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Research Article

MR Brain Image Segmentation based on Markov Random Field with the Application of ACO

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Abstract

Magnetic resonance (MR) medical image segmentation plays an increasingly important role in computer-aided detection and diagnosis (CAD) of abnormalities. MRI segmentation manually is time consuming and consumes valuable human resources. Hence a great deal of efforts has been made to automate this process. Markov Random Field (MRF) has been one of the most active research areas of MRI brain segmentation which seeks an optimal label field in a large space. The traditional optimization method is Simulated Annealing (SA) that could get the global optimal solution with heavy computation burden. Therefore great deal efforts have been made to obtain the optimal solution in a reasonable time. In this paper, we conduct a comparative study with the traditional minimization approach Simulated Annealing (SA) and a novel proposed method: MRF-Hybrid Parallel Ant Colony Optimization (MRF-HPACO) with Fuzzy C-Means (FCM) Algorithm for the segmentation of MR images. Comparing with Simulated Annealing (SA) and MRF with Improved Genetic Algorithm (MRF-IGA) that is often used in the image segmentations based on Markov Random Field (MRF) models, HPACO has been used in reducing the computation complexity of optimization. There are M colonies, M-1 colonies treated as slaves and one colony for master. Each colonies visit all the pixels without revisit. Initially, initialize the pheromone value for all the colonies. Posterior energy values are computed by Markov Random Field. If this value is less than global minimum, the local minimum is assigned to global minimum. The pheromone of the Ant that generates the global minimum is updated. At the final iteration global minimum returns the optimum threshold value for select the initial clustering the FCM implementation in the brain Magnetic Resonance Image (MRI) segmentation.The qualitative and quantitative results of each system are investigated as well.

Keywords: *Ant colony optimisation, Fuzzy C Means algorithm, Image segmentation, Magnetic Resonance Image, Markov Random Field, Simulated Annealing.*

1. Introduction

Segmentation is an important process in digital image processing which has found extensive applications in several areas. It aims to find the homogeneous regions for labeling objects and background. In other words image segmentation is a process of grouping the pixels of an image into regions with respect to certain features based on different segmentation methods. It is the process of partitioning an image into disjoint and homogeneous regions. A homogeneous region refers to a group of connected pixels in an image which share a common feature. This feature could be colour, texture, intensity, etc. Image segmentation is a critical step of any image analysis application as it has a significant influence on the quality of subsequent treatments as it isolates and extracts the pertinent features needed by image analysis processes.

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Extracting the information from an image is called image analysis. The first step in image analysis is to segment the image. Segmentation subdivides an image into its constituent parts (or) objects. Segmentation algorithm is based on one of the two basic properties of gray level values discontinuity and similarity. In discontinuity, the approach is to partition an image based on abrupt changes in gray level. Using this we can detect isolated points, lines and edges in an image. Using similarity approach we do thresholding, region growing and region splitting and merging.

Image segmentation is a wide-ranging domain with a rich literature describing unnumbered set of methods. Automatic Magnetic Resonance Image segmentation is one of the most important steps in computer –aided detection and diagnosis (CAD) of abnormalities such as lesions, cancers or tumors. On CT and MRI are based on tissue characteristics like calcifications, fat, cystic components, contrast enhancement and signal intensity on T1WI, T2WI and DWI. Most brain tumors are of low signal intensity on T1WI and high on T2WI. Therefore high signal intensity on T1WI or low signal on T2WI can be an important clue to the diagnosis. Finally to detect the possibility with a lesion that simulates a tumor - like an abscess, MS-plaque, vascular malformation, aneurysm or an infarct with luxury perfusion. Roughly one-third of CNS tumors are metastatic lesions, one third is gliomas and one-third is of non-glial origin. Glioma is a non-specific term indicating that the tumor originates from glial cells like astrocytes, oligodendrocytes, ependymal and choroid plexus cells. Astrocytoma is the most common glioma and can be subdivided into the low-grade pilocytic type, the intermediate anaplastic type and the high grade malignant glioblastoma multiforme (GBM). GBM is the most common type (50% of all astrocytomas). The non-glial cel tumors are a large heterogenous group of tumors of which meningioma is the most common. To detect and diagnose the CNS tumor Initial and most important process is segmentation of timorous tissue and then to classify the types based on classification techniques. This paper proposed the improved methodology to segment the CNS tumor. The various segmentation methods have been proposed in the literature. Numerous segmentation methods have been proposed in the research literature, e.g., thresholding methods (P.K. Sahoo et al,1988), clustering methods(A.K. Jain et al,2000), edge-based methods, region splitting and merging methods, and multi-resolution techniques. This paper addresses a Markov random field (MRF) based segmentation approach for segmenting tumour tissue in MR Brain image.

