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Research Article

Analysis of Water Parameters Using Haar Wavelet (Level 3)

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Abstract

Today, we are living in a water starved world. Water is an essential element for life. Most of countries fulfill the requirement of water from river water and ground water. Pollution in river water draws attention of government, public, NGO's and environmentalists in India and world over. Wavelet analysis of water quality parameters COD (Chemical Oxygen Demand), BOD (Biochemical Oxygen Demand), DO (Dissolved Oxygen), WT (Water Temperature), AMM (Free Ammonia), TKN (Total Kjeldahl Nitrogen), TC (Total Coliform), FC (Fecal Coliform) and PH (Potential of Hydrogen) monitored at Nizamuddin bridge-mid stream (Delhi) of Yamuna River in India have been studied for last 10 years. 1-D Discrete and Continuous Wavelet transform using Haar Wavelet at level 3 have been analyzed for each water parameter. Discrete Haar Wavelet (level 3) analysis decomposes each data in 5 parts namely s, a_3, d_1, d_2 and d_3. The first part 's' represents signal or raw data, second part 'a_3' correspond to amplitude of signal for Haar Wavelet at level 3. d_1, d_2, d_3 represents details of signal or raw data at three different levels. 1D continuous wavelet analysis has four parts; first part on top represents the analyzed signal or raw data, second part contains the scalograms values. The scale of color changes from minimum (dark color) to maximum (light color). The vertical axis shows the frequency values while the horizontal axis represents the number of days. The third part shows the daily variation of coefficients. The fourth part depicts the local maximums of related parameters.

Keywords: *Wavelet Analysis, Water Pollutant, Statistical Analysis, Daubechies Wavelet.*

1. Introduction

Water is essential element of life and rivers are most important resources of water. In old times many civilizations developed on the banks of rivers only for the availability of fresh and pure water, but unfortunately now the rivers are affected by urbanization, industrialization and other human activities. Many rivers have been dying at an alarming rate due to pollution created by factories. Parmar, et al. (2009) studied the pollution level and water quality indexing of main rivers of Punjab. Pollution of rivers first affects its Physico-Chemical Quality later on it systematically destroys the community disrupting the delicate food web. Kumar, A. and Dua, A. (2009) studied water quality of river Ravi, using Water Quality Index (WQI) technique, which provides a single number that expresses overall water quality at a certain location and time based on several water quality parameters. Rangarajan, G. and Sant, D. A. (2004) investigated the Indian climatic dynamic using fractal dimensional analysis and analyzed time series data of three major dynamic components of the climate i.e. temperature, pressure and precipitation. It has been observed that regional climatic models would not be able to predict local climate as it deals, with averaged quantities and that precipitation during the south-west monsoon is affected by temperature and pressure variability during the preceding winter. Shukla, J.B. et al. (2008) studied the simultaneous effect of water pollution and eutrophication on the concentration of dissolved oxygen (DO) in a water body. The analysis of the model shows that the decrease in the concentration of DO due to simultaneous effect of water pollution and eutrophication is much more than when only single effect is present in the water body, thus leading to more uncertainty about the survival of DO-dependent species. Isaac M. et al. (2004) discussed the complex Morlet Wavelet Transform is used to identify long period oscillations in the horizontal component (*H*) of the geomagnetic field over the magnetic equatorial location of Trivandrum (8.5°N; 77°E; dip lat. 0.5°N) during the solar maximum period 1990/1991 and solar minimum period 1995/1996. The Morlet WT of the geomagnetic data set indicates the presence of multiple timescales, which are localized in both frequency and time. This analysis also brings out clearly the merging of long period oscillations with short period oscillations during conditions of geomagnetic disturbance. Torrence C. and Compo G.P. (1998) discussed new statistical significance tests for wavelet power spectra are developed by deriving theoretical wavelet spectra for white and red noise processes and

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using these to establish significance levels and confidence intervals. Andreo B. et al. (2006) studied annual periodicity of the rainfall and temperature distributions was constant over more than 100 years, although weaker (6-month) periodicities have also been observed, as well as rainfall and temperature periodicities of 5 and 2.5 years by means of spectral and correlation analyses and continuous wavelet analysis. Partal T. and Kucuk M. (2006) determined the possible trends in annual total precipitation series by using the non-parametric methods such as the wavelet analysis and Mann–Kendall test. Using discrete wavelet components of measurement series, the aimed to find which periodicities are mainly responsible for trend of the measurement series. Watkins L. R. (Article in press) studied the Mathematical theory underlying the 1 Dimension and 2 Dimensional Continuous Wavelet transforms. The phase or instantaneous frequency of fringe patterns with spatial or temporal carriers can be recovered from the wavelet ridge, a path that follows the maximum modulus of the CWT. The relative merits of these two approaches, termed the phase and gradient methods, respectively, are discussed. Common 1-D wavelets are listed and their broad scope of applicability is indicated. Popular 2-D isotropic and directional wavelets are given and the advantages of 2-D wavelet methods over 1-D are discussed.

