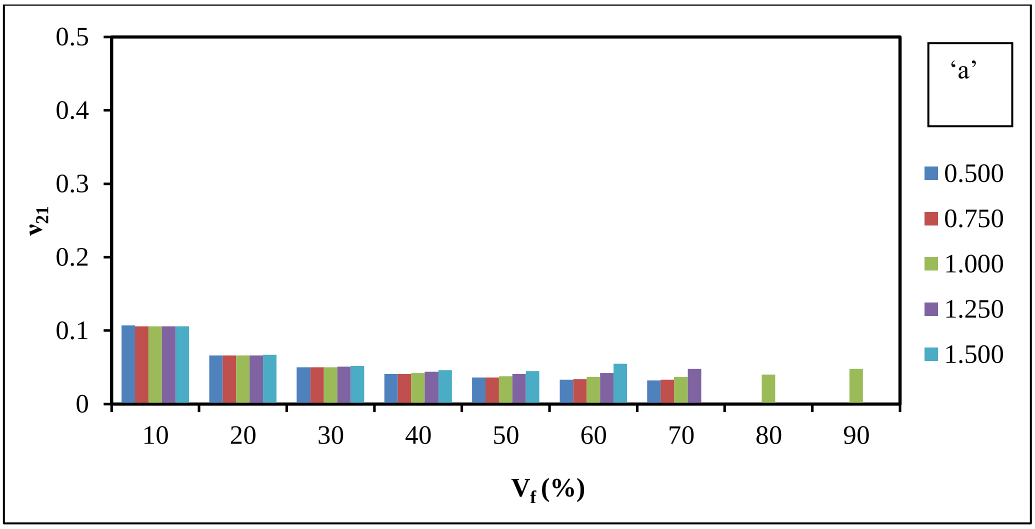

Fig. 8 Variation of $\nu_{21}$ with respect to $V_f$ for different 'a'

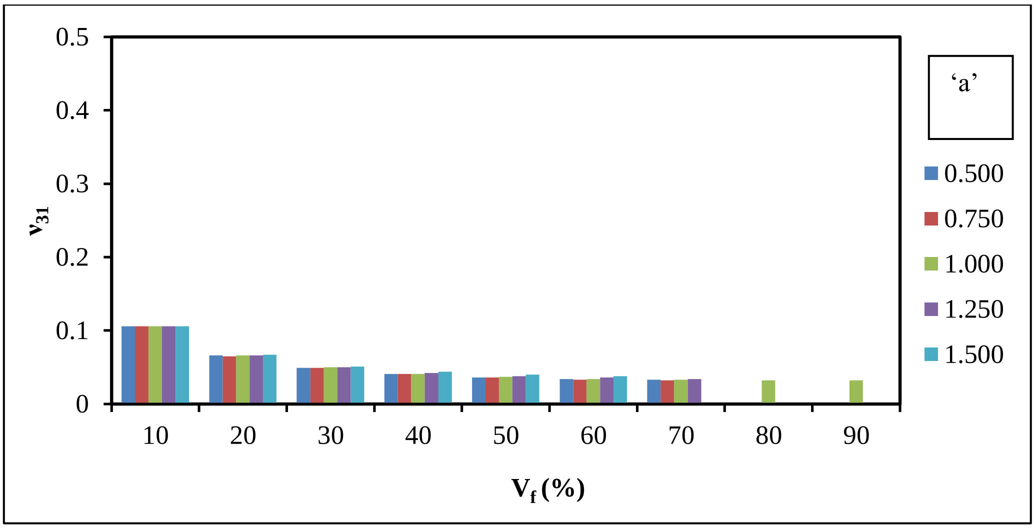

Fig. 9 Variation of $\nu_{31}$ with respect to $V_f$ for different 'a'