There are various MRF based segmentation models that have been developed. Cohen and Cooper (F.S. Cohen et al,1987) proposed a doubly MRF model for segmenting range images and natural scenes. The doubly stochastic representation uses a Gaussian MRF to model textures and an auto-binary MRF to model a priori information about the local geometry of textured image regions. Won and Derin (C.S. Won et al, 1992) developed a hierarchical MRF model for segmenting noisy and textured images. The model assumes the texture process also as a Gaussian MRF and can be used to segment images with GMRF-modelled textures very well. Geman and Geman (D. Geman et al,1990) proposed an algorithm based on simulated annealing to find the MAP (Maximum A-Posteriori) estimate of the true image, which minimizes the energy function over all possible labelling. They were the first to apply the methods of statistical mechanics to image segmentation. They use an a priori probability model for neighbouring voxels and some additional, hidden edge elements. But they do not take account of nonparametric intensity distributions and the inhomogeneities that are important for MR images.

MRF is a statistic model which seeks the optimal label field of the image pixels (Stan Z. Li et al,2004). Markov Random Field based image segmentation is based on region based segmentation technique.The MRF is a stochastic process that specifies the local characteristics of an image. The MRF itself is a conditional probability model, where the probability of a pixel depends on its neighbourhood. The MRF is a discrete stochastic process whose global properties are controlled by means of local properties.

In this work the Magnetic Resonance (MR) Brain image with tumour has been considered. The main aim of this work is to segment the tumourous tissues from the brain image using three different algorithms for seeking the optimal solution .The first is classical MRF method which is based on Simulated Annealing (SA). This method converges to the global optima asymptotically but requires a great deal of computation. The second applies a hybrid of simulated annealing (SA) and (HPACO) in order to optimize the problem which is formulated by MRF. The proposed method is also compared with MRF with Improved Genetic Algorithm (MRF-IGA) for the segmentation of MR images.

This paper is organised as follows: In Section 2 image datasets, MRF model is introduced briefly, and describes the optimization algorithms in order to seek optimal solution. Image segmentation experiment results are presented in Section 3 and performance evaluation in section 4 and the conclusion is obtained in Section 5.

2. Materials and methods

2.1 image datasets

In this work totally 100 clinical MR Brain images of T1, Contrast enhanced T1, and T2 images with CNS tumor are considered for analysis.

2.2 Markov random field

MRF model poses image segmentation as a labelling problem[9] in which a set of labels are assigned to the set of image pixels. In the MRF [10]model, we consider two random fields for *X* and Y.

The segmentation results we want to obtain is the realization *X*=*x* of the field *X*. The measured data, i.e., the set of multispectral MR images, is a realization *Y*=*y* of *Y*. The joint distribution of the data *Y* and segmentation *X* is

$$p(x,y) = p(x)p(y|x) \quad (1)$$

where is the prior distribution of assumed to be stationary and Markovian, and is the posterior distribution.

A clique is a set of sites in which all pairs of sites are mutual neighbours. If C_s denotes the set of

Cliques containing site *s*, a neighbourhood system is the ordered class $\{C_1, C_2, C_3 \ldots\ldots C_{nm}\}$.

Generally, cliques are not pair wise disjoint. Cliques are defined by (S, N) where S denotes a set

of pixels in an image and N denotes the neighborhood pixels.