In this paper the 1 D Discrete Wavelet transform and 1 D Continuous Wavelet transform of water quality parameter have been calculated at Nizamuddin bridge-mid stream (Delhi) of Yamuna River in India. Haar Wavelet level 3 used to study Wavelet analysis.

2.1 1-D Discrete wavelet analysis of water quality parameters

Wavelet Transform T_ψ decomposes a signal into several groups (vectors) of coefficients. Different coefficients vectors contain information about characteristics of the sequence at different scales. The mother wavelet Ψ (Haar) satisfying the conditions

$$\psi_{a,b}(t) = a^{-\frac{1}{2}}\psi\left(\frac{t-b}{a}\right), \text{where } a > 0 \qquad 1$$

$T_\psi f(a,b)$ is called the Wavelet Transform of $f(t)$ and

$$T_\psi f(a,b) = a^{-\frac{1}{2}}\int_{-\infty}^{\infty} f(t)\,\psi\left(\frac{t-b}{a}\right)dt \qquad 2$$

$$= \left\langle f, \psi_{a,b}\right\rangle$$

= inner product or dot product of f and $\psi_{a,b}$

It may be observed that the wavelet transform is a prism which exhibits properties of signal such as points of abrupt changes, seasonality or periodicity. The wavelet transform is a function of 'a' and 'b' where

a = the scale of frequency

b = spatial position (translation) or time

The plane defined by the variables (a, b) is called the scale-space or time frequency plane. The wavelet transform $T_\psi f(a,b)$ measures the variation of f in the neighborhood of b. For a compactly supported wavelet (for a wavelet vanishing outside a closed and bounded interval) the value of $T_\psi f(a,b)$ depends upon the value of f in a neighborhood of b of size proportional to the scale a. At small scales, $T_\psi f(a,b)$ provides localized information such as localized regularity (smoothness) of f. The local regularity of a function (or signal) is often measured with Lipschitz exponent. The global and local Lipschitz regularity can be characterized by the asymptomatic decay of wavelet transformation at small scales.

$$\psi_{j,k}(t) = 2^{\frac{j}{2}}\psi\left(2^j t - k\right) \qquad 3$$

is a discrete version of $\psi_{a,b}(t)$ where j,k are integers.

$$d_{j,k} = \int_{-\infty}^{\infty} f(t)\,\psi_{j,k}(t)\,dt \qquad 4$$

are called the wavelet coefficients of $f(t)$ and

$$\sum_{j,k} d_{j,k}\,\psi_{j,k}(t)$$

is called the wavelet series of f with every orthonormal wavelet. $\phi(t)$ called scaling function such that

$$\phi_{j,k}(t) = 2^{\frac{j}{2}}\phi\left(2^j t - k\right) \qquad 5$$

One dimensional discrete wavelet analysis of water quality parameter such as pH, BOD, COD, DO, AMM, TKN, WT, TC and FC for Yamuna river at Nizamuddin bridge-mid Stream, Delhi (India) have been discussed.

2.2 1-D continuous wavelet analysis of water quality parameters

A wavelet is a function, $\psi \in L^2(\mathbb{R})$ centered at t =0 with a few oscillations and zero mean Mallat (2001) and Daubechies I.(1992).

$$\int_{-\infty}^{\infty}\psi(t)\,dt = 0 \qquad 6$$

It is usual to normalize ψ such that $\|\psi\|$ =1A family of function $\psi_{a,b}(t)$ is generated from this "mother wavelet" (Haar wavelet) by translates and dilates (Lilly L.(2010) and Watkins L.R (Article in press)) according to

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right), \quad a,b \in \mathbb{R}^+ \times \mathbb{R} \qquad 7$$