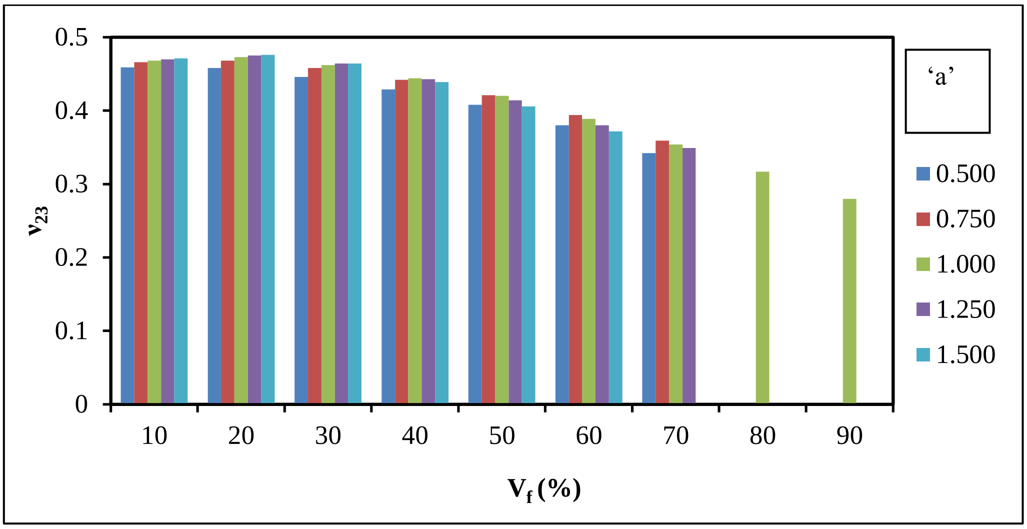

Fig. 10 Variation of $\nu_{23}$ with respect to $V_f$ for different 'a'

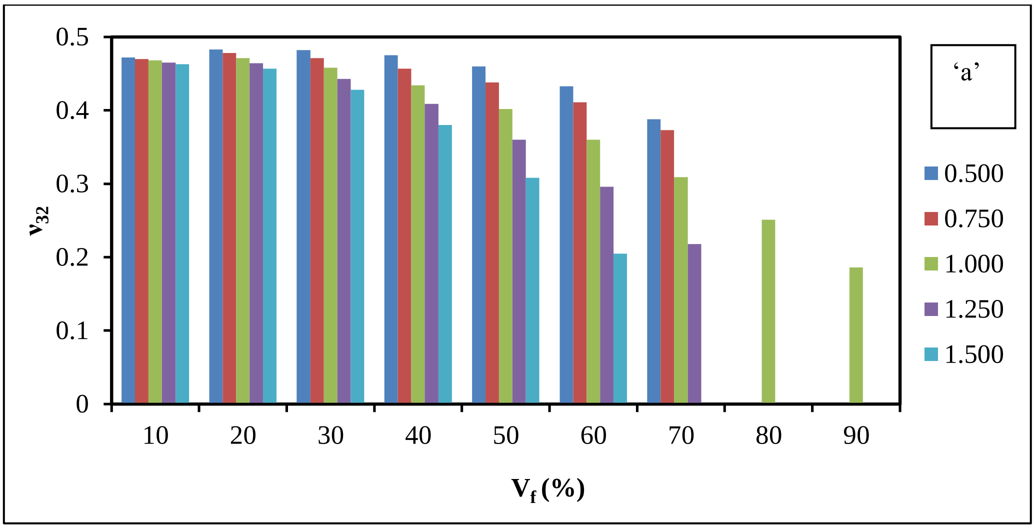

Fig. 11 Variation of $\nu_{32}$ with respect to $V_f$ for different 'a'

## Conclusions

Mechanical properties such as Young's modulus and Poisson's ratios are predicted by applying 3-D finite element method to solve an hexagonal RVE of an hybrid FRP composite consisting of T-300 and S-Glass fibers in a polymer matrix. The influence of fiber content and fiber cross-sectional arrangement in composite on the predicted properties is analyzed. The following conclusions are drawn.

- Increase in $V_f$ results in increase in Young's moduli.
- Ellipse aspect ratio influences transverse Young's moduli at higher $V_f$ and Poisson's ratios at almost all values of $V_f$.
- The idea of the present analysis gives the scope to choose various arrangements of materials in a composite in view of material stiffness.

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