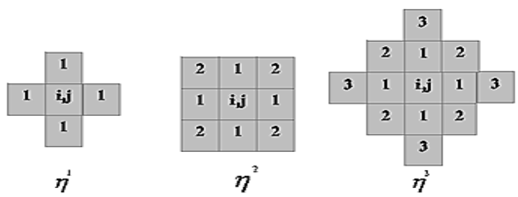

Figure 1. First Order Clique,Second Order Clique,Third Order Clique

$$c1 = \{i, i'i' | i' \in N, i \in S\} \quad (2)$$

$$c2 = \{i, i', i'' \; | i', i'' \in \; N, i \in \; S\} \quad (3)$$

$$c3 = \{i, i', i'', i''' | i', i'', i''' \in \; N, i \in \; S\} \quad (4)$$

A set of random variables is said to be a Gibbs random field with respect to neighborhood if and
only if it obeys Gibbs distribution

$$p(x|y) = \frac{1}{Z} \; e^{-U(x|y) \;/T} \quad (5)$$

where Z is the normalising constant, $U(x|y)$ is the energyfunction and T is the temperature.

$$U(x|y) = \sum_{c \in C} V_c \; (x|y) \quad (6)$$

where $V_c\ (x|y)$ is the clique potentials.

$$\hat{y} = \arg\max U(x \; |y) \quad (7)$$

Optimisation algorithms

This section defines the MRF-based segmentation algorithms compared in this paper. These iterative algorithms attempt to optimize a statistical criterion by approximating the MAP Maximum A-Posteriori estimate.

2.3.1Simulated Annealing

Geman and Geman(D. Geman et al,1990) proposed an algorithm based on simulated annealing to find the MAP estimate of the true image, which minimizes the energy function $U(x|y)$ over all possible labelings x. An exhaustive search for a global optimum creates an impossible computational burden because the labels for all pixels must be estimated simultaneously. Although simulated annealing is theoretically guaranteed to find a globally optimal labeling, it can fail in actual problems because compromises are needed to overcome the computational burden (P. J. M. Van Laarhoven et al,1987). The steps of the SA algorithm are described below.

2 Choose an initial temperature T.
3 Initialize $\hat{x}$ by maximize $ft(yt|xt)$ (This is the maximum-likelihood *estimate* of pixel label.))
4 Perturb $\hat{x}$ into $\hat{z}$ let
$\Delta = U(\hat{z}\; |y) - U(\hat{x}\; |y)$

If Δ>0 then replace $\hat{x}$ into $\hat{z}$. Else replace $\hat{x}$ into $\hat{z}$ with probability $e^{\Delta/t}$

5 Repeat (3) N_{inner} times.
6 Replace T by Φ(T) Φ in monotonically decreases function
7 Repeat (3) – (5) K_{max} times

2.3.2 MRF with Improved Genetic Algorithm

Comparing with Simulated Annealing (SA) that is often used in the image segmentations based on Markov Random Field (MRF) models, Genetic Algorithm (GA) (U. Maulik at al,2009)has been applied into reducing the computation complexity of optimization (E. Y. Kim et al,2000). However many scholars used GA as an optimal tool that based on the gray-scale values of pixels as individuals and limited the mutations and crossovers with the gray-level coding, which caused these algorithms sensitive to noise especially to the multiplicative noise. To avoid trapping into the low-grade limitations of canonical GA with gray-level values of pixels, the labels coding of individuals in a neighbourhood instead of the gray-scale values coding is proposed in this paper. And the mutations and crossovers with labels coding in a neighbourhood increased the efficiency of searching optimal and preserved the original information of images(Sahar Yousefi et al,2010).

1) Coding and individuals: In order to consider the correlation between pixels, we use a new 2-D label
field as individual which is noise insensitive against the gray level coding (LuXiaodong et al,2010).

2) Individual fitness function: The fitness value of each individual indicates its survival ability. Since the purpose of the problem is minimizing the energy function *U(y)* a stronger individual corresponds to having lower energy and consequently higher fitness value. Therefore, the fitness function is defined as *F(y) = c / U(y),* where c is a constant.

3) Crossover: In the proposed algorithm we have used single point crossover on 2-D individuals. For this goal, two individuals are selected as *Parent1* and *Parent2* by considering their fitness values. Then, crossing points are selected randomly and crossover is performed.

4) Mutation: Mutation is an operation by which the degree of population diversity could be enhanced. *5) Selection:* In order to achieve the idea of 'Survival of the fittest', selection is performed.