The factor $\frac{1}{a}$ ensures that $\psi_{a,b}$ maintains its norm with scaling factor. The continuous wavelet transform (CWT) of a function is $f \in L^2$ □

$$W_f(a,b) = \langle f, \psi_{a,b} \rangle = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{a}} \psi^* \left(\frac{t-b}{a} \right) dt \qquad 8$$

The function f can be reconstructed from its wavelet transform via the resolution of the identity

$$f = \frac{1}{c_\psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_f(a,b) \frac{1}{\sqrt{a}} \psi \left(\frac{t-b}{a} \right) db \frac{da}{a^2} \qquad 9$$

Where the constant C_ψ depends only on ψ and given by

$$c_\psi = \int_{-\infty}^{\infty} \frac{\left| \hat{\psi}(\omega) \right|^2}{\omega} d\omega < \infty \qquad 10$$

The monthly average value of PH, BOD, COD, DO, AMM, TKN, WT, TC and FC were used for the period of last ten years. In figure1 Haar wavelet at level 3 contains five parts. The first part's' represents the signal or raw data and the second part 'a_3' corresponds to the amplitude of the signal. The last parts i.e d_1, d_2 and d_3 of these represent details of the signal or raw data at three different levels of respected water parameters. Figure 2 describes 1D continuous wavelet analysis, it has four parts; first part on top represents the analyzed signal or raw data, second part contains the scalograms values. The scale of color changes from minimum (dark color) to maximum (light color). The vertical axis shows the frequency values while the horizontal axis represents the number of days. The third part shows the daily variation of coefficients. The fourth part depicts the local maximums of related parameters.