The algorithm of the proposed method is as follows:

1. Initialization of temperature *(T)*, population number *(population size)*, crossover rate *(Pc)*, mutation rate *(Pm)*.
2. Choose $\dot{y}$ such that satisfies MAP criterion.
3. Generate population by deriving from $\dot{y}$.
4. Repeat until frozen:

a) Selection: use universal selection method to select parents: *parent1* and *parent2*.
b) Crossover: perform crossover between *parent1* and *parent2*, offspring are *offspring1* and *offspring2*.
c) Mutation: mutate *offspring1* and *offspring2*.
d) Replacing: compute *U=U (parent)-U (offspring)*
For two offspring, replace parent with offspring if $U \geq 0$ or U < *0* and ξ > *exp (-EU)*, ξ is a random Number between [0, 1] and T is the system temperature.
e) Replace *T* by ψ *(t)*, where ψ is the monotonically decreasing function.

2.3.3 MRF with Hybrid Parallel Ant Colony Optimisation

A novel approach to MRI medical Image segmentation

based on the Hybrid Parallel Ant Colony Optimization (HPACO) with Fuzzy C-Means (FCM) Algorithm have been used to find out the optimum label that minimizes the posterior energy function to segment the image. There are M colonies, M-1 colonies treated as slaves and one colony for master. Each colonies visit all the pixels without revisit (Karnan,M et al,2010). Initially, initialize the pheromone value for all the colonies. Posterior energy values or fitness values are computed by Markov Random Field. If this value is less than global minimum, the local minimum is assigned to global minimum. The pheromone of the Ant that generates the global minimum is updated. At the final iteration global minimum returns the optimum threshold value for select the initial clustering the FCM(S. Chen et al,2004) implementation in the Magnetic Resonance Image (MRI) segmentation.
The steps for HPACO-MRF are shown below.

1. Read the medical image.
2. Pixels with same gray value are labelled with same number.
3. For each kernel in the image, calculate the posterior energy U (x) value.
4. The posterior energy values of all the kernels are stored in a separate matrix.
5. Ant Colony System is used to minimize the posterior energy function.
6. Initialize number of iterations (N), number of ants (K), initial pheromone value (T_0), a constant value for pheromone update (ρ).Store the energy function values in S. Initialize all the pheromone values with T_0=0.001.

For N times
For each pixel in the image
For each ant update pheromone values

$$T_{new} = (1-\rho)^{*}\, T_{old} + \rho^{*} T_{old} \quad (11)$$

End
End
End
If the Slave value is less than the master value then the value is discarded.
Else, interchanged.

7. Select a random pixel for each ant, which is not selected previously.
8. Update the pheromone values for the selected pixels by all the ants.

The optimal value HPACO is used to select the initial cluster point. FCM- ACO Algorithm is the following:

1. Calculate the cluster centers.

$$C = \left(\frac{N}{2}\right) 1/2 \quad (12)$$

2. Compute the Euclidean distances.

$$D_{ij} = CC_p - C_n \quad (13)$$

3. Update the partition matrix

$$U_{ij} = \frac{1}{\sum_{k=1}^{c}\left(\frac{d_{ij}}{d_{ij}}\right)^{2/(m-1)}} \quad (14$$

and find the maximum of

$$U_{ij}(k+1) - U_{ij}(k) < \varepsilon \quad (15)$$

Calculate the average clustering points.

$$C_i = \sum_{i=1}^{c} \sum_{j=1}^{n} U_{ij}^{n}\, d_{ij}^{2} \quad (16)$$

4. Compute the adaptive threshold

Adaptive threshold =max (Adaptive threshold, C_i) i =1...n
In the MRI image, the pixels having lower intensity values than the adaptive threshold value are changed to zero. The entire procedure is repeated for any number of times to obtain the more approximated value.

3. Segmentation results

In this work the three algorithms such as Simulated Annealing (SA), MRF with Improved GA and MRF with Hybrid Parallel Ant Colony Optimization (HPACO) Algorithm for the segmentation of MR images are implemented using Matlab 2010a software under Windows 7 Operating System. The main aim of this work is to segment the tumourous tissues from the original MR brain image. The MR brain images with tumour have been shown. The two image segmentation algorithms have been used to segment the tumorous tissues. These algorithms are simulated and performance evaluation is done based on region non- uniformity and correlation among the neighbouring pixels

4. Performance evaluation

To evaluate the performance of the segmentation algorithm, there are many methods available to evaluate the performance of the techniques. In this work In this the performance evaluation for medical images have been done based on region non- uniformity, correlation between the ground truth image and segmented images. And compare the computation time for each algorithm and the results are tabulated as shown below.

4.1 Region Non- Uniformity

This is a standard method to evaluate the performance. This does not require ground truth information and is defined as

$$NU= [|F_T| / |F_T + B_T|]*[\sigma_f^{2} / \sigma^2] \quad (11)$$

where σ^2 represents the variance of the whole image, and σ_f^{2} represents the foreground variance. F_T and B_T denotes the foreground and background area pixels in the test image. It is expected that a well segmented image will have a non- uniformity measure close to 0, while the worst case corresponds to NU=1.

4.2 Correlation

Correlation between two quantities denotes how closely related those two are:

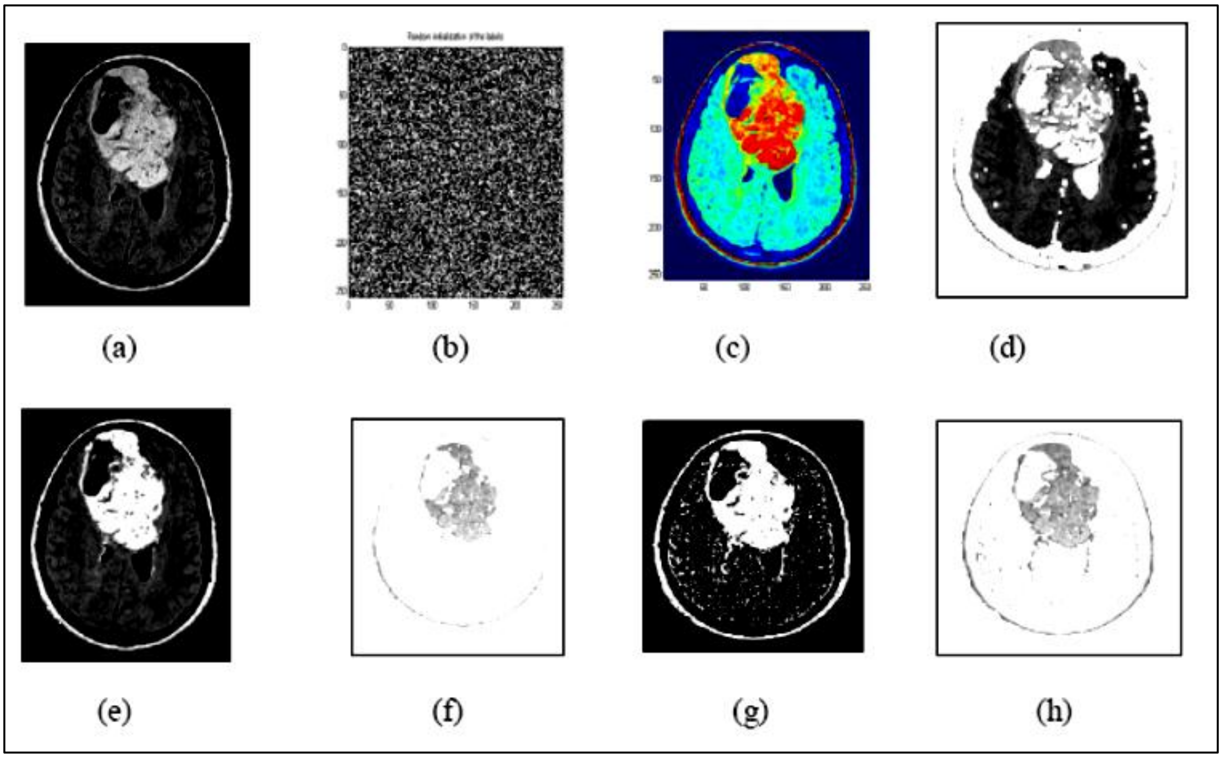

Figure 2. (a)MR Brain Image 1 with Tumour , (b)Random Initialisation of Labels, (c)Labeled Image after SA, (d)Extracted image after SA (e) Optimised Image after MRF-IGA (f) Extracted Image after MRF-IGA (g)Optimised Image after HPACO, (h)Extracted Image after HPACO.