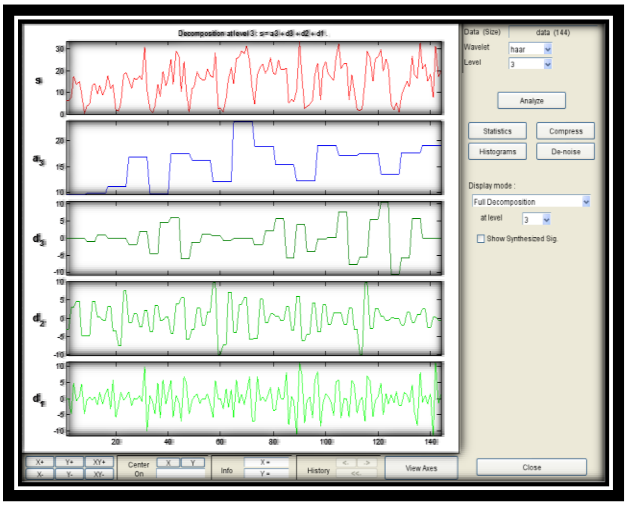

Fig.1 (a) 1D Discrete Wavelet Analysis of AMM

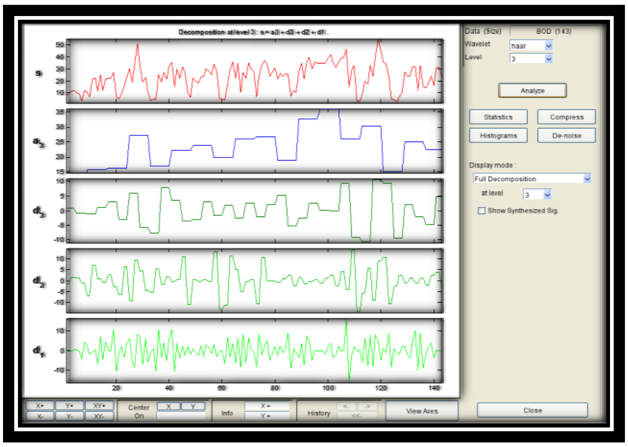

Fig. 1(b) 1D Discrete Wavelet Analysis of BOD

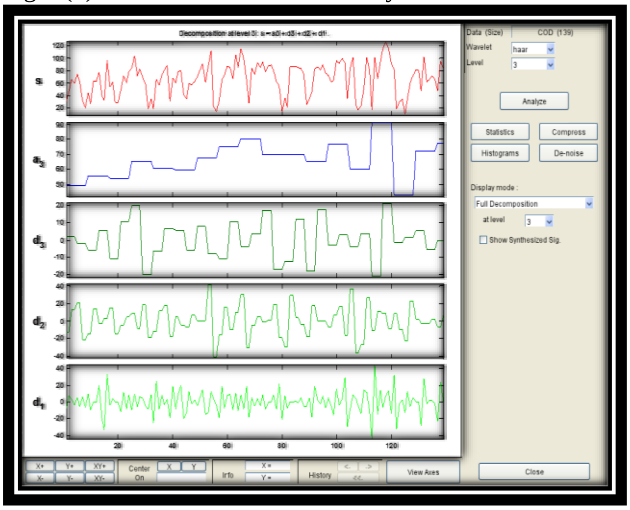

Fig. 1(c) 1D Discrete Wavelet Analysis of COD

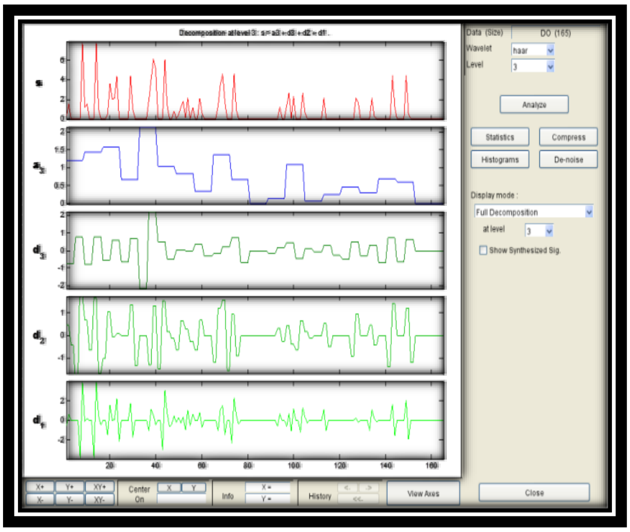

Fig. 1(d) 1D Discrete Wavelet Analysis of DO

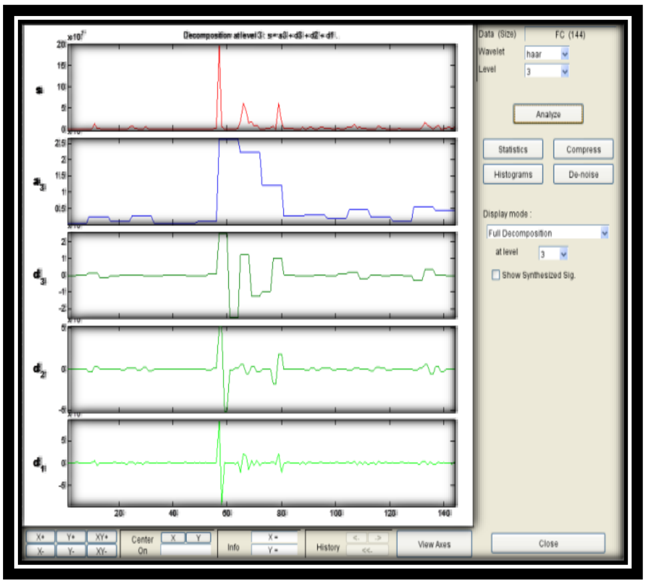

Fig. 1(e) 1D Discrete Wavelet Analysis of FC

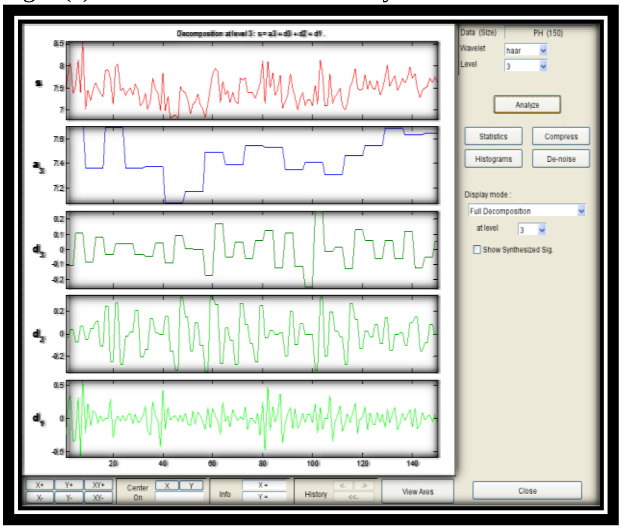

Fig. 1(f) 1D Discrete Wavelet Analysis of pH

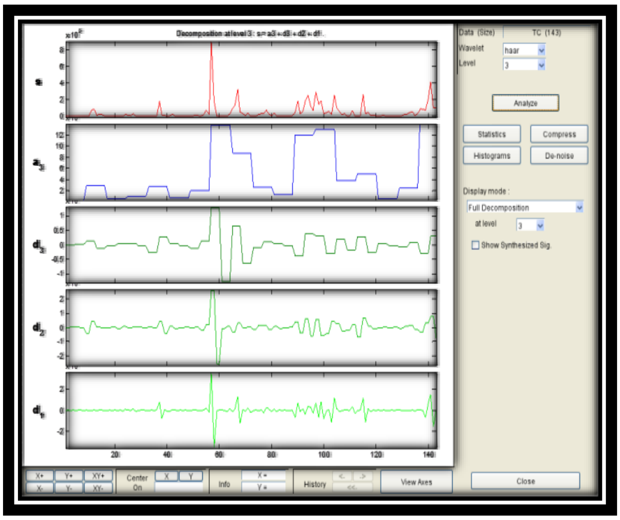

Fig. 1(g) 1D Discrete Wavelet Analysis of TC

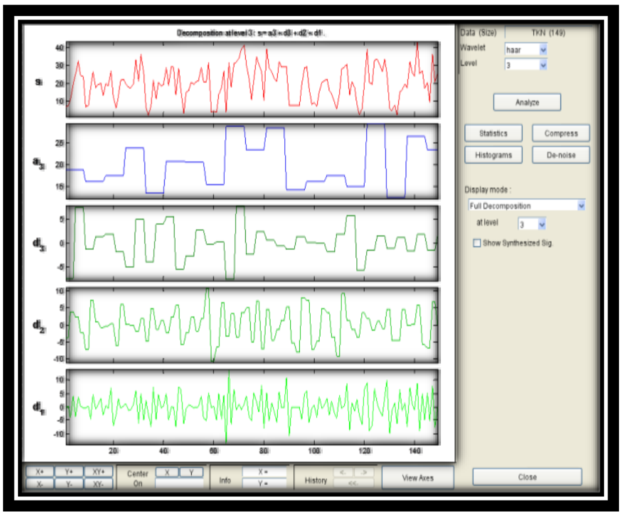

Fig. 1(h) 1D Discrete Wavelet Analysis of TKN

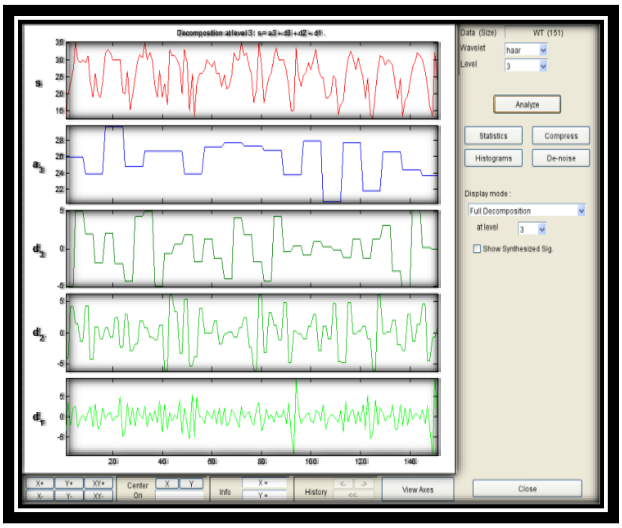

Fig. 1(i) 1D Discrete Wavelet Analysis of WT

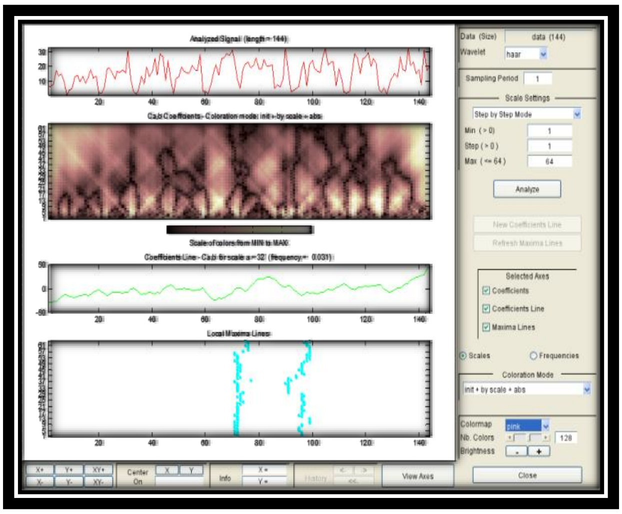

Fig. 2(a) 1D Continuous Wavelet Analysis of AMM

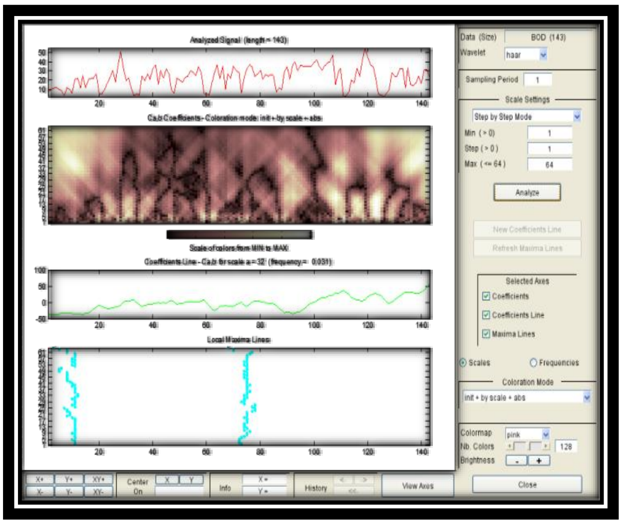

Fig. 2(b) 1D Continuous Wavelet Analysis of BOD