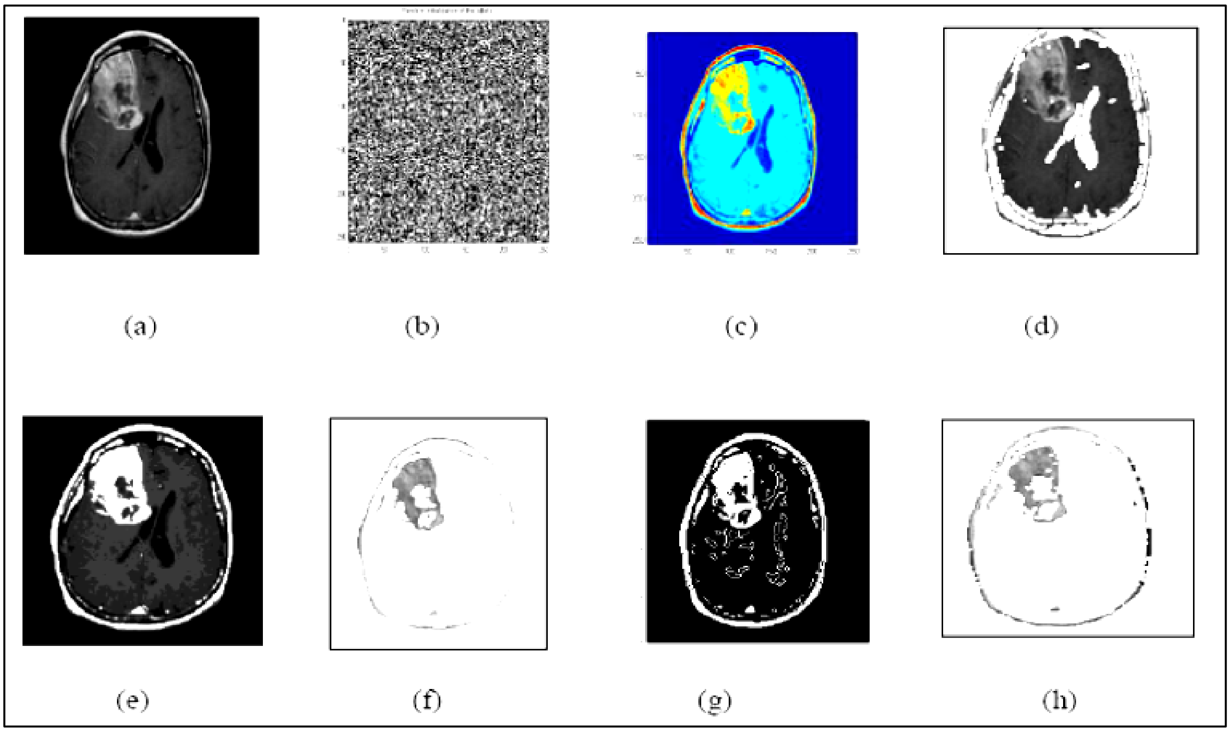

Figure 3. (a)MR Brain Image 2 with Tumour , (b)Random Initialisation of Labels, (c)Labeled Image after SA, (d)Extracted image after SA, (e) Optimised Image after MRF-IGA, (f) Extracted Image after MRF-IGA (g)Optimised Image after HPACO, (h)Extracted Image after HPACO