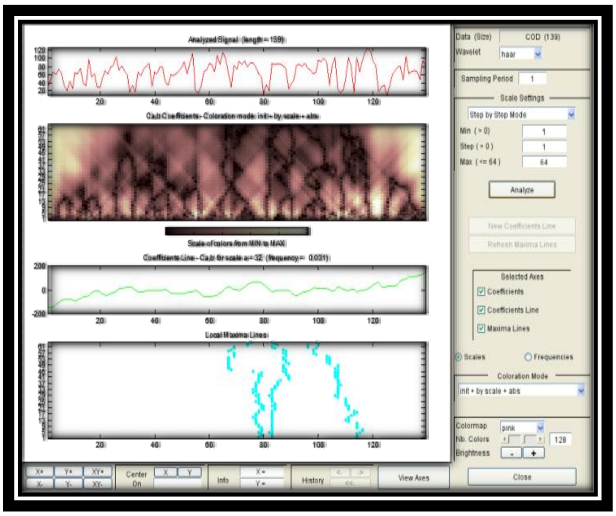

Fig. 2(c) 1D Continuous Wavelet Analysis of COD

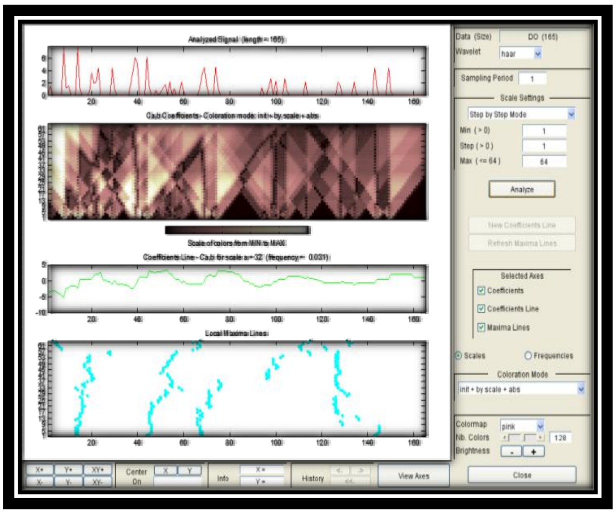

Fig. 2(d) 1D Continuous Wavelet Analysis of DO

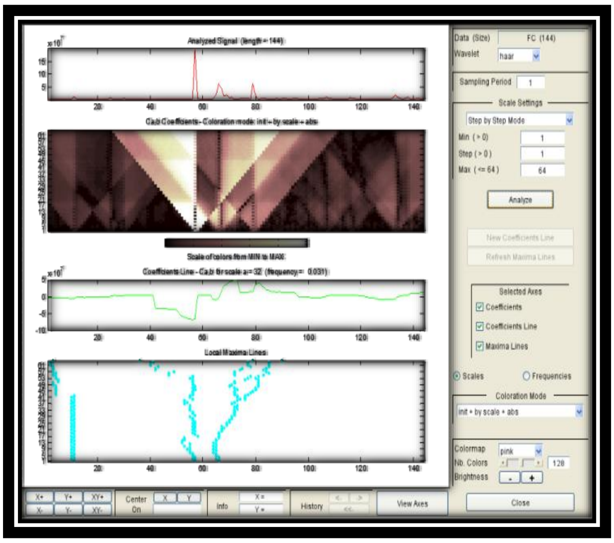

Fig. 2(e) 1D Continuous Wavelet Analysis of FC

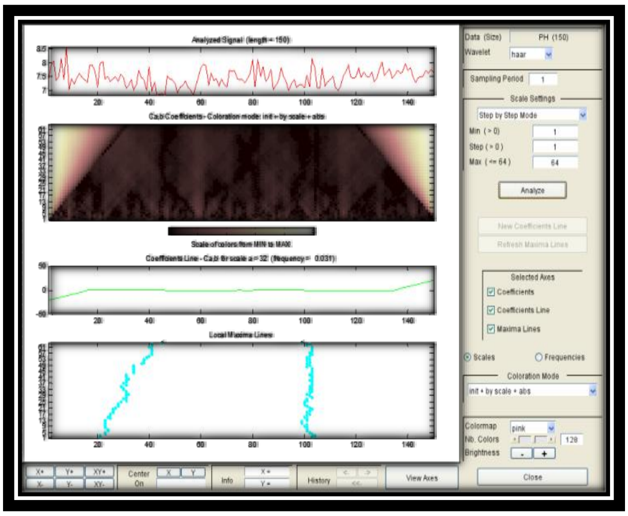

Fig. 2(f) 1D Continuous Wavelet Analysis of pH

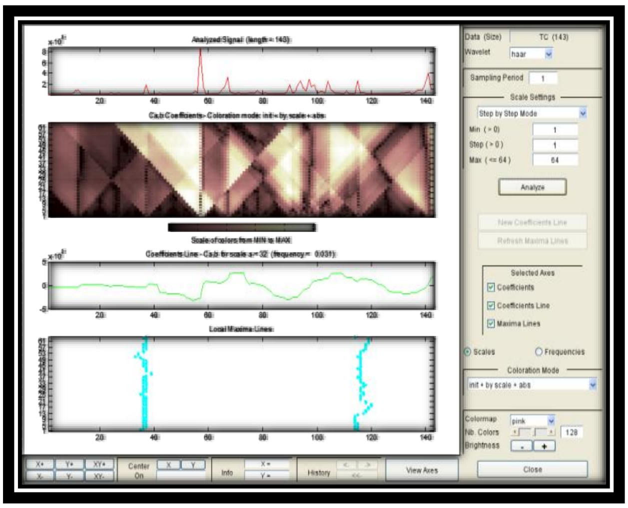

Fig. 2(g) 1D Continuous Wavelet Analysis of TC

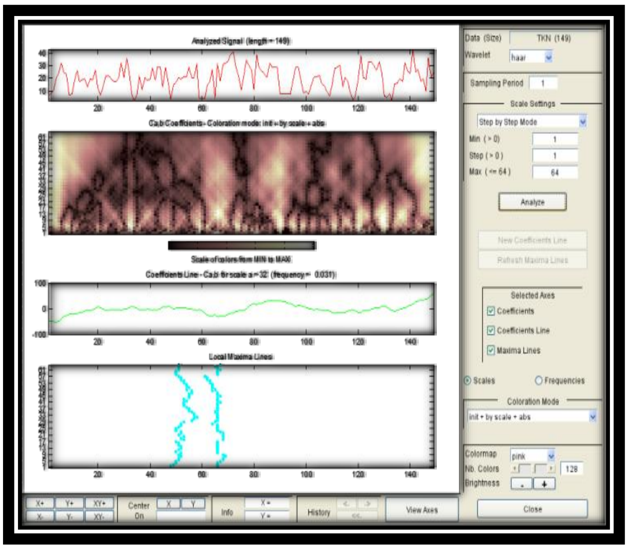

Fig. 2(h) 1D Continuous Wavelet Analysis of TKN

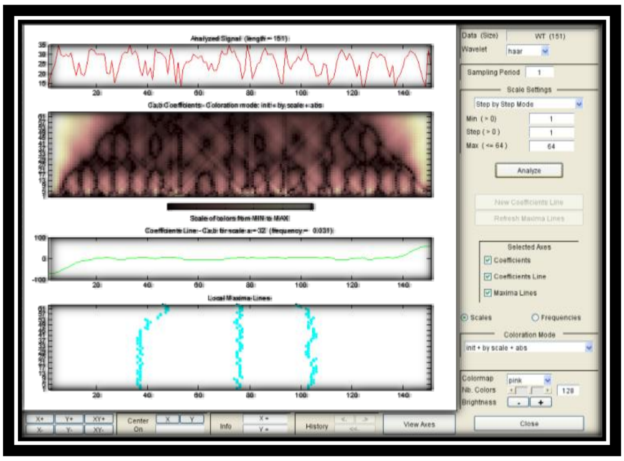

Fig. 2(i) 1D Continuous Wavelet Analysis of WT

3. *Results and discussion*

1D discrete and continuous wavelet analysis of water quality parameters AMM, BOD, COD, DO, FC, pH, TC, TKN and WT at Nizamuddin bridge, Delhi of Yamuna River has been analyzed using Haar wavelet. The 1D Discrete Haar Wavelet analysis and continuous wavelet analysis, it has been that parameters AMM, BOD, COD, DO, FC, pH, TC, TKN and WT have the maximum value at 30, 60, 120, 7, 20, 8.5, 8, 40 and 35 respectively and amplitude (a_5) varies between 10 to 18, 10 to 30, 54 to 76, 0.2 to 1.4, 0 to 14, 7.3 to 7.7, 0 to 10, 18 to 26 and 24 to 27 respectively. The values of d_3, d_2 and d_1 ranges between -40 to +40 for all parameters. The scale of color changes from minimum (dark color) to maximum (light color) and it also gives the local maxima values at different time periods. It has been analyzed that Water is not fit for drinking purpose and industrial use as it crosses the prescribed limits of WHO/EPA standard values.

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References

Andreo, B., Jiménez, P., Durán, J.J., Carrasco, F., Vadillo, I. and Mangin, A., (2006) ,Climatic and hydrological variations during the last 117–166 years in the south of the Iberian Peninsula, from spectral and correlation analyses and continuous wavelet analyses" *Journal of Hydrology*, 324(4), 24-39.

Bhardwaj, R., Parmar, K.S., Chuhg, P., Minhas, P.and Sahota, H.S. (2011), Seasonal variation of physico-chemical parameters and water quality indexing of Harike Lake, *Indian j.Env.Prot*, 31(6), 482-486.

Can, Z.; Aslan, Z., Oguz, O. and Siddiqi, A.H., (2005),Wavelet Transform of Metrological Parameter and Gravity Waves, *Annales Geophysicae* , 23, 659-663.

Daubechies I. Ten lectures on wavelets. Philadelphia, PA: SIAM; 1992.

Isaac, M., Renuka, G., and Venugopal, C., (2004) ,Wavelet Analysis of Long Period Oscillations in Geomagnetic Field Over the Magnetic Equator, *J. Atmos. Sol. Terr. Phys.*, 66, 919–925.

Kumar, A., Dua, A., (2009),Water quality index for assessment of water quality of river Ravi at Modhopur (India), *G.J.Env.Sci.*, 8(1), 49-57.

Lilly, L., Olhede, S., (2010), On the analytic wavelet transform, *IEEE Trans Inf Theory* ,56, 4135–4156.

Mallat, S., (2001) ,A wavelet tour of signal processing,2nd ed. San Diego, CA: Academic Press.

Parmar, K.S.; Chugh, P.; Minhas, P.; Sahota, H.S., (2009). Alarming pollution levels in rivers of Punjab. *Indian J. of env. prot.*, 29(11), 953-959.

Partal, T. and Kucuk, M. (2006), Long- term trend analysis using discrete wavelet components of annual precipitations measurements in Marmara region (Turkey),*Physics and Chemistry of the Earth, Parts A/B/C*, 31(18), 1189-1200.

Rangarajan, G. and Sant, D.A., (2004),Fractal dimensional analysis of Indian climatic Dynamics ,*Chaos, Solitons and Fractals*, 19, 285–291.

Shukla, J.B., Misra, A.K. and Chandra, P. (2008),Mathematical Modeling and analysis of the dissolved oxygen in eutrophied water bodies affected by organi pollutant, *Nonlinear Analysis: Real world Applications*, 9, 1851-1865.

Watkins, L. R. (Article in press), Review of fringe pattern phase recovery using the 1-D and 2-D continuous wavelet transforms, *Optics and Lasers in Engineering*.