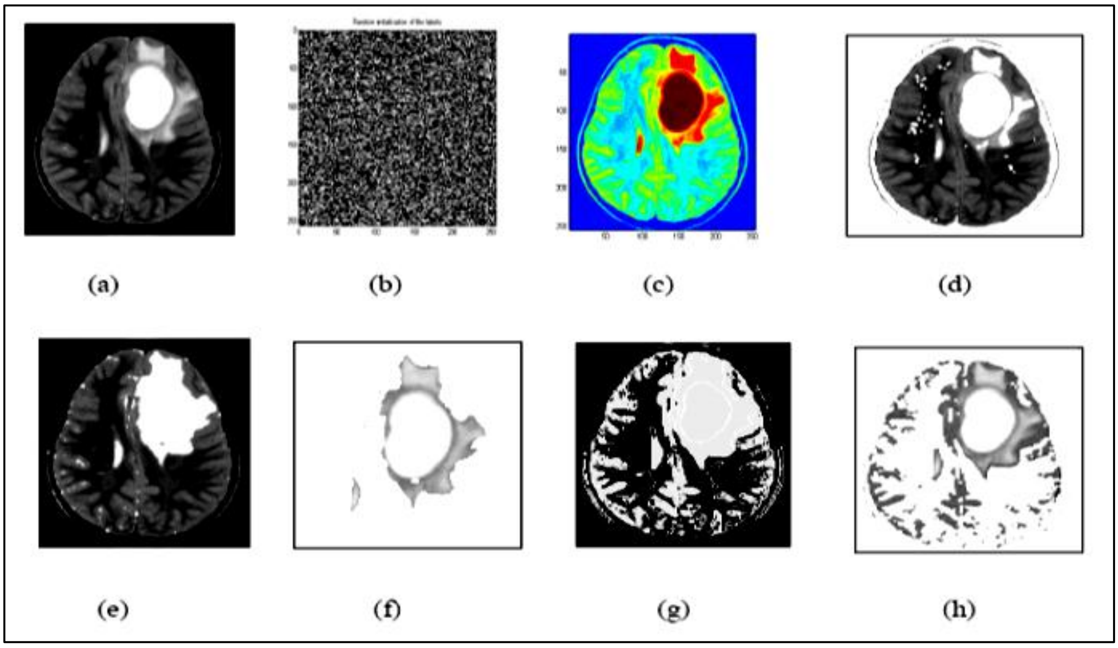

Figure 4. (a)MR Brain Image 3 with Tumour , (b)Random Initialisation of Labels, (c)Labeled Image after SA, (d)Extracted image after SA, , (e) Optimised Image after MRF-IGA, (f) Extracted Image after MRF-IGA , (g)Optimised Image after HPACO, (h)Extracted Image after HPACO.

Correlation between two images thus denotes how closely one image resembles the other. A measure of correlation is thus a really good quantity to prove proper segmentation. In this method, a correlation is done between the segmented image and the ground truth image.

MR Brain scan with Tumor of ten patients were acquired for study purpose. These images were recorded as DICOM (Digital Imaging and Communications in Medicine) format during the scan. These DICOM images are then converted to JPEG format for image segmentation.10 images per patient were taken and segmented. Totally 100 images were segmented and performance evaluation is done based on region non-uniformity ,correlation among the neighbouring pixels and computation time for each algorithm and the results are tabulated as shown below. Consolidated results were obtained for 100 images with 10 images per scan of each of 10 patients and tabulated in Table 1. And experimental results for sample 3 MR Brain images are shown in figure 2, 3, 4 for SA, IGA and HPACO.

The performance evaluation for medical images have been

Table 1 Performance Evaluation Results for Medical Images

IMAGE	REGION NON-UNIFORMITY			CORRELATION		
	SA	IGA	MRF-HPACO	SA	IGA	MRF-HPACO
1	0.1779	0.0553	0.0394	1	1	1
2	0.2469	0.012	0	0.9944	0.9945	0.99
3	0.1265	0.0002	0	0.996	0.9959	0.99
4	0.2111	0.0358	0	0.9951	0.9957	0.99
5	0.2521	0.0372	0	0.9931	0.9958	1
6	0.2447	0.0353	0	0.9941	0.9968	1
7	0.1678	0.0243	0	0.9941	0.9965	1
8	0.2346	0.0298	0	0.9931	0.9958	1
9	0.2223	0.0356	0	0.9964	0.9988	1
10	0.1804	0.0237	0	0.9935	0.9968	1
FOR	COMPUTATION TIME (S)					
ALL	SA		IGA		MRF-HPACO	
IMAGES	45 (Approx)		210(approx)		105(Approx)	

done based on region non- uniformity and correlation and the results are tabulated as shown above. From the table it is inferred that values closer to zero in region non-uniformity results in best segmentation. The variation in values is due to variation in gray levels of the pixels. The correlation of the segmented results using three algorithms with manually segmented result is inferred to be good as the values approaches to one. It is inferred from the table that the computation time for MRF-HPACO is low when compared to the other methods. Hence MRF-HPACO is preferred as inferred from the table 1.

4. Conclusion

In this work a medical image is segmented based on Markov Random Field using the Matlab R2010a . We have proposed a new tightly coupled hybrid framework to Segment MR Brain images. The efficiency of the proposed algorithm is compared with IGA algorithm. Performance of these algorithms are thus analysed based on region non-uniformity, correlation with ground truth image and computation time for each algorithm and the results are tabulated. MRF-HPACO is preferred as inferred from the table1. The further improvement of the performance and decrease the computational complexity can be done by using hidden MRF for this applications[19,20].

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